COMPLEXITY AND BEHAVIORAL ECONOMICS

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ABSTRACT: This paper will consider the relationship between complexity economics and behavioral economics. A crucial key to this is to understand that Herbert Simon was both the founder of explicitly modern behavioral economics as well as one of the early developers of complexity theory. Bounded rationality was essentially derived from Simon's view of the impossibility of full rationality on the part of economic agents. Modern complexity theory through such approaches as agent-based modeling offers an approach to understanding behavioral economics by allowing for specific behavioral responses to be assigned to agents who interact within this context, even without full rationality. Other parts of modern complexity theory will also be considered in terms of their relationships with behavioral economics. Fundamentally, complexity provides an ultimate foundation for bounded rationality and hence the need to use behavioral economics.

KEYWORDS: complexity, behavioral economics, bounded rationality

Introduction

The late Herbert A. Simon developed the idea of *bounded rationality* from his earliest works (Simon, 1947, 1955a, 1957), which is viewed as the foundation of modern *behavioral economics*. Behavioral economics contrasts with more conventional economics in not assuming full information rationality on the part of economic agents in their behavior. In this regard, it draws on insights regarding human behavior from other social science disciplines such as psychology and sociology, among others. Without question, one can find earlier economists[i] who argued that people are motivated by more than mere selfish maximization. Indeed, from the very beginnings of economics with Aristotle, who put economic considerations into a context of moral philosophy and proper conduct, through the father of political economy, Adam Smith in his *Theory of Moral Sentiments* (1759), to later institutional economists such as Thorstein Veblen (1899) and Karl Polanyi (1944) who saw peoples' economic conduct as embedded within broader social and political contexts. Nevertheless, it was Simon who coined both of these terms and established modern behavioral economics.

Simon's initiatives led to a flurry of activity and research over the next few decades, much of which became more influential in business schools and management programs as the rational expectations revolution conquered most of economics during the 1970s and 1980s. Assuming bounded rationality by economic agents led him to the concept of *satisficing*, that while people do not maximize they strive to achieve set goals within constraints. This became accepted in business schools as managers were taught to achieve levels of profit acceptable to owners.

Also arising out of his discovery of bounded rationality was his interest in pursuing more deeply how people think and understand as part of their making decisions. This led him to consider how this could be studied through the use of computers. This led him to become one of the founders of the field of *artificial intelligence* (Simon, 1969), and Simon more generally is regarded as one of the early leaders of computer science more generally. But it was his concern regarding the implications of bounded rationality that led him into this nascent field.

Simon would also become a leading figure in the early development of complexity theory, particularly of hierarchical complexity theory (Simon, 1962), although he only made an indirect link between this and bounded rationality. However, modern complexity theorists are much more willing to see a close and direct link between complexity of one sort or another and bounded rationality, and thus also with behavioral economics. Indeed, complexity can be seen as a, if not the, fundamental foundation for why people have bounded rationality. Complexity lies at the very heart of behavioral economics in this view, which this paper adopts, and Simon sought to understand how people decide in the face of such ineluctable complexity.

Forms of Complexity

A discussion regarding the relationship between "complexity" and something else clearly requires some discussion of what is meant by this term, or at least what this observer means by it. Indeed, this is arguably a weasel term, one that has no clearly agreed-on meaning more generally. The MIT engineer, Seth Lloyd, some time ago famously gathered a list of various different meanings, and this list was at least 45 before he stopped bothering with this effort, or at least making it publicly known (Horgan, 1997, p. 303). It may be useful therefore to refer to the broadest possible view of complexity that includes all of these and any others as being *meta-complexity*. The definition of this may simply amount to listing all possible meanings that any have ever claimed should be on the list.

If one seeks general definitions or concepts, something often appears in such general definitions is the idea that somehow something that is complex involves a whole that is "greater than the sum of its parts," as the old cliché puts it. Such an idea can be traced as far back as Aristotle, with many since contributing to it. We shall see below that not all the items on Seth Lloyd's list might agree with this, particular the many that relate to *computational complexity*, arguably the sub-category of complexity with more variations than any other. That those concerned with this sub-category might not have such a view might explain why John von Neumann (1966) did not distinguish complexity from mere *complicatedness*. While some may not wish to make this distinction, many do, with Israel (2005) noting that the two words come from different roots in Latin, *complecti* and *complicare* respectively, the former meaning "to enfold" and the latter "to entangle." Thus, while close and possibly from an identical deeper origin, the former implies some completing in a higher order whereas the latter implies more simply "to confuse" due to the bringing together of many different elements.

In any case, perusing Lloyd's list allows one to lump many of his definitions into higher order sub-categories. Arguably the sub-category with the most items on it can be considered forms of *computational complexity*, with at least as many as 15 of them fitting in this category, possibly more.[ii] If there is a linking concept through this set of definitions, it involves ideas of size or length, how long a program is or how many distinct units there are within the object such as bits of information. However, the many variations on this do not map onto each other readily. Nevertheless, many of these definitions have the virtue of being clearly measurable, even if there are many such definitions. Thus, if one gloms onto one of these, one can argue that it may have a stronger claim to being "scientific" due to this specific clarity than some other fuzzier alternatives. Interestingly, among those fuzzier alternatives listed by Lloyd is the *hierarchical complexity* concept introduced by Herbert Simon (1962), which is relevant to several disciplines.

Within economics and arguably several other disciplines the strongest rival to the varieties of computational complexity can be called *dynamic complexity*, although no item called precisely this appears on Lloyd's list, with perhaps the closest being "self-organization" and "complex adaptive systems." More precisely, Day (1994) defined (dynamic) complexity as arising in nonlinear dynamical systems that due to endogenous causes do not asymptotically approach a point, a non-oscillating growth or decline, or two-period oscillation. Thus such a system will exhibit some form of erratic dynamic behavior arising endogenously from within itself, not due to an erratic exogenous driver. Rosser (1999) adopted this definition for his "broad-tent" complexity that is clearly dynamic.[iii]

Within this broad-tent form of dynamic complexity one can observe four well-known sub-categories that were identified as being "the four Cs" of *chaoplexity*, according to Horgan (1997, Chapter 11). These were cybernetics, catastrophe theory, chaos, and "small-tent" or agent-based or Santa Fe complexity. Horgan argued that these have all constituted a succession of intellectual fads or bubbles, beginning in the 1950s with Norbert Wiener's cybernetics and moving on successively, with agent-based complexity simply the latest in this succession that was overhyped and then discarded after being shown to be overhyped. However, an alternative view is that these represent an accumulating development of knowledge regarding the nature of nonlinear dynamics, and that students of this development should take Horgan's ridicule and turn it on its head, much as such art movements as Impressionism were originally named critically, only to have them become widely admired. Let the "four Cs" be the focus of a successful ongoing intellectual system.

Norbert Wiener (1948) introduced *cybernetics*, which strongly emphasizes the role of positive and negative feedback mechanisms. Wiener emphasized issues of control, which made cybernetics popular in the Soviet Union and other socialist planned economies long after it had faded from attention in western economies. While Wiener did not emphasize nonlinear dynamics so much, certain close relatives of cybernetics, *general systems theory* (van Bertalanffy, 1968) and *systems dynamics* (Forrester, 1961) did so more clearly, with Forrester particularly emphasizing how nonlinearities in dynamical systems can lead to surprising and "counterintuitive" results. However, the discrediting of cybernetics and its relatives may have come most strongly from the failure of the limits to growth models based on systems dynamics when they forecast disasters that did not happen (Meadows, Meadows, Randers, and Behrens, 1972). Much of the criticism of the cybernetics approaches, which emphasized computer simulations, focused on the excessive levels of aggregation in the models, something that more recent agent-based models are not guilty of, with these arguably representing a new improved revival of the older cybernetics tradition.

Catastrophe theory developed out of broader bifurcation theory, and to the extent that formal catastrophe theory may not be applicable in many situations due to the strong assumptions required for it to be applied, broader bifurcation theory can analyze the same fundamental phenomenon, that of smoothly changing underlying control variables having critical values where values of endogenous state variables may change discontinuously. Formal catastrophe theory, based on Thom (1975), provides generic forms for these bifurcation conditions on equilibrium manifolds according to the number of control and state variables, and Zeeman (1974) provided the first application in economics to the analysis of stock market crashes using the cusp catastrophe model that has two control variables and one state variable. Empirical analysis of such models requires the use of multi-modal statistical methods (Cobb, Koppstein, & Chen, 1983; Guastello, 2009). A backlash developed as critics argued that the theory was applied to situations that did not fulfill the strict assumptions necessary for the application, but Rosser (2007) has argued that this backlash was overdone, with many avoiding its use who should not do so.[iv]

While *chaos theory* can be traced back at least to Poincaré (1890), it became prominent after the identification of sensitive dependence on initial conditions, aka "the butterfly effect," by the climatologist, Edward Lorenz (1963), probably the most important idea associated with the phenomenon. Applications in economics followed after an important paper by May (1976) that initially suggested some of them. Debates over empirical measurements and problems associated with forecasting have reduced some of the earlier enthusiasm for chaos theory in economics, which probably peaked during the 1980s. However, the fundamental insights derived from it continue to influence economic thinking as well as that in other disciplines.

Figure 1 shows the butterfly effect as found by Lorenz initially, with two very different trajectories arising from the same point, with these distinguished by only small differences in starting conditions from that point. Figure 2 shows a case the combines both catastrophic effects with chaotic dynamics in *chaotic hysteresis*, a figure due originally to Puu (1989), with the axes representing investment and the rate of change of investment for certain parameter values in a Keynesian style macroeconomic model. Rosser, Rosser, Guastello, and Bond (2001) estimated such phenomena for investment in the former Soviet Union over the post-World War II period.[v]

Figure 1

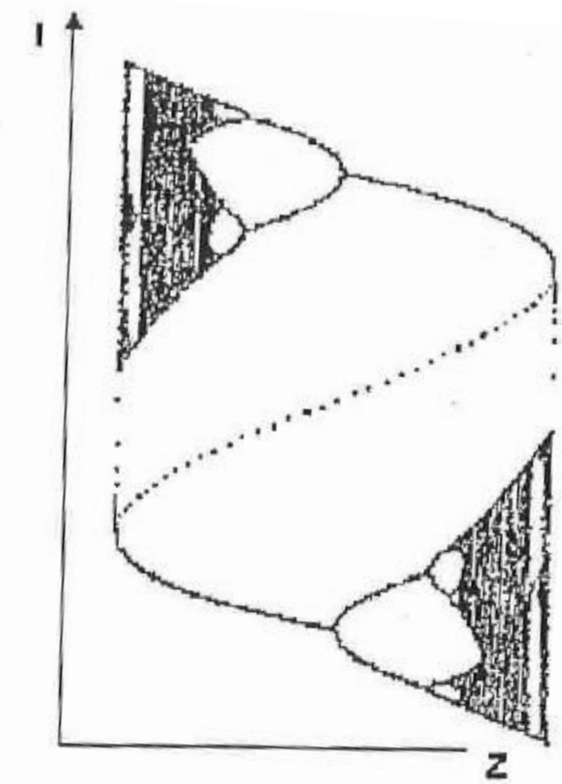


Fig. 1. A configuration of two globally stable states with a bifurcation manifold connecting them and an internal bifurcation mechanism within each of them.

Figure 2

Coming on the heels of the popularity of chaos theory would be agent-based (or “small tent”) dynamic complexity, strongly associated with the Santa Fe Institute. However, its origin is generally traced to the urban segregation model of Schelling (1971), who used a go board rather than a computer to work out the dynamics of a city starting out racially integrated and then segregating with only the slightest of incentives through nearest neighbor effects.[vi] Such systems are famous for exhibiting self-organization and do not generally converge on any equilibrium, also showing cross-cutting hierarchical interactions and ongoing evolutionary change (Arthur, Durlauf, and Lane, 1997). Substantial active research in economics using such models is ongoing.

We note that these are only a small subset of the full array of complex dynamics that nonlinear systems can exhibit. Others include *non-chaotic strange attractors* (Lorenz, 1983), *fractal basin boundaries* (Abraham, Gardini, and Mira, 1997), *flare attractors* (Hartmann and Rössler, 1999; Rosser, Ahmed, and Hartmann, 2003)), and more.

A central point that should be clear is that the presence of such dynamic complexities in economic systems greatly complicates the problem for economic agents of forming rational expectations regarding the future path of such systems. In their presence, it becomes highly unlikely that agents can fuflill the conventional assumption of full information and complete rationality in their decisionmaking.

Herbert Simon and Bounded Rationality

The late Herbert A. Simon is widely considered to be the father of *modern behavioral economics*, at least it was his work to which this phrase was first applied. He was also an early theorist of complexity economics, if not the father per se, and also was one of the founders of the study of artificial intelligence in computer science. Indeed, he was a polymath who published well over 900 academic papers in numerous disciplines, and while he won the Nobel Prize in economics in 1978 for his development of the concept of *bounded rationality*, his PhD was in political science and he was never in a department of economics. We must use the term “modern” before “behavioral economics” because quite a few earlier economists can be seen as focusing on actual human behavior while assuming that people do not behave fully in what we would now call an “economically rational” manner (Smith, 1759; Veblen, 1900).

We must at this point be clear that by "behavioral economics" we are not assuming a view similar to that of "behavioral psychology" of the sort advocated or practiced by Pavlov or B.F. Skinner (1938). The latter does not view studying what is in peoples' minds or consciousness as of any use or interest. All that matters is how they behave, particularly how they respond to respond to repeated stimuli in their behavior. This is more akin to standard neoclassical economics, which also purports to study how people behave with little interest in what is going on inside their heads. The main difference between these two is that conventional economics makes a strong assumption about what is going on inside peoples' heads: that they are rationally maximizing individual utility functions derived from their preferences using full information. In contrast, behavioral economics does not assume that people are fully rational and particularly does not assume that they are fully informed. What is going on inside their heads is important, and such subjects as *happiness economics* (Easterlin, 1974) are legitimate topics for behavioral economics.

In any case, from the beginning of his research with his path-breaking PhD dissertation that came out as a book in 1947, *Administrative Behavior* and on through important articles and books in the 1950s (Simon, 1955a, 1957), Simon saw people as being limited in both their knowledge of facts as well as in their ability to compute and solve the difficult problems associated with calculating optimal solutions to problems. They face unavoidable limits to their ability to make fully rational decisions. Thus, people live in a world of *bounded rationality*, and it was this realization that led him into the study of artificial intelligence in computer science as part of his study of how people think in such a world (Simon, 1969).

This led Simon to the concept of *satisficing*. People set targets that they seek to achieve and then do not pursue further efforts to improve situations once these targets have been reached, if they are. Thus a firm will not maximize profits, but its managers will seek to achieve an acceptable level of profits that will keep owners sufficiently happy. This idea of satisficing became the central key to the behavioral study of the firm (Cyert and March, 1963) and entered into the management literature, where it probably became more influential than it was in economics, for quite a long time.

Some economists, notably Stigler (1961), have taken Simon's position and argued that he is actually a supporter of full economic rationality, but only adding another matter to be optimized, namely minimizing the costs of information. People are still optimizing but take account of the costs of information. However, Stigler's argument faces an unavoidable and ineluctable problem: people do not and cannot know what the full costs of information are. In this regard they face a potential problem of infinite regress (Conlisk, 1996). In order to learn the costs of information, they must determine how much time they should spend in this process of learning; they must learn what the costs of learning what the costs of information are. This then leads to the next higher order problem of learning what the costs of learning what the costs of information are, and there is no end to this regress in principle.[vii] In the end they must use the sorts of *heuristic* (or "rule of thumb") devices that Simon proposes that people facing bounded rationality must use in order to answer the question. Full rationality is impossible, and the ubiquity of complexity is a central reason why this is the case.

Simon (1976) distinguishes *substantive rationality* from *procedural rationality*. The former is the sort of rationality traditionally assumed by most economists in which people are able to achieve full optimization in their decisionmaking. The latter involves them selecting procedures or methods by which they can "do their best" in a world in which such full optimization is impossible, the heuristics by which they manage in a world of bounded rationality. In this regard it is not the case that Simon views people as being outright irrational or crazy. They have interests and they generally know what those are and they pursue them. However, they are unavoidably bounded in their ability to do so fully, so they must adopt various essentially ad hoc methods to achieve their satisficing goals.

Among these heuristics that Simon advocated for achieving procedural rationality were trial and error, imitation, following authority, unmotivated search, and following hunches. Pingle and Day (1996) used experiments to study the relative effectiveness of each of these, none of which clearly can achieve fully optimal outcomes. Their conclusion was that each of these can be useful for improving decisionmaking, however, none of them is clearly superior to the others. It is advisable for agents to several of these and to move from one to another under different circumstances, although as noted above it may be hard to know when to do that and precisely how.

Imitation and the Instability of Markets

While this list of procedures that can support a boundedly rational pursuit of procedural rationality, a point not clearly made is that excessive focus on one of these rather than others can lead to problems. Clearly following authority can lead to problems when the authority is flawed, as many unfortunate examples in history have shown. Any of these can lead to problems if too intensively followed, but one that has particularly played an unfortunate role in markets is imitation, even though it is a widely used method by many people with a long history of being evolutionarily successful. The problem is particularly acute in asset markets, where imitation can lead to speculative bubbles that destabilize markets and can lead to much broader problems in the economy, as the crisis of 2008 manifestly shows.

A long literature (MacKay, 1852; Baumol, 1957; Zeeman, 1974; Rosser, 1997) has recognized that while agents focusing on long term fundamental values of assets tend to stabilize markets by selling them when their prices exceed these fundamentals and buying when they are below those, agents who chase trends can destabilize markets by buying when prices are rising, thus causing them to rise more, and vice versa. When a rising price trend appears, trend chasers will do better in returns than fundamentalists and imitation of those doing well will lead agents who might have followed stabilizing fundamentalist strategies to follow destabilizing trend chasing strategies, which will tend to push the price further up. And when a bubble finally peaks out and starts to fall, trend chasers can then push the price down more rapidly as they follow each other in a selling panic.

That such a tendency to engage in trend chasing speculation is deeply rooted in the human psyche was initially established by Smith, Suchanek, and Williams (1988), with many subsequent studies supporting this observation..[viii] Even in situations with a finite time horizon and a clearly identified payment that establishes the fundamental value of the asset being traded, in experimental markets it has been repeatedly shown that bubbles will appear even in these simplified and clearcut cases. People have a strong tendency to speculate and to follow each other into such destabilizing speculation through imitation. Procedures that can support procedural rationality in a world of bounded rationality can lead to bad outcomes if pursued too vigorously.

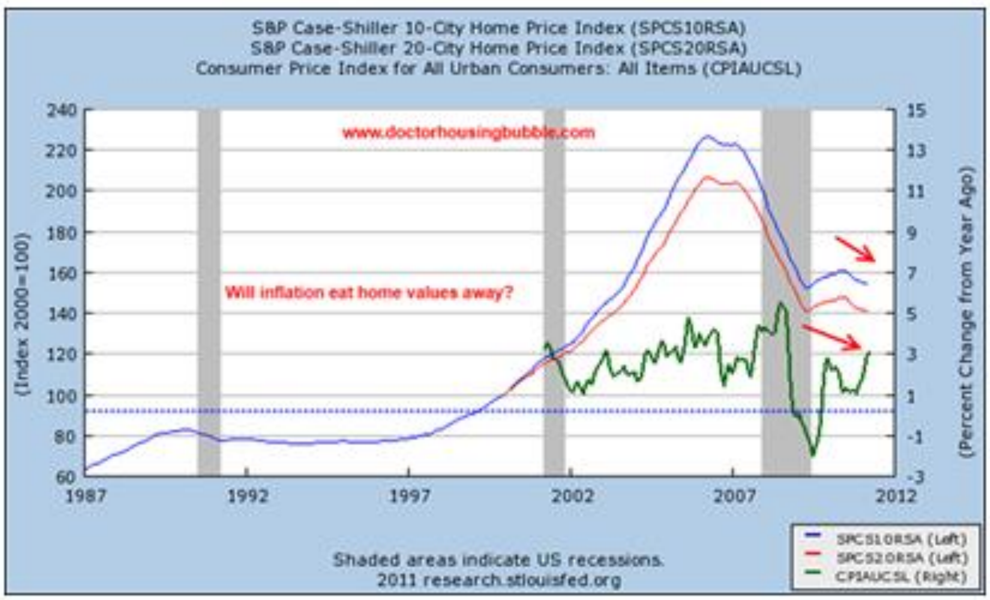
We note that such patterns regularly take three different patterns. One is for price to rise to a peak and then to fall sharply after hitting the peak. Another is for price to rise to a peak and then decline in a more gradual way in a reasonably symmetric manner. Finally, we see bubbles rising to a peak, then declining gradually for awhile, finally collapsing in a panic-driven crash. Kindleberger's classic *Manias, Panics, and Crashes* (2001) shows in its Appendix B that of 47 historical speculative bubbles, each of the first two have five examples, while the remainder, the vast majority, follow the final pattern, which requires heterogeneous agents who are not fully rational for it to occur (Rosser, 1997). This shows that complexity is deeply involved in most speculative bubbles.

Figures 3, 4, and 5 show the time path for prices of three bubbles before, during, and immediately after the 2008 crisis. They show the three patterns described above, taken from Rosser, Rosser, and Gallegati (2013). The first is for oil, which peaked at $147 per barrel in July 2008, the highest nominal price ever observed, and then crashed hard to barely over $30 per barrel in the following November. It seems that commodities are more likely to follow this pattern than other assets (Ahmed, Rosser, and Uppal, 2014).

Figure 3

The second pattern was followed by the housing bubble, which peaked in mid-2006 according to this figure, which shows to different indexes, the Case-Shiller 10-city one and their 20-city one as well. Looking closely one can see a bit of roughness around the peak making it look almost like the third pattern, whereas in fact if one looks at housing markets in individual cities, they look as posited by this pattern, with this roughness at the national level reflecting that different cities peaked at different times, with a final round of them doing so as late as January 2007 before they all declined.

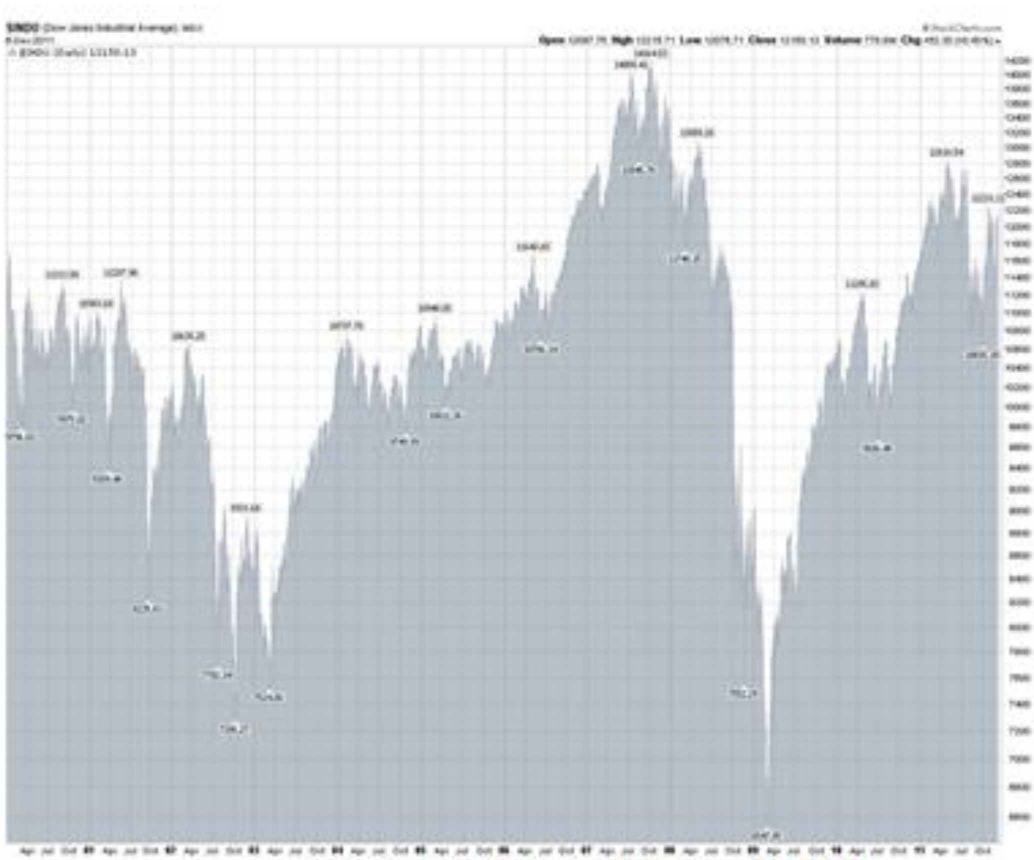
This sort of pattern historically is often seen with real estate market bubbles. The more gradual decline than in the other patterns, nearly symmetric with the increase, reflects certain behavioral phenomena. People identify very personally and intensely with their homes and as a result tend not to easily accept that their home has declined in value whey they try to sell it during a downturn. As a result they have a tendency to offer prices that are too high and then refuse to lower their prices readily when they fail to sell. The upshot is a more dramatic decline in volume of sales on the downswing compared to the other patterns as people hang on and refuse to lower prices.



Housing Prices in US, Case-Shiller Index, 1987-2011

Figure 4

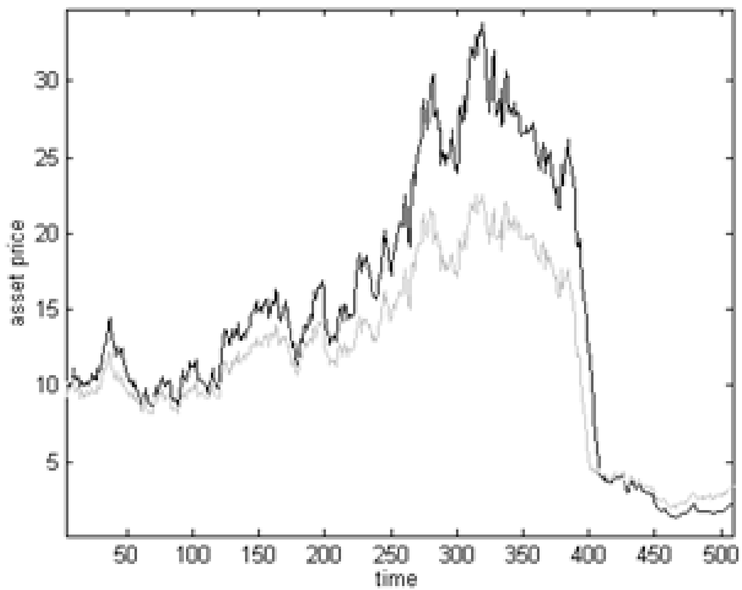
The third case shows the US stock market as exhibited by the Dow-Jones average, which peaked in October 2007, only then to crash in September 2008. Such patterns seem to be more common in markets for financial assets. Such patterns show heterogeneity of agents with different patterns of imitation, a smarter (or luckier) group that gets out earlier at the peak, followed by a less smart (or less lucky) group that hangs on hoping the price will return to rising, only to panic later en masse for whatever reason.



: Dow-Jones Industrial Average, 200-2011, Daily Data

Figure 5

Finally, Figure 6 shows how this pattern with its *period of financial distress* (Minsky, 1972) can be modeled in an agent-based model that has agents shifting from one strategy to another based on their relative successes, although not instantly (Gallegati, Palestrini, and Rosser, 2011). This model is based on ideas from Brock and Hommes (1997, 1998) that underlie the so-called Santa Fe stock market model (Arthur, Holland, LeBaron, Palmer, and Tayler, 1997). What triggers the delayed crash is agents running into financial constraints such as happens when individuals must meet margin calls in stock markets. The higher curve shows the pattern when agents imitate each other more strongly, as in a statistical mechanics model when there is a stronger interaction between particles.



Increase in J. The two time series share the same random numbers and same parameters but J. In the grey time series J = 0.5; in the black time series J = 3.

Figure 6

Hierarchical Complexity and the Question of Emergence

While we can see Herbert Simon's discovery of bounded rationality as an indirect claim to being a "father of complexity," his most direct claim, recognized by Seth Lloyd in his famous list, is his 1962 paper to the American Philosophical Society on “The Architecture of Complexity.” In this transdisciplinary essay he deals with everything from organizational hierarchies through evolutionary ones to those involving “chemico-physical systems.” He is much concerned with the problem of the decomposability of higher-order systems into lower level ones, noting that productions ones, such as for watchmaking, as well as organizational ones, function better when such decomposability is present, which depends on the stability and functionality of the lower level systems.[ix]

However, he recognizes that many such systems involve *near decomposability*, perhaps a hierarchical complexity equivalent of bounded rationality. In most of them there are interactions between the subsystems, with the broader evolution of the system depending on aggregated phenomena. Simon provides the example of a building with many rooms. Temperature in one room can change that in another, even though their temperatures may fail to converge. But the overall temperatures that are involved in these interactions are determined by the aggregate temperature of the entire building.

Simon also deals with what many consider to be the most fundamental issue involving complexity, namely that of emergence. His most serious discussion of the emergence of higher levels of hierarchical structure out of lower levels involves biological evolution, where these issues have long been most intensively discussed. He argues that how these higher levels emerged has not reflected teleological processes but strictly random processes. He also argues that even in closed systems, there need be no change in entropy in the aggregate when subsystems emerge within that system. But he also recognizes that organisms are energetically open systems, so that “there is no way to deduce the direction, much less the rate, of evolution from classical thermodynamic considerations” (Simon, 1962, p. 8). However, it is the development of stable intermediate forms that is the key for the emergence of yet higher forms.

Simon does not cite this older literature, but this issue was central to the British "emergentist" literature that came out of the 19th century to become the dominant discourse in the 1920s regarding the broader story of biological evolution, all embedded within a broader vision fitting this within the emergence of physical and chemical systems from particles through molecules to such higher levels above biological evolution in terms of human consciousness, social systems, and yet higher systems.[x] Simon dealt with this multiplicity of processes without drawing their interconnection as tightly as did these earlier figures. In the 1930s with the *neo-Darwinian synthesis* (Fisher, 1930; Wright, 1931; Haldane, 1932), the emphasis returned to near-continuous Darwinian process of gradual changes arising the level of probabilistic changes arising from mutations at the gene level, with the gene the ultimate focus of natural selection (Dawkins, 1976).

While Simon avoided dealing with this issue of emergence in biological evolution in 1962, when the reductionist neo-Darwinian synthesis was at the highest level of its influence, soon the emergence view would itself re-emerge, based on multi-level evolutionary process (Crow, 1955; Hamilton, 1964; Price, 1970). This would further develop with the study of nonlinear dynamics and complexity in such systems, with such figures as Stuart Kauffman (1993) and James Crutchfield (1993, 2003), who draw on computational models for their depictions of *self-organization* in biological evolutionary systems.

Figure 7 from Crutchfield (2003, p. 116) depicts how an initial genetic level mutation can lead to emergent effects at higher levels. On the right side are genotypes moving upwards from one basin of attraction to another, while on the left side phenotypes are also doing so in a parallel pattern. He introduces the concept of *mesoscales* for such processes, which clearly follow Simon's admonition about the necessity of stable intermediate systems emerging to support the emergence of yet higher order ones.

This view remains questioned by many evolutionists (Gould, 2002). While the tradition going through catastrophe theory from D'Arcy Thompson (1917) has long argued for form arising deep structures in organic evolution, critics have argued that such self-organizing processes are ultimately teleological ones that replicate old pre-evolutionary theological perspectives such as Paley's (1802) in which all things are in their place as they should be due to divine will. Others have criticized that such process lack invariance principles (McCauley, 2005). Others coming from more a more computational from such processes (Moore, 1990). There is no easy resolution of this debate, and even those advocating the importance of emergent self-organization recognize the role of natural selection. Thus, Kauffman (1993, p. 644) has stated, "Evolution is not just 'chance caught on a wing.' It is not just a tinkering of the ad hoc, of bricolage, of contraption. It is emergent order honored and honed by selection."

While the mechanisms are not the same, the problems of emergent self-organization apply as well to socio-economic systems. Simon's focus tended to be on organizations and their hierarchies. While he may well have sided with the more traditional neo-Darwinian synthesizers when it came to emergence of higher order structures in biological evolution, the role of human consciousness within human socio-economic systems means that the rules are different there, and the formation of higher order structures can become a matter of conscious will and planning, not mere randomness.

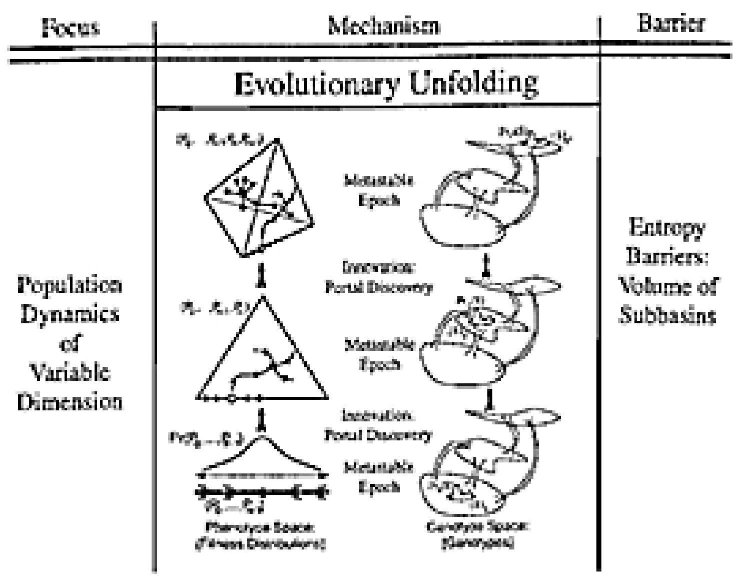


Figure 7

Bounded Rationality and Learning to Believe in Chaos

One of the greater ironies regarding bounded rationality is that it was colleagues of Herbert Simon's at Carnegie-Mellon, particularly John Muth (1961), who developed the idea of rational expectations while studying implications of bounded rationality. Muth in particular saw the assumption of rational expectations as a solution to the problems raised by bounded rationality. However, Herbert Simon would never have anything to do with this development, seeing it as a repudiation of bounded rationality. The idea that people not only know what is the true model of the economy, but that their subjective view of the probability distribution of exogenous noise in the system corresponded with the objective probability distribution of such noise, which was also conveniently Gaussian, simply was not acceptable in his view. Quite aside from the inability of boundedly rational agents to discern the "true model of the economy," he would never accept the idea that noise would be Gaussian. Indeed, he was a deep student of power law distributions that exhibit kurtosis or "fat tails" (Simon, 1955b), hence he did not join his colleagues in their elation at the development of this idea.

That said, under certain circumstances it can come to pass that simple heuristic rule of thumb behaviors may do well in a world of complex nonlinear dynamics at helping boundedly rational agents mimic underlying dynamics that may even be chaotic. This can arise if agents are able to achieve *consistent expectations* or CEE (Hommes and Sorger, 1998), an idea derived from work by Grandmont (1998) that had been done earlier, even though it was only published in the same year as theirs. An example of this was studied by Hommes and Rosser (2001) for fishery dynamics when these might exhibit chaotic patterns. Such patterns can arise due to the tendency of fisheries to exhibit backward-bending supply curves due to the carrying capacity limits of most fisheries. When prices go beyond a certain level that is consistent with *maximum sustained yield* the amount of fish will decline and fewer will get caught.

Figure 8 summarizes the fundamentals for an intertemporally optimizing fishery from Rosser (2001a, p. 27). X is the biomass of fish in the fishery, with F(X) being the growth rate of X, which in turn equals steady state harvest yields from the fishery, h, which in turn equals Q in the supply-demand diagram in the upper right portion of the figure. The bionomic portion is in the lower right part of the diagram and reflects a Schaeffer (1957) yield function, with r being the unconstrained natural growth rate of the fish population and K the carrying capacity of the fishery:

$$Q = h = F(X) = rX(1 - X/K). \qquad (1)$$

This logistic is well known to be able to exhibit chaotic dynamics when in a discrete form from the work of May (1976). Following Gordon (1954) with E = catch effort measured by time boats are out, q = catchability per vessel per day, C = cost, with constant marginal cost = c, p = price of fish, and δ the time discount rate, then cost is given by

$$C = c/qX, \tag{2}$$

and the basic harvest function can be given by

$$h(X) = qEX. \tag{3}$$

Drawing on Clark (1990), Hommes and Rosser (2001) derived a full supply curve that varies with δ. This slopes upwards for $\delta = 0$, asymptotically approaching the output level associated with maximum sustained yield, but bends backwards for $\delta > 0.02$, reaching a maximum backward bend at $\delta = \infty$, at which point the supply curve is identical to the open access equilibrium due to Gordon (1954) given by

$$S(p) = rc/pq(1 – c/pqK), \tag{4}$$

with linear demand curve given by

$$D(p) = A – Bp. \tag{5}$$

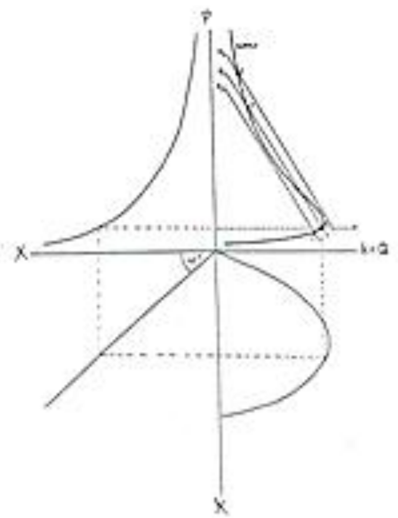


Figure 3 here: Gordon-Schaefer-Clark Fishery Model

Figure 8

Hommes and Rosser (2001) describe the cobweb dynamics of such a fishery under adaptive expectations by means of a discrete function

$$P_t = [A - S_\delta(p_{t-1})]/B. \qquad (6)$$

Hommes and Rosser (2001) show that this can be chaotic for given values of δ as S varies with it. This will occur when S is backward-bending in those portions.[xi]

The question of boundedly rational fishers arises if we allow them to base their expectations on a simple heuristic, p^e representing expected price, of a one-period autoregressive process given by

$$P^e(t) = \alpha + \beta(p_{t-1} - \alpha). \qquad (7)$$

This AR(1) process can change according to sample autocorrelation learning in which the agents over time adjust the two control parameters, α and β, based on the performance of the fishers. Based on the CEE and assuming that the underlying chaotic dynamic for the optimizing fishery is given by an asymmetric tent map, Hommes and Rosser (2001) show that these parameters can converge on values such that this simple AR(1) heuristic will reproduce the underlying chaotic dynamic, which will be a CEE.

This is shown in Figure 9, where the fishers start out catching a given level of X assuming a constant p, but as β in particular initially changes, a two-period motion appears, which then goes chaotic after later adjustment by both of the parameters occurs. This process has been called *learning to believe in chaos*, with Figure 9 from Rosser (2001a, p. 28). The horizontal is time while the vertical is X. We note that this dynamic remains bounded as are all chaotic dynamics, thus avoiding catastrophic collapse, a case of chaos preventing catastrophe. While this replicates to some extent standard figures showing period-doubling bifurcations to chaos, this is not one of those that involve a growth parameter varying. Rather this is a process of converging on a behavioral pattern based on autoregressive parameters adjusting in real time, not the same thing, even if it resembles it.

Figure 4: Learning to believe in chaos

Figure 9

Behavioral Economics and Keynesian Fundamental Uncertainty

Herbert Simon largely avoided directly addressing macroeconomic implications of his ideas, beyond expressing his disapproval of the rational expectations hypothesis that many claimed derived from his work, with this even being asserted as an something so fundamental that it was axiomatic and could not be challenged for deep theoretical and philosophical reasons, despite its obvious and well known failure to follow empirical reality, a point that Simon was fully aware of. Given that his concept of bounded rationality violates full rational expectations, and also the deep connection with nonlinear dynamic complexity that has been presented earlier in this paper, although not as fully as it might have been, the question arises, pushing beyond just bounded rationality to behavioral economics more broadly, what is the relationship between these ideas and the deep Keynesian (and Post Keynesian)[xii] idea of fundamental uncertainty?

The conventional view is that in 1921 Frank Knight and John Maynard Keynes both published books that established the distinction between *risk* and *uncertainty*, with Knight having clearly coined this distinction, but with Keynes's work exploring the distinction more deeply as he adopted the same terminology later (Keynes, 1936; Rosser, 2001b). "Risk" is quantifiable in terms of being able to identify a probability distribution that is relevant to understanding a problem. "Uncertainty" means that there is no such identifiable probability distribution. In contrast to Knight, Keynes was more aware of the possibility of various intermediate possibilities arising from ability to estimate the quantitative measure for either data availability or other reasons, as well as recognizing the difficulty of separating a variety of probability distributions possibly appropriate. This latter is a matter that has become more heavily discussed particularly since the 2008 financial crisis as the role of kurtosis or "fat tails" in financial returns has become more publicized.

The range of possibilities has been heightened by such observers as Nassim Taleb (2010) who distinguishes *grey swans* from *black swans* . The former involve probability distributions that show fat tails and are known, which can potentially explain extreme outcomes in financial markets and other situations. The latter involve true Keynesian/Knightian uncertainty, where it is impossible to assign a probability distribution, and where the events described "come out of nowhere" without any possibility of forecasting or expecting them. In this regard, Taleb argued that the 2008 crisis was a mere grey swan, an extreme outcome, that nevertheless was obviously coming and to be expected by any reasonable observer, in contrast with the October 19, 1987 crash of the stock market, 22% for the Dow-Jones average, to this day the largest one day decline ever, which was predicted by nobody and had no obvious cause, which "came out of nowhere," and which was a true black swan, an example of true and fundamental uncertainty.[xiii]

Rosser (1998, 2006) has argued that complexity provides a fundamental foundation for the reality of fundamental uncertainty. Paul Davidson (1996) has argued that this is not the case, that not only complexity, but such notions as Simonian bounded rationality are not proper or fundamental foundations of fundamental uncertainty. He distinguishes *ontological* uncertainty from *epistemological* uncertainty, arguing that true Keynesian uncertainty is the former based on the reality of non-ergodicity in most dynamic relations in the real world (Davidson, 1982-83). In contrast he sees bounded rationality and the various variabilities arising from nonlinear complex dynamics as being merely epistemological. If only people had really accurate and precise knowledge and forecasting systems, they could overcome these difficulties. Simon's emphasis on knowledge limitations and computational limitations by individuals come under special scrutiny and criticism in this regard. The foundation of bounded rationality (and complexity) is not fundamental uncertainty, but mere inability to compute and know. If only we had supercomputers with superknowledge, all would be well.

There is no ultimate resolution of this debate, although it must be noted that a major source of non-ergodicity within many systems is nonlinearity of the underlying dynamical relationships that leads to complexity. But as is well known in the econometric study of chaotic dynamics, it is profoundly difficult to distinguish deterministic chaotic dynamics from random noise (Dechert, 1996). This debate faces this deep uncertainty of its own.

As it is, while behavioral economics may or may not be the foundation of true Keynesian/Knightian uncertainty,[xiv] Talebian black swans, but it may provide a possible way to deal with policy in a world subject to such uncertainty from whatever source. Thus, while it remains absurdly ignored officially, George Akerlof's (2002) *behavioral macroeconomics* is almost certainly strongly affecting policymakers in practice, even if they do not speak openly of its influence.[xv] Real world central bankers and other macroeconomic policymakers are following heuristic behavioral patterns as recommended by the late Herbert A. Simon, even if few of them will admit to doing so.

Conclusions

The late Herbert A. Simon was widely recognized as being the "father of behavioral economics," who in this regard discovered the ideas of *bounded rationality* and *satisficing*, along with many ideas in many other disciplines such as artificial intelligence, cognitive science, and management. He was also a founder of complexity analysis in its transdisciplinary formulation, in particular hierarchical complexity, with there being many different forms of complexity, with dynamic complexity especially important in economics and deeply connected to nonlinear dynamics. This branch of complexity theory cuts across many disciplines, with perhaps its most serious implications relating to evolutionary processes and the problem of the evolutionary emergence of higher order structures in nature. This goes beyond biology to a broader view of the universe, with such emergent evolutionary processes extending to emergence of atoms from sub-atomic particles to molecules to organic molecules to multi-cellular organisms to human consciousness to societies and to higher order structures beyond those.

A fundamental link between the two concepts is that the existence of complexity provides a foundation for the limits to knowledge and rationality that humans face, and thus why they must operate using bounded rationality. Satisficing is an alternative approach derived from bounded rationality in that one seeks to achieve goals that seem achievable and are socially acceptable rather than striving for all-out optimization. This leads to the use of heuristics such as following authority, imitation, trial and error, and unmotivated search. While these can clearly be useful when facing limits on information and computational ability, overly excessive focus on one of these can lead to problems, as shown by the role of excessive imitation of others in the appearance of damaging speculative bubbles and crashes.

While behavioral economics heuristics may even be able to mimic underlying nonlinear complex dynamics following the theory of consistent expectations equilibria, they also face deep limits to the extent that nonlinear dynamical complexity provides a foundation for fundamental uncertainty that is not quantifiable as is risk. While debates continue regarding this and many other matters involving the relation between behavioral economics and complexity, the emergence of a behavioral macroeconomics and its apparent influence on policymaking is something to be encouraged.

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[i] It must be noted that while Simon received the Swedish Bank Prize in Economic Science in Memory of Alfred Nobel in 1978, usually simply called the "Nobel Prize in Economics," he was not officially an economist in any way. His PhD from the University of Wisconsin-Milwaukee was in political science, and he never was in an economics department during his academic career. At his death in 2001, he was in four different departments at Carnegie-Mellon University, where he had been based since 1949 when it was still the Carnegie Institute of Technology: computer science, psychology, cognitive science, and management, and he had earlier been in the philosophy department as well. This author remembers well from personal communication with him how much Simon disdained conventional economics, and a number of prominent economists expressed public displeasure when he received his prize in 1978.

[ii] These would include at a minimum those that use the words "algorithm," "information," or "code length."

[iii] Velupillai (2011, p. 553) has referred to this form of complexity as "Day-Rosser complexity," even as he strongly advocates the use of more computationally based forms of complexity as being more useful and scientific. For a fuller presentation of Velupillai's perspective on computational complexity, see Velupillai (2000).

[iv] Vladimir Arnol'd (1992) provides a clear and reasoned overview of the mathematical issues involved while avoiding the controversies.

[v] Ralph Abraham (1985) coined the term *chaostrophe* to describe such combinations, although that has not caught on especially. He also coined the term "chaotic hysteresis" (Abraham and Shaw, 1987).

[vi] It is often claimed that Schelling used a chess board, however his board was 19 by 19, which makes it a go board, with go's use of simple black and white stones also fitting the model he developed.

[vii] This is a problem that central planners faced: how much time and in what way should planners spend thinking about how they should plan? This problem was discussed in the French and Russian literature on planning, with the French applying the word *planification* to this process of "planning how to plan," although that word was also used for both planning in general as well as for the more specific question of dealing with the problem of aggregating micro level plans into a coherent macro whole (Rosser and Rosser, 2004, p. 10).

[viii] This result is especially significant in that Vernon Smith (1962) has long been an advocate of the idea that free markets work well and lead to rapid convergence in properly structured markets such as double auction arrangements. This insight was a major basis for his receipt of the Nobel Prize in economics, although he clearly understands that markets can behave badly under certain circumstances.

[ix] See Rosser (2010) for a discussion of relations between *multidisciplinary, interdisciplinary, and transdisciplinary* viewpoints. For a discussion of variations on hierarchical relations see Rosser, Folke, Günther, Isomäki, Perrings, and Puu (1994).

[x] This tradition derived from J.S. Mill's (1843) *heteropathic laws* that focused on basic chemical interactions where two molecules come together to form a completely different molecule. Lewis (1875) coined the term *emergence* for such phenomena, with C. Lloyd Morgan (1923) representing its culmination in biological evolutionary theory. In the 1930s this approach would be pushed aside by the neo-Darwinian synthesis (Rosser, 2011), which emphasized a reductionist approach to the gene.

[xi] We note also that when supply is backward-bending the fishery is also subject to catastrophic collapses as demand smoothly increases, an outcome first argued by Copes (1970).

[xii] In contrast with post-Walrasian economics (Colander, 2006), Post Keynesian economics (sometimes spelled "post-Keynesian" and other variations) tends to admire the work of Keynes, although to varying degrees among Post Keynesians. See Harcourt and Kreisler, 2013a, 2013b for a discussion of the various schools of Post Keynesian economics.

[xiii] Of course there is no definitive separating these cases out. Even the Gaussian allows for extreme outcomes, if less frequently than some rivals that allow for kurtosis. In Tom Stoppard's *Rosencrantz and Guildenstern are Dead* (1967), the opening sequence has the ultimately doomed lead characters arguing about flipping coins when one of them keeps flipping heads "against all odds" successfully 92 times in a row, even as such an outcome is allowed by the probability distribution, and for each subsequent toss the probability is just one half, with even Keynes agreeing on the fact that a "fair coin" will toss tails with a probability of one half. After all, as he noted, insurance companies do generally make profits betting on identifiable and measurable probability distributions, for all his advocacy of fundamental uncertainty.

[xiv] Knighitan uncertainty has had a major influence on Austrian economics views of this, with Kirzner (1985) following up on Knight's argument that uncertainty is a foundation for entrepreneurship. For the classic Austrian view of complexity, see Hayek (1967).

[xv] This author will state that from personal knowledge that Akerlof and those near him are well informed regarding the relations between complexity and behavioral economics as presented in this paper, if not in all the precise details.

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