**A CONCEPTUAL HISTORY OF ECONOMIC DYNAMICS**

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**Introduction**

 “Economic dynamics” is a large topic. In the space allotted here it is impossible to fully cover it. So, I shall begin by clarifying what will not be covered in this essay. While the focus will be more on macroeconomic than microeconomic dynamics, long run growth will not be a main topic, although it will enter in as it is connected to economic fluctuations.

Also, there will be less focus on models of dynamics that rely mainly upon exogenous shocks as their main driver, sometimes argued to represent classical approaches. It must be recognized that much of current modeling in macroeconomics follows such an approach.[[1]](#footnote-1) Business cycles arise from shocks to productivity or to the desire to work on the part of labor. Little effort is made to model these shocks, and the result is that such models have performed poorly in explaining such events as the crisis of 2008 and the events following it. However, it must be recognized that the wisecrack of William “Buz” Brock that the only truly exogenous force in the economy is the sun contains considerable truth.

All this means that we shall mostly be concerned with fluctuations that arise endogenously from an economy, or with models that exhibit such endogenously driven fluctuations, even as there may be stochastic noise ultimately driving the system. Though we shall not be focusing on growth per se, we shall consider fluctuations that may occur over longer periods of time than short period business cycles, which may indeed involve supply side processes and effects.

Another issue that will not be focused on is that of microfoundations of macroeconomics. This is an unimportant issue, but models that arise from microfoundations or directly from the macro level will not be distinguished. Probably most macro fluctuations either resemble or arise from micro phenomena, with such examples as lags in production leading to cobweb dynamics or speculative bubbles that usually initially appear in particular markets but can spread to affect an entire economy. However, there are sources of macro fluctuations that arise largely at the macro level directly, particularly related to monetary and financial markets as well as fiscal policies, even though this essay will not focus on policy issues either.[[2]](#footnote-2)

**The Early History of Macroeconomic Fluctuation Models**

 The history of debates about the sources of macroeconomic fluctuations began with the first appearance of actual fluctuations within the newly industrializing economies after the Napoleonic Wars. There had always been economic fluctuations, but before the industrial revolution they were driven by agricultural production fluctuations that were in turn clearly driven by exogenous forces, particularly those related to climate. Although these were sometimes seen as having elements of periodicity, such as the Jevons (1878) sunspot theory or the much earlier Joseph story from Genesis in the Bible about the seven fat years followed by the seven lean years. What was missing in these earlier phenomena was any sort of endogeneity of the cycles that might be there or not. The idea that such might be the case arose with the fluctuations of the industrializing economies where capital investment would play the leading role.

 There had been commercial crises prior to the post-Napoleonic wars recession, but they had not led to major disruptions of employment, nor did they lead any economists to discuss where they came from, although the discussion of speculative bubbles in particular had gotten the attention of such figures as Cantillon (1755) and Adam Smith (1776, pp. 703-704) due to the large-scale and dramatic nature of the linked Mississippi and South Sea bubbles of 1719-1720. But these discussions focused more on the stupidity of those participating in such events than in any underlying processes or broader economic repercussions, although Cantillon (1755, Chap. 10) particularly emphasized the need for bullionism in order to avoid speculation. As it was, the post-Napoleonic wars years after 1815 saw considerable disruptions and the appearance of unemployment in several core nations involved, particularly Britain and France. These events triggered a debate between Malthus and Ricardo over its causes that also involved such figures as Sismondi and Say on the sidelines.

 Ricardo (1817, p. 265) argued that the disruptions were temporary adjustments arising from exogenous shocks, in particular, the beginning or ending of major wars that engender a microeconomic intersectoral misallocation. Essentially given the time needed to make adjustments, this can lead to a “distress in trade.”

“It changes in a great degree the nature of employment to which the respective capitals of countries were before devoted; and during the interval while they are settling in the situations which new incentives have made most beneficial, much fixed capital unemployed, perhaps wholly lost, and labourers are without full employment.”[[3]](#footnote-3)

While this is the foundation for modern classical views, it is also perhaps more in the spirit of the Austrian school, which argues that fluctuations in interest rates lead to intersectoral misallocations. In any case, the disruption is temporary, and the system should adjust on its own in some reasonable time without any tendency to a longer term general glut due to demand being insufficient to supply. On this latter point, the “law of markets” identified with J.B. Say (1803)[[4]](#footnote-4) would be invoked throughout the 19th century, even if the discussion by Keynes (1936) of “Say’s Law” and its role can be argued to be somewhat misguided.

 On the other hand, Malthus argued for the possibility of general gluts due to an insufficient aggregate demand, ultimately pointing to income inequality as a source of the underconsumption.

“Commodities would be everywhere cheap. Capital would be seeking employment, but would not easily find it; and the profits of stock would be low. There would be no pressing and immediate demand for commodities.” (Malthus, 1836, p. 415)

Ricardo and Malthus would go back and forth on this in their correspondence, but other elements came into the discussion.

In particular Sismondi (1819) became the first to hint at the possibility of endogenous cycles, or to be more precise, periodic crises that were linked, with one laying the groundwork for the next. His argument had many similarities to that of Malthus, particularly emphasizing the role of income inequality more than Malthus, even coining the term “class struggle” in this emphasis, but he made more of an effort to build a broader model with these general gluts chronically appearing without the need for some exogenous shock such as the beginning or end of a war.

Probably the most thorough codifier of the Ricardian defense against this Malthus-Sismondi critique was John Stuart Mill (1871). More than Ricardo he invoked Say, but he also allowed for temporary disruptions that could lead to temporary unemployment and output decline. He brought in other factors that could serve as sources of exogenous shocks besides war, such as bad crops or obstructions to imports. The major element he added to Ricardo was to emphasize the role of the financial sector and to see speculative bubbles as bringing about the disruption of the financial sector when they crash, thus leading to a fall in real investment that reduces output and employment termporarily. Many would say that speculative bubbles contain a strongly endogenous component, and once started a particular speculative bubble will clearly feed upon itself. However, Mill retained his classical perspective by emphasizing that the speculative bubble would be triggered by some initial shock to supply that would push prices up, this then triggering the bubble dynamic (Mill, 1871, p. 526). After the crisis and crash, it would be simply a matter of time for the financial sector to reorganize and revive for things to return to normal. There would be no need for a “diminution of supply” but rather for “the restoration of confidence” (Mill, 1871, p. 561). Each such event stands on its own with no cyclical aspect linking it to others.

Curiously, while Malthus did not pose a cyclical model in his response to the events after the Napoleonic wars, he had earlier posed the possibility of longer run fluctuations tied to relations between population growth and the broader economy, although with these fluctuations possibly being erratic and combining both exogenous and endogenous elements. Thus, from the first edition of his *Essay on the Principle of Population* (1798, pp. 33-34) we get:

“Such a history would tend greatly to elucidate the manner in which the constant check upon population acts; and would probably prove the existence of the retrograde and progressive movements that have been mentioned; though the times of their vibration must necessarily be rendered irregular, from the operation of many interrupting causes; such as, the introduction or failure of certain manufactures; a greater or less prevalent spirit of agricultural enterprise; years of plenty, or years of scarcity; wars and pestilence; poor laws; the invention of process for shortening labour without the proportional extension of the market for the commodity; and particularly the difference between the nominal and the real price of labour; a circumstance, which has perhaps more than any other, contributed to conceal this oscillation from common view.”[[5]](#footnote-5)

 One who drew on elements from both Mill and Malthus, although by way of Sismondi for the latter given his intense dislike of Malthus, was Marx, particularly in Volume III of *Capital* (1894) and also in Part II of his *Theories of Surplus Value* (1969). While Volume I laid out a vision of collapse of capitalism as a whole leading to revolution and socialism, Volume II concentrated more on equilibrium models of steady state or expanded reproduction in which the consumption and capital goods sectors remain in balance. It was in Volume III where Marx concerned himself more with the matter of the periodic outbreak of crises and the resulting pattern of repeated fluctuations. He expressed doubt that one ever sees equilibrium in micro markets except accidentally for a second as a market moves from excess demand to excess supply and back again. Malthus was muddled in his formulation of the general glut, nevertheless, the problem of “surplus realization” periodically arose, and he dismissed Mill’s dismissal of the possibility of general gluts. Like Mill, he was aware of the problem of speculative bubbles appearing in capitalist financial markets and how they could lead to “regular and periodic” crises (Marx, 1969, p. 500). Ultimately the problem would be that “the demand for the general commodity, money, exchange-value, is greater than the demand for all particular commodities” (Marx, 1969, p. 505). A particular idea that he first developed that has reappeared since in many models of macro fluctuations on down even to modern real business cycle models is that of the echo boom, wherein a wave of investment at one time leads to the wearing out of that capital stock at the same time that then engenders another wave of replacement investment (Kydland and Prescott, 1982).

**Haberler’s Taxonomy of Business Cycle Models in the Early 20th Century**

 The appearance of depressions in the 1870s and 1890s (with some arguing for the entire period to have been a depression), along with more crises in the early 20th century and finally the Great Depression led to many further developments in the theory of how macroeconomic fluctuations could occur. One year after Keynes published his most important work attempting to explain the Great Depression as arising from failures of aggregate demand, Gottfried Haberler (1937) published the first edition of his *Prosperity and Depression*. Whereas Keynes sought to lay out a specific theory that incorporated a number of ideas that had been developed earlier by others, such as the multiplier effect (Johansen, 1903; Kahn, 1931), Haberler reviewed broad schools and views that had been developing since the end of the 19th century, some of them also adopted by Keynes, but some of them disagreeing with him to various degrees.

 The theories reviewed by Haberler are the purely monetary theory, the over-investment theories (three of them), changes in cost or horizontal maladjustments or over-indebtedness, underconsumption, psychological theories, and harvest theories (which largely amounts to a discussion of the already mentioned sunspot theory of Jevons, 1878). Of these he spends the most time on the three over-investment theories. It must be noted also that not all of these theories are mutually exclusive and that in any particular fluctuation the effects described by more than one of them may be simultaneously at work.

For Haberler, the leading exponent of the purely monetary theory was R.G. Hawtrey (1913, 1932) who relies upon the quantity equation of exchange of Irving Fisher (1911)[[6]](#footnote-6), but who more strongly emphasizes the role of credit and interest rates and their impact on merchants. He believed that there was instability in the credit system that led to cumulative movements away from equilibrium in either direction, accompanied by either deflation or inflation. Indeed, in his earliest writing he recognized the possibility of a complete freezing of credit in a sufficiently deflationary situation, in which monetary policy will be ineffective to stimulate the economy, rather resembling the liquidity trap of Keynes. Later the greatest champion of Hawtrey’s approach would be Milton Friedman ( 1956; Friedman and Schwartz, 1963). While Friedman agreed with Hawtrey about shorter term impacts of money on the economy, he also strongly emphasized the longer run neutrality of money associated with the monetarist approach.

The leaders of the monetary theory of over-investment tended to be of the Austrian School, notably von Mises (1928) and Hayek (1933), although they drew on the work of the Swedish economist Wicksell (1898).[[7]](#footnote-7) While there is overlap with the monetarist theory, there is even greater emphasis on interest rates and their control by central banks. So, Wicksell posed the idea of the natural rate of interest. If interest rates were held below this rate, then there would be over-investment, particularly of longer time horizon projects, with this being followed by interest rates above the natural rate, which would lead to a crisis and decline of output while there was a shutting down of many now unprofitable projects.

The non-monetary over-investment theory’s leading advocate was Spiethoff (1902), who saw investment in fixed capital respond to a rise in demand for final goods, but that the boom that develops runs into an outright shortage of capital goods as that sector hits capacity limits. There is an element of a production lag element to this such as one finds in the cobweb theorem dynamics of microeconomics (Cheysson, 1887; Ezekiel, 1938) with the crucial lag being in capital investment in the capital goods sector. However, this theory has roots in some of Marx’s arguments, and his follower Tugan-Baranowski (1901) argued for a periodicity and regularity to such a cycle, likening it to a steam engine.

The third over-investment theory depends on the acceleration principle that has investment responding to changes in consumer final goods demands, but overshooting the increase in capacity. Among those developing and applying this would be Carver (1903), Aftalion (1913), J.M. Clark (1917), and Harrod (1936). The related multiplier concept due to Johansen (1903)[[8]](#footnote-8) and Kahn (1931) would be combined in the form of multiplier-accelerator models to show mathematically complete models of periodic and endogenous macroeconomic fluctuations (Samuelson, 1939a).[[9]](#footnote-9)

 The horizontal maladjustment theory, or “error theory,” much resembles the explanation by Ricardo of what happened after the Napoleonic Wars. As argued by Mitchell (1924), this involved over-investment in particular sectors, and did not necessarily lead to general decline, although it could.

 Irving Fisher (1933) developed the debt-deflation theory, which has influenced Minsky (1986) and Bernanke, Gertler, and Gilchrist (1996) since. Fisher did not support any ideas of periodicity of cycles, but saw each as an individual event. A deflation could expand the real value of debt, which in turn could drive down output as firms fail due to their rising real indebtedness. For later writers this would become the “financial accelerator.”

 We have already encountered the under-consumption theory earlier from the work of Malthus and Sismondi. A later more detailed presentation was due to Hobson (1909), who placed special emphasis on the role of income inequality leading to the poor being unable to buy consumer goods. Haberler notes that at least conceptually this theory is similar to the over-investment theory, although the emphasis in the latter tends to be on the behavior of the capitalists making overly large investments, whereas the under-consumption view focuses on the would-be consumers who do not buy enough to justify the investments that have taken place.

 Finally we have the psychological theory. Many observers mention psychological factors, including Spiethoff and Mitchell. But the main developers of this theory were Pigou (1927) and Keynes (1936). This theory more directly confronts the issue of expectations formation, with waves of mania and depression driving investment up and down, Keynes adopting the term “animal spirits” for these oscillations of mood. This theory also plays an enormous role in the work of Minsky (1986), who sees the swelling of optimism in an initially favorable equilibrium situation to destabilize it as a bubble develops leading to a crash. Kindleberger (1978) and Shiller (2005) have been followers of this approach, which easily fits in with some of the other theories, particularly the over-investment ones. An unresolved issue with this approach is the degree to which the psychological factors are exogenous or endogenous, with them conceivably exhibiting both characteristics. The modern theory of psychological variations being driven by arbitrary exogenous effects that themselves do not directly affect production are the sunspot theories of dynamic fluctuations (Azariadis, 1981; Cass and Shell, 1983), which contrast to the Jevons theory of direct influence on agricultural production from sunspot cycles.

**The Question of Periodicity**

 While many models of fluctuation patterns suggest possible mechanisms for endogenously driven repetition, if only through “echo effect” bunching of investments in similarly depreciating capital stocks, probably a majority of economists are skeptical regarding regular periodicities occurring with respect to most macroeconomic fluctuations. The general problem is that even when such endogenous cycling happens, it is often argued that there are sufficient variations from recession to recession of the mechanisms involved that strictly regular periodicities are unlikely or are likely to break down or change over time if there is a tendency for them to exist. Thus, it may be that previously observable inventory cycles, usually viewed as the shortest of periodic cycles, may have largely disappeared from more advanced economies in recent decades due to improved, “just-in-time,” inventory management systems tied to computerization and improved information transmission. However, there has long been a fascination with the possibility of periodic cycles, with different periodicities being proposed and studied.

 Although he would be followed soon by Jevons (1878) with his exogenously driven but regular sunspot cycles working through agriculture, the first to identify a possible regular periodic macroeconomic cycle was Juglar (1862). He argued that these were driven by cycles of fixed capital investment along echo boom lines, replacement waves of investment coinciding with a periodicity of about 7-11 years, depending on time period and place. Kitchin (1923) would label this the “major cycle” and posited a shorter 40 month cycle that he called the “minor cycle.” He found this looking at commodity prices, interest rates, and bank clearings, and explained it as due to mass psychology, and argued that the major cycle was simply an aggregation of two or three minor cycles. Later observers would argue that inventory fluctuations due to cobweb-like lags[[10]](#footnote-10) were more likely to be responsible (Metzler, 1941), with these being examples of the acceleration principle at work.

 Next up in periodicity is the “Long Swing,” first argued for by Simon Kuznets (1930a), with a period of 15-17 years, possibly twice the length of a Juglar major cycle. Kuznets focused on demographics and residential construction as the main sources of this hypothetical cycle, although infrastructure investments have been suggested as well.

 Probably the most controversial of hypothesized cycles is the “Long Wave,” usually associated with Nikolai D. Kondratieff (1926), although he was preceded by the obscure Dutchman, van Gelderen (1913), with others making vague arguments along such lines even earlier (van Duijn, 1983). Kondratieff would be executed by the Soviet government, at least partly for his work on this idea. The cycle was argued to be about 50 years in length, and Kondratieff focused on prices and interest rates like some others in looking for cycles.[[11]](#footnote-11) From the beginning there was debate about what might be a driving mechanism, with many arguing that even if it was there in the data it was an artifact with no clear basis for a consistent regularity (Kuznets, 1940). Some argued that it was a monetary phenomenon, with the major discoveries of gold in California and Alaska a half century apart responsible for it, or even a sort of long period accelerator tied to investment waves in the capital goods sector responding slowly to those in the consumer goods sector (Forrester, 1977). But the major argument put forward by Kondratieff and strongly supported by Schumpeter (1939) and most later discussants of the cycle (Freeman, 1996) has been technological change, particularly the idea that major innovations bunch together during major downturns. Schumpeter (1939) went further and argued that the Kondratieff was constructed out of the two lower cycles, the Kitchin and the Juglar (ignoring the Kuznets Long Swing), although few have supported this view even among those who argue for the reality of the Kondratieff. Many have spent much effort identifying particular innovations associated with particular Kondratieffs, such as cotton textiles and steam power during a 1787-1800 boom, followed by a railroadization boom in 1843-1857, and an electricity and automobile boom in 1898-1911 (Kuznets, 194, p. 261).

 Unsurprisingly, the empirical debate on this has been lengthy itself without a clear conclusion. Typical is the exchange between Solomou (1986) and Bieshaar and Kleinknecht (1986) with many issues involved in this. One of the most recent efforts to consider these matters has been by Korotayev and Tsirei (2010) who test for all four of the hypothesized cycles mentioned above at the level of global GDP. They claim to find both the Juglar and the Kondratieff, but neither the Kitchin nor the Kuznets.[[12]](#footnote-12)

 While they emphasize elements that go beyond economics, and economists have largely ignored their work, others have posited even longer cyclical patterns. Modelski (1987) sees economic developments tied to larger political ones, particular the rise and fall of hegemonic powers at the global level, with the economies of the hegemons rising and falling along with their power. He argues that such a cycle may be around 120 years.

 Even longer is the “longue durée” or “geographical cycle of Fernand Braudel (1949). Associated with the Annales school of history, he sees deep cycles driven by demographic forces in relation to the geographical environment that work themselves out on a scale of 300-400 years, with collapses in Europe in the 6th, the 10th, the 14th, and the 17th centuries. It can be noted that the dynamics described by Braudel are somewhat similar to those modeled by Day and Walter (1989). It can be noted that if one wishes to be pessimistic about future economic growth in the coming decade or so, some have argued that we could be experiencing a simultaneous downturn of the fifth Kondratieff wave since the Industrial Revolution (Korotayev and Tsirei, 2010) with a downturn of a Braudel longue durée that began its early upturn in the late 17th century, with the pessimists pointing to a possibly substantial slowdown of serious new innovations as the key to this (Cowen, 2011).

**Endogenous Irregular Fluctuations**

 We have already noted that probably the vast majority of economists do not accept that macroeconomic fluctuations exhibit definite periodicities of any length, including many of those who accept that such fluctuations are substantially endogenous in character, with only a few of those stressing exogenous sources seeing any possibility of this, notably Jevons (1878) who saw sunspots as the periodic exogenous driver. However, gradually many came to realize that if crucial relationships, particularly those involving capital investment, are nonlinear, then endogenous fluctuations can arise that may be not only periodically regular, but aperiodically erratic in a variety of ways. Ragupathy and Velupillai (2012) argue that the first to realize this possibility was Hamburger (1930), who was inspired by the physics models of relaxation oscillations of van der Pol (1926).[[13]](#footnote-13)

Hamburger was followed by Le Corbeiller (1933), who in turn personally influenced the crucial work of Goodwin (1951) whose nonlinear accelerator model was shown by Strotz, McAnulty, and Naines (1953) to be capable of generating endogenous erratic fluctuations that could possibly be chaotic. However, probably the first to develop a model capable of generating chaotic dynamics was Palander (1935) whose regional dynamics model exhibited three-period cycles, shown by Sharkovsky (1964) and Li and Yorke (1975) to be sufficient in one-dimensional models to produce chaotic dynamics. While none of these realized at the time what they had shown, Goodwin (1990) would later become a deep student of endogenous chaotic dynamics in nonlinear economic models.[[14]](#footnote-14)

Independently various economists associated with Keynesian economics developed models with nonlinearities that they realized could endogenously generate periodic cycles (Kalecki, 1935; Kaldor, 1940; Metzler, 1941; Hicks, 1950[[15]](#footnote-15)), with later economists realizing these models could generate various forms of complex aperiodic dynamics. Variations on the Kaldor model, particularly by Chang and Smyth (1971) have been shown to exhibit a wide variety of complex dynamics, including catastrophic[[16]](#footnote-16) discontinuities (Varian, 1979), chaotic dynamics (Dana and Malgrange, 1984), and transient chaos as well as fractal basin boundaries with multiple attractors (Lorenz, 1992). Key to the Kaldor model dynamics is a nonlinear sigmoid function relating output with investment, with this investment function shifting over the course of a business cycle as the capital stock varies. As the investment function shifts, discontinuous changes in investment happen. This is depicted in Figure 1.



 Figure 1: Shifting Kaldorian Nonlinear Investment Functions

Full recognition of how wide the conditions are under which deterministic chaotic dynamics can happen only came about in the 1980s.[[17]](#footnote-17) Among the first to be explicitly studied in this way were overlapping generations models (OLG) that followed the formulation of Gale (1973). Benhabib and Day (1980) showed that a sufficient degree of substitutability in the intergenerational offer curves can lead to endogenous chaotic dynamics, although presumably these would be fluctuations more along Kuznetsian Long Swing time periods rather than shorter term fluctuations. Grandmont (1985) extended this, bringing in risk aversion and interest rates (and thus implicitly potentially monetary policy) to show that chaotic dynamics can arise if older agents have a sufficiently higher marginal propensity to consume leisure than younger agents. Keeping in mind that in one-dimensional systems a three-period cycle is a sufficient condition for the existence of chaotic dynamics, Figure 2 depicts such a cycle where μ represents real balances today and x(μ) real balances tomorrow.



Figure 2: Chaotic Grandmont Monetary Dynamics

Chaotic dynamics have been shown to be possible in neoclassical models of agents with multiple sectors and infinitely lived agents, hence not just OLG models where things can happen (such as rational speculative bubbles) that do not in representative agent models. Day (1982) provides one such example that involves an upper limit on the capital-labor ratio, suggested by him to be due to a pollution limit. This leads to a standard Solow growth model turning into one with a logistic function determining the dynamics of the sort shown to exhibit chaotic dynamics by Robert May (1976). This model by Day can serve as a driver for a model of much greater financial volatility through the mechanism of “flare attractors” (Rosser, Ahmed, and Hartmann, 2003) as first posed by Rössler and Harmann (1995).

Another take on this without any “ad hoc” variation from the neoclassical approach is to find that with a sufficiently high discount rate, “every possible” sort of behavior can happen in a conventional growth model (Boldrin and Montrucchio, 1986; Mitra, 1996; Nishimura and Yano, 1996; Khan and Piazza, 2011). We note that these models all assume full optimization as well as perfect knowledge by an infinitely lived agent. However, generally the discount rates at which chaos appear are very high, although lower as the number of factors of production increases.

Returning to the world of more Post Keynesian models without representative infinitely lived fully informed and rational agents, more complex dynamics have been observed in other variations of models. Goodwin (1967) proposed a model of endogenous regular business cycles driven by class struggle dynamics using Lotka-Volterra predator-prey equations, with the workers ironically performing the role of the “predators” in the model as their wages fluctuate over the course of the cycle. Pohjola (1981) showed that a discrete version of this model could produce chaotic dynamics and Soliman (1996) shows that it can also lead to fractal basin boundaries between multiple basins of attraction.

Goodwin’s (1951) version of the nonlinear accelerator has also been shown capable of being modified to produce dynamics that exhibit both catastrophic discontinuities along with chaotic dynamics in a phenomenon dubbed “chaotic hysteresis”[[18]](#footnote-18) (Puu, 1997). More particularly Puu (1997, Chap. 8) posits that the investment function be non-monotonic rather than merely nonlinear. An example of chaotic hysteretic dynamics can be seen in Figure 3 that holds for certain parameter values of the Puu model, with the horizontal axis being output and the vertical one being the change of output.



 Figure 3: Chaotic Hysteresis in a Puu Nonlinear Accelerator Model

Another route by which complex endogenous dynamics can arise in economic models is through the self-organized criticality phenomenon (Bak, 1996). Bak, Chen, Scheinkman, and Woodford (1993) present such a model, which depends on a lattice structure of multiple pathways and stages of production in a sandpile model. This can allow a Gaussian distribution of exogenous shocks (sand dropping on the sandpile) to generate non-Gaussian distributions of final outcomes.

Unsurprisingly adding a financial sector to many of these models simply increases the likelihood that one can observe some form of complex dynamics. Among those finding chaotic dynamics include Foley (1987), Woodford (1989), and Delli Gatti, Gallegati, and Gardini (1993). A variation on these models involves the role of speculative bubbles in the financial sector, which harks back to our earlier discussions of the role that such can play in macroeconomic dynamics going back to Mill and even to the origins of economic dynamics discussions with Cantillon. This links to more modern Post Keynesian concerns such as those involving financial fragility following the work of Minsky (1986). That bubbles might be chaotic was first shown by Day and Huang (1990), and Keen (1995) has shown this in the context of a model more directly based on Minsky.

**Where Do We Stand Now?**

 The financial market crash of 2008 and the subsequent Great Recession has substantially altered the discussion of economic dynamics. The long Great Moderation beginning in the mid-1980s (at least for a set of the higher income market capitalist economies) had underpinned the rise to dominance of the use of DSGE models based on rational expectations and real business cycle approaches emphasizing stochastic exogenous shocks (Woodford, 2003). These models would be modified somewhat by the introduction of various frictions such as sticky prices to lead to New Keynesian variants that came to be widely used (Smet and Wouters, 2002). However, the complete failure of these models to either model or to forecast showed up their severe limitations. While there is no clear successor (and sufficiently modified versions of these may well yet continue to dominate policy discussions), several alternatives drawing on some of the traditions discussed above have emerged as possible contenders for replacements. We note two of these to conclude this essay. Both of these clearly involve great emphasis on the financial sector, generally with an emphasis on Minskyan style potential fragility.[[19]](#footnote-19)

 One of these involves drawing together elements from the models of Marx, Keynes, Metzler, Goodwin, and occasionally even Schumpeter. This has been done principally by economists either based at or visiting Bielefeld University over the last couple of decades, with a series of articles and substantial books, a group that can arguably be called the “Bielefeld School.” It is not simple to characterize these models, but their elements do include emphasis on financial sector dynamics in connection with the real economy, lags and inventory dynamics, nonlinear accelerators affecting investment decisions, along with concerns about distributional effects, with their models able to show the full array of complex dynamics discussed above. An incomplete list of works includes Asada, Chiarella, Flaschel, and Franke (2003), Chiarella, Flaschel, and Franke (2005), Flaschel (2009), and Semmler and Bernard (2012).

 The other approach more specifically tries to model heterogeneous agents from the ground up and their interactions. One branch of this has followed on a more explicitly econophysics approach focusing on statistical mechanical stochastic processes in the presence of nonlineariities (Aoki, 1996). The other has moved to more explicitly model individual parts of the economy in an effort to build up a microfoundation to obtain macroeconomic results, although without tying that microfoundation to a rational or optimizing model of agents in the DSGE tradition (Delli Gatti, Gaffeo, Gallegati, Giulioni, and Palestrina, 2008). While there have been efforts to introduce heterogeneity into DSGE models (Krusell and Smith, 1998), these have usually taken the form of assuming a distribution of characteristics over an interval, with the interval then in effect behaving as a homogeneous agent. The agent-based approach attempts to more specifically model the interactions of the agents directly to find how extreme outcomes can endogenously arise from within an economy.[[20]](#footnote-20)

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1. Frisch (1933) provided the archetype of a formal model that is driven by random exogenous shocks that lead to stabilizing adjustment following the “rocking horse” principle. Later such models have relied on the rational expectations assumption (Lucas and Sargent, 1981; Kydland and Prescott, 1982; Long and Plosser, 1983), although Sargent (1993) would later abandon reliance on the strict rational expectations assumption. Zambelli (1992) critiques the original Frisch model. [↑](#footnote-ref-1)
2. This essay in terms of overall perspectives draws broadly on several overview works, particularly Haberler (1963), Freeman (1996), Rosser (1999a, 1999b, 2000, 2004, 2011), Gandolfo (2009), Ragupathy and Velupillai (2012). [↑](#footnote-ref-2)
3. Bhaduri and Harris (1987) have shown that the adjustment path to a Ricardian equilibrium may be chaotic. [↑](#footnote-ref-3)
4. Ironically, Say himself recognized that his “law” did not always hold, with a glut possibly arising from individuals hoarding, as in the Ottoman Empire where people did not wish to draw attention to themselves by the tax authorities through displays of conspicuous consumption, and financial markets were not so well developed that savings could be channeled easily to real capital investment. Say also supported public works as a solution to the problems of the post-Napoleonic wars recession, in this agreeing with Malthus and Sismondi over Ricardo. [↑](#footnote-ref-4)
5. Day (1983) and Day and Walter (1989) have shown how a Malthusian style demo-economic model can generate chaotic dynamic fluctuations over long periods of time. [↑](#footnote-ref-5)
6. While Fisher’s formulation is the most widely used, Humphrey (1984) documents that forms of this equation were written from the late 1600s, with that of Lubbock (1840) arguably more sophisticated than Fisher’s in allowing different velocities for different components of the money supply. [↑](#footnote-ref-6)
7. Ironically Wicksell would inspire the Swedish sequence analysis school generally regarded as providing an independent version of the Keynesian approach (Lundberg, 1937). [↑](#footnote-ref-7)
8. Johansen authored this work under the pseudonym, “J.J.O. Lahn.” [↑](#footnote-ref-8)
9. The less well-known Samuelson (1939b) posed a nonlinear consumption function, flattening out for higher income levels consistent with data, which made this model able to generate fluctuations of irregular periodicity. Ezekiel (1938) was aware that with nonlinear supply and demand curves his cobweb dynamics could become irregular. [↑](#footnote-ref-9)
10. While much doubt has been raised about the empirical validity of the various macro cycles, Ezekiel’s observation of a 5-6 year corn-hog cycle and a 14-17 year cattle cycle has been strongly supported for the latter case for the period 1875-1990 (Rosen, Murphy, and Scheinkman, 1994). Ezekiel posited the possibility of erratic cobweb cycles, and the first to show the possibility of chaotic cobweb cycles was Artstein (1983). Chavas and Holt (1993) claim to have found chaotic patterns in milk price dynamics over time. [↑](#footnote-ref-10)
11. One reason such variables were looked at rather than GDP was that data for them was more available in the 19th century, with the concept of the GDP not even being clearly invented until the 1930s, much less accurate data being gathered for it. [↑](#footnote-ref-11)
12. It is somewhat ironic that much of the study of Kondratieff waves has been by the Dutch, following on van Gelderen, and since the fall of the Soviet Union by Russians, at least possibly out of an effort to revive the reputation of this victim of Stalin’s purges. [↑](#footnote-ref-12)
13. Curiously, van der Pol and van der Mark (1927, p. 364) may have been the first to actually observe chaotic dynamics in the form of “an irregular noise heard in the telephone receivers…[that] strongly reminds one of the tunes of a bagpipe”as they adjusted frequency ratios in telephone receivers. But they did not understand mathematically what they heard, despite earlier development of the mathematical theory by Poincaré (1908). [↑](#footnote-ref-13)
14. Chaotic dynamics in particular exhibit sensitive dependence on initial conditions, also known as the “butterfly effect,” in which small changes in either a control parameter value or a starting point lead to large changes in the dynamic pattern. For more on chaotic dynamics, see Rosser (2000, Chap. 2). [↑](#footnote-ref-14)
15. The Hicks model is one of a nonlinear accelerator somewhat like that of Goodwin (1951), based on the idea of floors and ceilings for investment. Blatt (1983) was the first to suggest that it could generate chaotic dynamics, an outcome studied more thoroughly by Hommes (1991). [↑](#footnote-ref-15)
16. Although it has much earlier roots, catastrophe theory was substantially developed by René Thom (1972). The first economics model using it was to explain stock market crashes (Zeeman, 1974). See Rosser (2000, Chap. 2) for the mathematics of catastrophe theory and Rosser (2007) for applications in economics. [↑](#footnote-ref-16)
17. For a more comprehensive review of these models than will be done here, see Rosser (2000). [↑](#footnote-ref-17)
18. The term “chaotic hysteresis” is due to Abraham and Shaw (1987). [↑](#footnote-ref-18)
19. Some have argued for the use of Austrian business cycle models as another alternative. While the overbuilding of housing during the prior bubble suggests that their emphasis on intersectoral misallocation may have some applicability, the repeated predictions by advocates of an outbreak of hyperinflation that has not happened undermines the usefulness of their models in the current circumstances. Garrison (2001) provides a high quality discussion of the Austrian approach to modeling macroeconomic dynamics. [↑](#footnote-ref-19)
20. An example showing more subtle dynamics described for financial markets by Minsky is Gallegati, Palestrina, and Rosser (2011). [↑](#footnote-ref-20)