**COMPLEXITY AND AESTHETICS: HOW ARTS, SCIENCES, AND ECONOMICS COEVOLVE**

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Abstract: This paper presents the ideas of the late Tönu Puu regarding how arts, sciences, and economics interact and coevolve. This involves complex nonlinear dynamics with critical bifurcations occurring at crucial points that transform the paradigm or system involved. Centrally important is how influences from one area influence another, even from the arts to the sciences and economics and back again. Thus the role of generalists is important in progress of humanity and must be encouraged.

Keywords: Renaissance art, Baroque music, fractals, incompleteness theorem, development tree

**Introduction**

 The late economist Tönu Puu, the first economics editor of the journal *Chaos, Solitons & Fractals*, propounded a deeply insightful vision of how arts, sciences, and economics evolve in similar patterns, with developments in one interpenetrating into another to form a vast coevolution. These connections lie largely beneath the surface and are not noticed by most participating in these various sub-parts of this grand process. Puu has laid this argument out in a book named *Arts, Sciences, and Economics: A Historical Safari*, which went through two editions [1], [2], following on an early article in the *Journal of Cultural Economics* [3]. A crucial element of this vision is the argument that various forms of nonlinear dynamics, including chaos and fractals among others, play deep and crucial roles in both the forms these evolutionary processes take, as well as in the ways these links between these different broad fields operate.

 An important aspect of Puu’s advocacy of this vision is that he himself personally characterized it and assisted it himself personally. His main profession was that of an economist who would apply ideas of complex nonlinear dynamics to such fields as urban and regional economics spatial patterns [4], macroeconomic fluctuations [5], and industrial organization dynamics [6]. But he also led another life in the world of music. This partly took the form of for many years organizing an annual festival that featured Renaissance and Baroque music in Umeä, Sweden, where taught for most of his career at the university there. However, he also made musical instruments played in those eras, including viols and even harpsichords, with these of such high quality that professional musicians paid him to purchase these from him. His discussion of the history of harpsichords in his book in both editions reflects an intimate and personal knowledge based on this experience. He was himself a player in this interchange between at least music and economics.

**Philosophical Themes Underlying Aesthetic Evolution**

 In his “historical safari” Puu lays out various themes coursing through these evolutionary processes, with most of his initial discussion dealing with these issues as they operate in aesthetic evolution. But even as he focuses on aesthetic evolution, the theme of how arts relate to sciences and other matters are upfront as important.

 A particular example is his discussion of the role specialists versus generalists in making profound innovations. Unsurprisingly, Puu sees generalists as playing a crucial role, but he also recognizes that often they fail to get credit for their achievements. And for all his advocacy of the importance of generalists, unsurprising given his own personal history of working across music and economics, he nevertheless recognizes that the vast sweep of evolution and development at least in economies involves greater and greater specialization. While he at times complains about Adam Smith [7] and his focus on the apparently narrow topic of pin factories, he also understands that Smith was right about the fundamental importance of “the division of labor” in increasing productivity and allowing the high material wealth our modern world economy exhibits. And that division of labor has continued to proceed far beyond what it was in 1776 when Smith described it.

 In effect Puu sees the role of generalists as more important for generating critical breakthroughs in arts and sciences and economics, the dramatic changes that in economies Schumpeter [8] would identify as “saltationalist” that fundamentally redefine the nature of an economic system. These generalists may be “amateurs,” not just they may not be getting paid for what they are doing, although often they are, but because they are motivated by the original meaning of that word from its root meaning “love,” that generalists may be significantly motivated by love for what they are doing. And again, Puu himself was clearly so motivated in his efforts to produce old musical instruments, even though he was so good at doing so led professional musicians to pay him for his labors doing so. Puu identified the ideal of such world-shaking generalists as the great figures of the Renaissance who strove to know all things and could do many things, especially in Florence.

He singles out Filippo Brunellescchi and Leonardo da Vinci. The former was the most important figure in developing perspective in painting. But he also was an engineer as well as architect. He was genuinely revolutionary in the latter category when he designed the cathedral of Florence, whose dome was larger than any ever built before it, with this depending on new ideas. But in order to complete this grand project he invented construction machinery. Leonardo’s achievements across painting and science and inventions were so great that he indeed has long been the very model of the universally capable Renaissance Man, including many activities and skills not so widely known, such as that he was an excellent musician.

He sees this ideal of the full generalist as being expressed in the highly influential book published in 1528 by Baldassaro Castiglione [9], *Il Cortigiano*, (The Book of the Courtier). Castiglione was a diplomat who spent several years shortly after 1500 in the court in Urbino of the last independent Duke Montefeltro and his wife, Elisabetta Gonzaga, whose famous portraits by Piero della Frencesca are in the Uffizzi in Florence. Raphael was from Urbino and painted Castiglione’s portrait with this court featuring numerous notable figures depicted in Castiglione’s book. Many see this work as expressing the culmination of this ideal of the Renaissance Man, with the courtier supposed to be capable of many things, from military activity to poetry and philosophy and civic activity. These talents were to be fused in courteous manner that allowed them to be displayed naturally in a subtle way. The skilled courtier would have *sprezzatura* that is the “art that conceals art.” This book would be translated into many languages and spread across Europe, having an especially strong influence in the court of Elizabeth I in England, with it continuing to influence the ideals of European elites arguably for hundreds of years afterwards until nearly our current day.

Nevertheless, for Puu this matter of the amateur who was essentially an outsider was especially important, even when such a person was not necessarily a full generalist like the brilliant figures of the Florentine Renaissance or Vienna around 1900, another time and location he noted. He noted several who were quite successful, such as Heinrich Schliemann, a successful businessman from St. Petersburg who made himself into an archaeologist and successfully found the ruins of Troy and Pierre de Fermat, who was a judge in Toulouse but who also was a mathematician on the side, proposing his famous “last theorem” that would only finally be proven 350 years later. These amateurs were successful in obtaining recognition for their achievements in their amateur activities while maintaining their main successful careers. But for some who stepped too far out of the box only trouble would come. Puu notes the case of Evariste Galois, who was rejected for entry into the École Polytechnique to study mathematics because his answers to questions in an interview were over the head of his interviewer. While he made one of the most dramatic discoveries in the history of mathematics with group theory, his frustration with his outcast status would lead him to engage in a duel at age 21 that many view as having been essentially a “scientific suicide.”

In any case, this phenomenon of influences across fields with these generalists generating dramatic breakthroughs and innovations highlights the link between complexity and aesthetics that play a deep role many of these dramatic developments. As we shall see these depths involve such matters as incompleteness and fractals, with layers of meaning as inevitable imperfection remains in the search for ultimate truth and beauty.

**The Evolution of Science and Mathematics and their Limits in Aesthetic Complexity**

 The problem of shifts in schools of art resembles that of paradigm shifts in science, as Puu notes, bringing up Thomas Kuhn’s *The Structure of Scientific Revolutions* [10] and his contrasting of “normal science” with the sudden profound shifts that happen when deep structures of understanding transform. He links this with the view of Schumpeter [8], who saw this conflict between periods of continuous and slow change with those moments when major changes happen, those discontinuities that can arise in nonlinear dynamical systems. So Schumpeter, described those periods of what Kuhn later called normal science when they seem to reach a state of apparent perfection as “classical situations…like a Greek temple.” Of course, Schumpeter like Kuhn understood that such situations would not and could not remain as they were. There are always loose ends and phenomena that resist explanation in the existing paradigm, and as troubling empirical data appears the pressure increases, which can bring down even the most apparently perfect of Greek temples, most of which are in ruins today, as new paradigms arise. Curiously, even the most perfect seeming of such temples had built into them imperfect details that added to their apparent perfection. Thus with the most famous of them all, the Parthenon in Athens, the columns are not perfectly straight, but vary in width as one moves up and down them, with this variation adding to the illusion from a distance of perfection.

 Puu argues that in science, logical empiricism came to dominate with such figures as Galileo, Kepler, and Newton. Of course, Galileo suffered as a heretic in the eyes of the still-dominant Roman Catholic Church, and Kepler was fearful of straying to the new Copernican paradigm, this struggle over Euclidean geocentrism and its epicycles with Copernican heliocentrism, which would have the advantage of a greater simplicity, especially once Newton came along with his laws of gravitation and motion that explained the elliptical orbits that Kepler understood.. This was indeed the paradigmatic paradigm shift that Kuhn [10] wrote about in his influential book on scientific revolutions. But as logical empiricism became the keystone of normal science, it would become institutionalized in the form of such entities as the Royal Society in Britain, which would be imitated in other nations, with Boyle playing a leading role in its foundation. But as with Newron’s law, Boyle’s Law regarding temperature, pressure, and volumes, only truly held for an ideal state that does not exist. Empirical reality only approximates these ideal states where the mathematical equations precisely hold. And in the case of Boyle’s Law, it would encounter greater difficulties due to the phenomenon of phase transitions between states of matter as solid, liquid, or gas, the nettlesome problem of discontinuities arising even as underlying variables varied continuously.

 Puu sees that this effort to seek perfection and to move to a higher perspective led to ever more grandiose projects. Thus out of the entrenchment of logical empiricism came efforts to formulate “supertheories.” A dramatic example of this would be Auguste Comte who around 1830 would attempt to present a system of how all fields of thought relate to each other in a hierarchical system that has fields relating as what they study relates in the real empirical world. Thus physics was at the top as it determines fundamental physical forces that underpin chemistry, which in turn underpins biology, with this leading to social sciences. This founder of positivism has also often been viewed as the founder of sociology, which lay at the opposite end of this structure from physics at the peak of human activities. While his view would face much criticism, it had enormous influence on the development of academia later, especially in France. In the twentieth century Comte’s positivism would fuse with logic in the philosophical doctrine of logical positivism, which would dominate the philosophy of science for much of the century, facing criticism eventually from many who drew on the ideas of Kuhn.

 This drive for all-encompassing supertheories would show up within certain disciplines as well. In mathematics and logic this culminated in the early twentieth century with the program of Hilbert to develop a set of axioms that would provide a complete logical foundation for mathematics. This would be furthered by work of Whitehead and Russell [11] in their *Principia Mathematica*. This work confronted various apparent paradoxes arising from set theory, but developed ways to get around them. This program came to a crashing halt with the fundamental Incompleteness Theorem of Gödel in 1931 [12]. While this theorem closed the door to the dream of a universally complete mathematics, it opened the door to certain fundamental themes of complexity that have ricocheted across not only mathematics, but the arts and economics. Central to this is notion of reflexivity or self-referencing implying self-consciousness. Self-referencing is key to the paradoxes that Gödel would use as to prove his theorem.

 It is curious that is immediately after discussing Gödel’s Incompleteness Theorem that Puu brings up chaos and predictability. He notes Laplace’s vision of perfect predictability as a precursor to the program of Hilbert and then brings up Poincaré and his study of the three body problem. From there it is a short jump to Lorenz and his famous attractor, which Puu recognizes as aesthetically beautiful, as effectively art. This extends also to biological evolutionary dynamics as argued in *Growth and Form* by Wenworth-Thompson [13]. He even brings in the “beauty contest” among mathematicians carried out in 1990 in the *Mathematical Intelligencer* that was won by Euler’s famous equation that eiπ = -1.

 Hofstadter in his *Gödel, Escher, Bach: An Eternal Golden Braid* [14] openly argues the link based on self-referencing across apparently completely different fields of endeavor, from mathematics through visual art in the form of the work of Escher through J.S. Bach in music, a key player also in Puu’s view. This dazzling work indeed may be the closest one out there that is similar in perspective to that of Puu with its argument of these sorts of deep links, or at least since perhaps the Renaissance. An especially profound example is a particular work by Escher that both Hofstadter and Puu highlight, although with Puu making more of it than Hofstadter and noting its multi-layered mathematical linkages, even as reportedly Escher himself did not know the mathematics that he was using, creating his prints out of sheer intuition.

Anyway, this particular print is in a self-referential way titled *Print Gallery*. With an empty hole in the middle, it shows a man in the lower left one sees looking through a window into the print gallery who is looking at a print on the wall of a city. As one follows the image of the city up and to the right and around the central hole, where Escher put his name, the image expands and on the lower right focuses on a building that expands as one moves along and has a long window in its front that eventually it becomes clear is the print gallery with the window through which one is looking at the man again in the lower left who is looking at the print on the wall that contains the image that contains him looking at it. Other prints on the wall to the right of this one happen to be ones by Escher himself, more self-referencing. This print effectively depicts the infinite set of images that are implied by the paradoxes of self-referencing, the infinite loops that arise in computer programs that self-reference and thus a la Turing have a stopping problem and go on forever. This infinite set of images is also implied in the fractal nature of chaotic dynamics as studied by Mandelbrot [15] that repeat themselves as one moves ever downward in an infinite chase to smaller and smaller scales, with many fractal images themselves being works of art, something brought out very clearly at a conference Puu organized in Umeä in the 1990s that I and Mandelbrot attended in which he organized a visit to a local art museum where the museum director was overjoyed to meet Mandelbrot, viewing him as more a great artist than a great mathematician.

The infinite layers of self-referencing implied in Escher’s *Print Gallery* also invoke the reflexivity argument that would later be taken up by many in economics who would see such reflexivity or self-referencing as key to understanding various phenomena in economic dynamics such as the emergence and then crashes of speculative bubbles in financial markets, agents thinking about what other agents are thinking about what they are thinking about those other agents and so on as discussed by Rosser [16, 17] . For Hofstadter this view points ultimately to the ultimate foundation of human consciousness as a self-referential or reflexive “strange loop,” like a pair of mirrors facing each other generating an infinite set of images of images of images and so on without end.

**The Frustrated Search for Perfection in Music**

Given his own involvement in making Renaissance and Baroque instruments for professional musicians, as well as regularly organizing Baroque festivals, it is not surprising that some of Puu’s most insightful discussion of these issues involves music, with him seeing these matters relevant to other fields including in sciences and economics. Thus one issue is this matter of inevitable imperfection implicit in everything from factals and the cascading images of Escher’s *Print Gallery*, to the inevitable distance the real physical world is from the abstract ideals formulated in the equations of physical laws. This inevitable imperfection Puu sees in perhaps the greatest musician of them all in his view, Johann Sebastian Bach. This involves the matter of tuning or tempering for keyboard instruments and others as well. For any given note there is a perfect harmonic sequence. Doubling the frequency of a tone raises it an octave, tripling it raises it a perfect fifth above that, quadrupling raises it two octaves, and so on. Having all instruments keyed to a single note will have them all having higher notes above a fundamental correspond to these perfect frequencies, and a purity of sound is obtained by having this, and holds for a group of instruments if they are all similarly tuned.

 The problem is that when one takes other notes and examines their harmonic series, the notes that appear as one moves up a series will not precisely correspond with notes that appear in the upper portions of the harmonic series of other notes. If one has an musical group consisting strictly of either keyless wind instruments or fretless string instruments, then skilled players can adjust what notes are by changing their lip positions or moving their fingers slightly to different locations for various notes, thus maintaining the harmonic purity for each key. Puu notes that Baroque trumpet players had skills for doing this that modern trumpet players with their keyed instruments do not possess, and so no modern performance of Baroque pieces for trumpet will sound as they did at the time, The skills have been lost.

 But these skills are of no use for a keyboard instrument, such as a clavichord or harpsichord or pianoforte. Each not is tied to a key that either plucks or strikes a string of a particular length that generates a particular frequency and tone with its note. There is no way for all of the keys to play notes that will fit perfectly the harmonic series of other notes. Making things closer to the harmonic ideal doe one note or key will make them farther off for another note of key. There is an inevitable disharmony or imperfection, with this shifting around as one tunes a keyboard instrument differently. It was J.S. Bach who confronted this problem and sought to resolve it, even if in the end it would be a student of his who would generate the exact system now widely in use. But Bach proposed a system that sought to minimize the overall degree of these disharmonies, attempting to more or less evenly distribute the “errors” across all the notes and keys. His famous work, *Das Wohltempierte Klavier* (The Well-Tempered Clavier), which had pieces in every key, was composed for demonstrating how “well-tempered” his compromise system was, or if one prefers, how minimally distempered it was.

 This ties to two further concerns that link especially to his views on evolutionary economic development patterns. One is this matter of indeed the existence of major shifts that happen. These are like the shifts in schools of painting from medieval to Renaissance with the discovery of perspective, or the revolutionary saltationalist technological inventions that as Schumpeter noted transform an economic system, or dramatic paradigm shifts in scientific thought as from Ptolemy to Copernicus in astronomy. Bach’s invention of well-temperedness paved the way for modern keyed orchestral instruments, which would be associated with the dramatic change from the Classical style of Haydn and Mozart to the Romanticism of Beethoven, which he at times expressed disapproval of. He in fact liked modern music such as the serial compositions of the Viennese Arnold Schoenberg, who was also a painter, partly because this school abjured Romanticism and at times harked back to the earlier Classical and Baroque styles at times. But an important link especially with economic development is that these shifts of styles and schools involved technical changes in the instruments themselves.

 For Puu an important element in his invocation of ongoing technical change is how sometimes it involves casting back to the past to bring forward something that previously existed, often in a new form. Thus, it came to pass during the nineteenth century that the knowledge of how to make harpsichords in the manner they were previously was forgotten. Early in the twentieth century there was an effort to revive in a reinvented form the harpsichord, but many realized that it was a flawed effort. This would lead to deeper studies that rediscovered the old technique, and by the middle of the century people were again making harpsichords in the way that they had been in the past. Puu himself contributed to this and was proud of his ability to make older instruments as they were supposed to be made.

**The Development Tree and the Intercalation of Influences**

Which brings us to the matter more explicitly of technological change and economic development. Here Puu recognizes the inevitable tension between continuity and discontinuity, and also admits the importance of Adam Smith’s celebration of specialization, although Smith emphasized the specialization of labor while Puu points to the specialization of tools. So he describes the development of modern tools out of primitive stone originals. A basic stone tool for splitting and smoothing would evolve into modern saws and axes. He notes that by 1867 over 500 kinds of hammers were being produced in Birmingham, all these descended from some simple original. Each splitting into yet more specialized forms occur at bifurcation points along the developmental path, possibly describable by catastrophe theory. Often key to the new changes are back older forms that are then combined with current ones, or drawing on some other tool or idea to modify the existing one. These patterns of cutting across boundaries to generate something new is a central piece of the entire process. The entire pattern of evolution from these simple primitive stone tools to the vast array of modern specialized ones constitutes the development tree.

 At the same time, despite all this emphasis on Schumpeterian saltationalist shifts, he emphasizes the importance of continuity in this evolutionary process. These dramatic shifts arise from the accumulation of small changes that go on all the time. He lays this out in a discussion of how steam power machinery developed, with such breakthroughs as the Newcomen steam engine and the later Watt one involved combining already existing technologies and machine parts in new ways that had been gradually improved. He pushes this to invoke processes simultaneously operating at different time speeds, slow and fast. This is already implicit in catastrophe theory, where underlying control variables change slowly, bringing about the discontinuous changes in fast variables at critical bifurcation points. He further invokes the ideas of Hermann Haken and his synergetics [18] to extend this analysis of the relation between slow and fast variables, although with this approach allowing for greater differences in dimensionality in the different processes. Thus it can be changes in a fairly small number of dimensions that can bring about discontinuous structural shifts in a larger number of ones, the “grear changes” that form the greater pattern of economic development over time, as well as that of scientific and aesthetic forms.

**The Importance of Generalists Redux**

 To conclude this safari through Puu’s own safari across these many ideas and fields, it is worth returning to Puu’s invocation of the importance of generalists who can cut across many fields for generating breakthrough ideas, much as he was able to do across music and mathematical economics. He became concerned that the hyper-specialization of modern academia and modern thought more generally could lead to a stagnation. He expressed this especially regarding academic culture in a discussion of the influence of modern US academic culture as it has changed that of an older more generalist European one, a theme of Rosser, Holt, and Colander [19} in their book on *European Economics at the Crossroads* in which they interviewed Tönu Puu. A quote by him was discussed that initially appeared in the first edition of his *Arts, Sciences, and Economics: A Historical Safari* [1. pp.31-32]. This paper concludes with that quote.

“European university culture until the 1960’s, heavily depended on seminars, where various members of the staff, working with entirely different topics, communicated their results. For that reason the staff members had to keep a broad perspective on their disciplines. Relatively little was regarded as being worth publishing, and national and local “schools” were established, which made visits to other environments really interesting.

We tend to look down on the precious generation as they published relatively little. This fact, however, does not signify that they were more choosy about what they regarded as bein significant enough to merit publication.

After large scale production ideals from the US overtook the European style, everything is produced for immediate publication, even the most tiny little idea. The number of journals, which has exploded accordingly, conveniently provides for the space. We still have seminars, but we read already published or accepted papers, which we do not want criticized and we hardly expect anybody else at the department to understand our whole message. Travel and change of department only results in new personal relations, not new ideas.

 It may be that we would urgently need more of interdisciplinary for a in the future just in order to provide for encounters with the unexpected ideas we need to secure creativity.”

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