Book Review of *Behavioral Rationality and Heterogeneous Expectations in Complex Economic Systems*, by Cars Hommes, Cambridge University Press, 2013, ISBN 978-1-107-01929-4 Hardback, pp. 253, Subject Index.

 This is the second book by Cars Hommes, his first in 1991 being his PhD thesis, and it provides an excellent summary of the work he has been doing over the past two decades both by himself and with numerous coauthors, most importantly, William (“Buz”) Brock, by people in the Center for Nonlinear Dynamics, Econometrics, and Finance (CeNDEF) at the University of Amsterdam that he founded in 1998 and long directed, and also in papers that have appeared in recent years in the *Journal of Economic Dynamics and Control* (JEDC) of which he was a coeditor for several years and which reflect the influence of this research, as well as related research outside this circle. It can be viewed as a graduate textbook for a course on nonlinear economic dynamics, and he states upfront in the Preface that indeed it is based on material that he has used in teaching just such a course over many years. Nevertheless, as with the best textbooks, it stands as an excellent summarizing monograph on the topic of the title, with the added virtue of being right up to date on the topic as well. That it is well and clearly written and accompanied by high numerous high quality figures, along with exhibiting an admirable and substantial comprehensiveness, makes it work well for either purpose.

 While it deals with a broad topic, the book seems to be substantially motivated by the recent crises in the financial markets that have been interconnected with the Great Recession and subsequent difficulties in recovering from it. Hommes does not accept rational expectations and clearly states that the crash and related events disprove macroeconomic theories of the Frisch-Slutsky-Tinbergen “rocking horse” type that only fluctuate due to exogenous shocks and are otherwise stable and well-behaved. Nevertheless, an important theme of the book is to avoid going hog wild on possible ad hoc alternative behavioral theories and to attempt to bound the bounding of rationality by imposing certain conditions, the most important of these being consistency. He favors studying what he calls “simple complex systems.” While he generally supports agent-based computational economics (ACE), most of the discussion, particularly of financial market models, involves only small numbers of types of agents, often only two: fundamentalists and trend-chasers or some other single alternative to fundamentalists. The emphasis on consistency fits with his having been one of the main originators of such concepts as *consistent expectations equilibrium*. It is this insistence on consistency even in the absence of rational expectations that is what he means by *behavioral rationality.* He clearly has an ambition to be both realistic and rigorous, with an eye ultimately to influencing monetary policymakers, or at least those in the research departments of the central banks who advise the policymakers. Indeed, quotations from Ben Bernanke and Jean-Claude Trichet on the very first page of the book declaring the economy to be complex and chaotic seem to indicate that he sees quite accurately that the policymakers would themselves support more such research coming out of their research departments than has been doing so in the recent past, where the ubiquitous DSGE approach continues to generally dominate.

 The book does an excellent job of making clear the mathematical foundations involved in this subject. It also provides at least some linking and citing to historical origins of most of the major ideas found in it. However, it eschews deeper philosophical, psychological, or methodological issues, even as it delves into a number of controversies in the topic, making reasonably clear the positions of the competing sides.

 The book contains 8 chapters. The first is a well-crafted introduction that provides a decent summary of the rest of the book, along with covering some historical and background issues. The second and third present the basic nonlinear dynamical mathematics of such complex systems. Chapter 4 discusses the nonlinear cobweb model with alternative formulations of expectations: naïve, rational, adaptive, along with the consistent expectations concept introduced. The fifth chapter also studies the cobweb model most useful in agricultural and natural resource markets assuming heterogeneous expectations, with much of this based on important work he did with Brock, particularly in the mid and late 1990s. Chapter 6 considers financial market models with heterogeneous expectations, with the emphasis in these models being on ones that are evolutionary in nature as agents switch strategies over time based on their relative performance, with the agents’ willingness to switch being a crucial control parameter regarding bifurcations and dynamics in these systems. Chapter 7 discusses some empirical evidence for the models already presented, and the final chapter focuses on experimental evidence about different systems of learning expectations, with much of this research having been carried out at CeNDEF under his direction or at least support. This final chapter may be the most innovative and cutting edge one in the book with most of this research done in only the last few years, whereas some portions of the book, such as the second and third chapters largely rehash well known material, if in a highly competent and clear manner.

 I shall now proceed to discuss some more detailed matters, both items that are especially important or well done, as well as some places where I either have questions or feel that some sources or ideas have been left out or insufficiently developed. I shall only provide specific citations in my References to items not contained in the book.

 The opening chapter does an excellent job of setting the stage and summarizing the book. His basic framework and approach are made quite clear. While he seeks to follow Herbert Simon in limiting the scope of behavioral heuristics to reasonable ones according to his consistency criteria, the presence of heterogeneous agents necessarily introduces nonlinearities into the evolutionary dynamics of the system that induce the complexities considered. He emphasizes that a key to these nonlinear dynamics are the appearance of *homoclinic orbits* as developed originally by Poincaré in the 1890s that involve intersections of stabilizing and destabilizing manifolds due as some agents may behave in stabilizing and others in destabilizing ways. This is a central point and argument, and one that this observer thinks is very important for understanding what we have seen in many economic phenomena, both at the micro level in certain kinds of markets as well as at the macro level and in financial markets.

 The mathematical presentation in the second and third chapters reflects Hommes’s solid training in mathematics from his professors, Helena Nusse and the late Floris Takens, both important developers of modern nonlinear dynamics in mathematics, with Takens in particular a deep student of turbulence and the role of homoclinicity in chaos theory, a deep theme in the work of Hommes, particularly that he did with Brock. In the second chapter he runs through various forms of stability, many bifurcations that appear in these models, and various aspects of chaos theory. An interesting point involves the matter of distinguishing topological from true chaos, with the latter requiring that there be a space for initial starting points leading to chaotic dynamics that is of positive Lebesgue measure, although there are some further technical controversies regarding the definition of chaos that he does not deal with. However, most of these are not really important for applications in economics. He also notes the rather curious nature of classic Cantor sets in that while they are uncountable infinite, they have Lebesgue measure zero, a deeply insightful mathematical point.

 Most of my complaints about this book involve not issues of commission but ones of omission: the author has left out or barely discussed some ideas of potential interest. The book would not have been harmed if it were another 50 or even more pages longer than it is. In this chapter when he discusses bifurcation theory, he could easily spent some time discussing one of its special cases, catastrophe theory. He does later in the book cite the financial market model from 1974 by Zeeman that uses catastrophe theory to look at instability in markets with fundamentalist and chartist traders, but never uses the term “catastrophe theory.” I think this lacuna is unfortunate.

I also note that while gives appropriate credit to Poincaré, his history jumps to Edward Lorenz and his discovery on a simulated computer model of climate of *sensitive dependence on initial conditions*, aka the “butterfly effect,” moving on to the work of Steve Smale, while ignoring completely the numerous contributions of the Russian School along the way by such figures as A.N. Kolmogorov (1941), A.A. Andronov and C.E. Chaikin (1949), and particularly A.N. Sharkovsky (1964) whose theorem both preceded that of Li and Yorke of 1975 discussed in this book and is also more general than Li and Yorke’s, although Lyapunov gets a shout-out for his famous exponent. Also missing from the historical account are the figures who almost certainly first observed chaotic dynamics while studying radio receiver frequency ratios, van der Pol and van der Mark (1927), although it can be argued that unlike Lorenz they never really understood what they had discovered.

The third chapter extends the discussion to two-dimensional systems, including a discussion of Smale’s horseshoe map and also a brief discussion of codimenion two bifurcations such as the degenerate Hopf bifurcation, also known as a *Chenciner bifurcation*. This can lead to coexisting attractors, sometimes with fractal basin boundaries between them that constitute another serious form of complex dynamics, with an example appearing later in Chapter 6 for a model of volatility clustering in a financial market model with heterogeneous agents and also a figure in Chapter 5 showing a figure with fractal basin boundaries between such coexisting attractors. My only complaint here is that this is a topic that more material could have been provided for. There is a large literature on this topic, much of it drawing on the work of Ralph Abraham, Laura Gardini, and Christian Mira (1997) and their many coauthors. , only a few pieces of which are briefly cited in the book. An uncited example due to Foroni, Gardini, and Rosser (2003) that is even an extension of a model discussed in the book on fishery dynamics that shows how different kinds of expectational learning dynamics lead to different patterns of coexisting attractors would have been an obvious example to add to the existing discussion.

Chapter 4 discusses the nonlinear cobweb model and how different sorts of expectations affect its dynamics. He then introduces the important concept of consistent expectations equilibrium along with that of *learning to believe in chaos* that he studies using sample autocorrelated learning, one of the more intriguing and deep ideas to emerge from chaos theory. Cobweb dynamics are very familiar territory for Hommes as this was a major topic in his dissertation and first book from 1991, with many papers since then on it.

Chapter 5 continues the focus on nonlinear cobweb models but rather than looking at in effect homogeneous agent models with alternative expectations models, agents with heterogeneous agents that evolve through responsive learning are considered, with this chapter presenting the core of the highly influential papers that Brock and Hommes coauthored in the late 1990s, which can be characterized as showing various “rational routes to randomness,” to quote the title of perhaps their most influential paper . This is excellent material presented very well, with the role of homoclinicity strongly emphasized and examined carefully.

Chapter 6, tied with the first one for being the longest in the book, uses these the previously presented ideas to develop an asset pricing model with heterogeneous beliefs that can evolve through learning and related dynamics, what he labels *adaptive belief systems* (ABS), again mostly drawing heavily on his work with Brock. A theme that emerges is how markets can oscillate back and forth due to noise between stable limit cycles where more fundamentalist strategies dominate and more volatile dynamics where more trend-chasing type strategies dominate, with this arising for a case of Chenciner bifurcation with coexisting attractors. The relatively new idea due to Brock and Hommes with their more recent coauthor, F.O.O. Wagener, of *large type limit* (LTL) dynamical systems. This allows for studying generic characteristics of systems with many trader types, in contrast with the focus on a few trader types that is found in most of the book.

This may be the moment to mention one other major lacuna that I found disappointing in the book, the absence of any discussion of applications of these ideas to macrodynamic models, something that Hommes has written on extensively, particularly involving applications of the cobweb model and particularly in connection with the 1950 trade cycle model of Hicks, which is briefly mentioned in a footnote in the first chapter, but with no development and which has a nonlinear investment function in it. Now it may be that he did not include this due to his having focused on this in his previous book, or perhaps because most of these models do not involve any evolutionary dynamics of expectations and learning. Nevertheless, many of these models would become the first to be studied for their ability to exhibit many of the complex dynamics such as chaos and coexisting attractors with fractal basin boundaries that appear in this book. Among those of importance besides that of Hicks are the very early one of Kalecki (1935), the much-studied one of Kaldor (1940), and the Goodwin (1951) nonlinear multiplier-accelerator model that was shown by McAnulty, Naines,and Strotz (1953) to be capable of highly erratic endogenous, possibly chaotic, dynamics, maybe one of the first such outcomes for an economic model ever, although they did not know what they had found, somewhat as in the case of van der Pol and van der Mark. It should also be noted that late in life Goodwin (1990) himself turned to studying chaotic dynamics in economic models as he learned that such could arise from several models he developed over the years.

Curiously the following Chapter 7 on empirical validation is the book’s shortest, which may reflect that Hommes is not all that interested in time-series econometrics testing of these sorts of models, preferring the approach in the final chapter of conducting laboratory experiments. Nevertheless he reviews a lot of studies, also noting some of bubble and crash dynamics and noting that due to the difficult econometrics involved neither the presence nor absence of chaotic dynamics in economic data has been neither proven nor disproven.

As noted already, this reviewer considers the final chapter to be one of the most important in the book and at the cutting edge of current research. The focus is *Learning to Forecast Experiments* (LtFEs). Many cases are studied, but among the topics examined are how do individuals form expectations and then learn and adapt from mistakes, how do individual rules interact at the micro level to co-create outcomes at the macro level, how coordination of expectations occurs and if heterogeneity of expectations persists, what the aggregate effect of coordination expectations is , and finally when does learning enforce convergence to rational expectations equilibria (REE)? Stylized facts coming from these studies include that participants only rarely move towards REE, that price patterns can vary even for similar setups between slow monotonic convergence through persistent oscillations of equal amplitude to oscillations of initially large amplitude that then gradually dampen. Near the end reactions to feedback mechanisms in cobweb models are studied with different feedback mechanisms. A general outcome is that while negative feedback leads to rapid price convergence with persistence of heterogeneous expectations, whereas positive feedback is characterized by convergence of expectations but with slow price discovery. The concluding section briefly discusses the important paper by Brock, Hommes, and Wagener from 2009 in the JEDC on how expanding the number of Arrow securities can destabilize a financial market. This could use further study.

To conclude let me say that this is a well-done and important book. My main complaint is that it could have covered more material and some of it in more depth. However, I hope that it is successful, perhaps particularly as a textbook, which could lead the publisher to ask the author to do a second edition in which he could expand on these topics as I have noted, along with adding new material that will surely be forthcoming in this hot and interesting area of research.

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J. Barkley Rosser, Jr.

James Madison University

Email: rosserjb@jmu.edu

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