**REFLECTIONS ON REFLEXIVITY AND COMPLEXITY**

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Abstract: This essay considers the relationship between reflexivity and complexity. Reflexivity has a considerable history as an idea in the social sciences, with many specific meanings and applications, although it generally has involved a mutual interaction between at least two separate agents or groups. Complexity also has many meanings, although often these involve some higher level emergence, the idea of wholes being greater than the sum of their parts. It has been argued by some in economics especially that there may be a relationship between these two as the dynamic interactions in reflexive systems may be more likely to bring about forms of complex emergence. The ideas of John B. Davis on this will be especially considered, but those of others will be examined as well, including some of those more critical of the usefulness of these concepts. A new idea put forth in this paper is that some forms of reflexivity may be more conducive to bringing about patterns of complex emergence than others. This may involve more subtle interactions of indirect self-referencing through reflexive system such as those that underlay proofs of incompleteness. An artistic analogy can be seen in the work of M.C. Escher, with many writing about reflexivity citing his “Drawing Hands” (Figure 2) as an example, which depicts two hands drawing each other. But this may show the sort of reflexivity that is not so associated with complexity. Rather another may do so better, Escher’s “Picture Gallery” (Figure 3) that shows a man standing in a picture gallery and looking at a picture of a city that contains a picture gallery that turns out to be the one in which he is standing.

**Introduction**

There is no single agreed upon definition of reflexivity within the social sciences, and some meanings of the word do not fit with how it has generally been used in social science discussions. Thus it can mean some sort of automatic “reflex” response to something, as in a knee jerking when tapped. However, more generally the term has been used in ways tied more to the word “reflection,” as in an image being reflected back and forth, possibly many times. The idea has involved agents interacting with an other (or others) that then reflects back onto them in some way. While many have seen the idea operating earlier, with some pointing to Marx and his class struggle system as being an example of a reflexive system, probably the first to use the term *reflexivity* in social science was sociologist, Robert K. Merton (1938). A leader of the functionalist sociology movement, he was concerned with how sociologists think about sociology and how their own roles and positions affect that. Merton would foreshadow ideas showing up later in economics, such as the idea of self-fulfilling prophecies (1948). But much of his focus was on what would become the scientific study of knowledge, a methodological movement that provided an early use of the idea in economics by D. Wade Hands (1994a.b; 1998). However, prior to economists getting into studying reflexivity, sociologists would consider a wide variety of meanings and applications of the term, as well as other social scientists and philosophers getting involved.

 Prior to Hands, successful financier and self-styled “failed philosopher” George Soros (1987) applied the term to “the alchemy” of financial markets, effectively highlighting Merton’s idea of the self-fulfilling prophecy to promulgate how agents respond to each other in markets to bring about various dynamics including bubbles and crashes, with his work attracting much greater attention following the 2008 financial crash (Soros, 2013). While Soros recognized the priority of Merton, his more important influences were Karl Popper (1959) in philosophy whose student he was, as well as the financial arguments of J.M. Keynes, especially from Chapter 12 of *The General Theory* (1936), where the beauty contest idea both implies the possibility of self-fulfilling prophecies and is certainly an early example of reflexivity thinking, even if Keynes did not use the term.[[1]](#footnote-1) As it was, Popper was almost a negative influence in the sense that his emphasis on falsifiability and fallibility led Soros to question standard rationalistic economic theory. In any case for Soros, the crucial element of reflexivity is not just the reflecting off others but the eventual self-referencing that occurs as one considers also their reflecting off oneself, with Soros recognizing the link of this idea to old paradoxes in mathematical logic.

 Economic philosopher John Bryan Davis initially began discussing reflexivity along the lines of Hands as more of a methodological issue related to the scientific study of knowledge within economics (Davis and Klaes, 2003). He posed a hierarchy of forms of reflexivity, with some sort of self-referencing at the core, but with other perspectives then being added on. However, in the wake of the financial crash and the attention paid to the arguments of Soros, Davis (2013, forthcoming a,b) expanded his notion and brought it in line with other discussions and concerns he had been dealing with previously, including the philosophical foundations of Keynes’s work as well as the nature of evolutionary economics as conceived by Veblen and Hodgson. It was from this stew that Davis began linking reflexivity to *complexity*.

 As with reflexivity, there is no universally accepted single definition of complexity. Indeed, there are groups of types of complexity, with computational, dynamic, and hierarchical among those most appearing in economics. Themes cutting across these include the idea of wholes being greater than the sums of their parts. Complex systems often involve emergence with higher of levels implied by or appearing out of lower levels. A centerpiece of thinking about the link between reflexivity and complexity is that the self-referencing part of the former may play a role in the phenomenon of *emergence* of higher levels or forms or structures within complex systems.

 A particular theme we shall pursue here is that it may matter how this self-referencing may take place. This will draw on ideas of Douglas Hofstadter (1979, 2006), who has probably influenced sociologists thinking about these matters more than he has economists. Inspired by the role that self-referencing plays in the paradoxes of mathematical logic, which in turn are connected especially to questions of computational complexity, Hofstadter focuses on the nature of the feedback loops that can arise in such systems, with a focus on how these relate to the emergence of consciousness, a problem that concerned Hayek (1952) also. Hofstadter emphasized that indirect, or “strange,” loops involving such reflexive self-referencing may be particularly important, drawing on insights from the Gödel (1931) incompleteness theorem. This then may relate to dynamic complexity as well, with this approach offering a possible way to synthesize the main branches of complexity, often seen as distinct or even in competition with each other, with these strange self-referencing loops possibly making it easier for emergence of higher order structures in dynamically complex systems. Thus the link between reflexivity and complexity may be deep indeed.

 **Forms of Reflexivity Before Economists Got to it**

An excellent overview of what sociologists have had to say about reflexivity, as well as some philosophers, anthropologists, and political scientists, is due to Michael lynch (2000). He identifies more kinds of reflexivity than economists have since discussed, although some of the ideas economists have had about it were foreshadowed at least to some extent by these non-economists. He identifies six broad types of reflexivity, with most of these having sub-categories, all of which together he labels “the Reflexivities.”

 The first broad category he calls *Mechanical reflexivity*, which in turn has three sub-categories: *knee-jerk*, *cybernetic loopiness,* and *reflections ad infinitum*. The second (Bateson, 1972) involves feedback loops in human communication systems that lead to *self-reflection*. He identifies the third specifically with Hofstadter’s work, particularly his famous 1979 book, *Gödel, Escher, Bach: an Eternal Golden Braid*, mentioning such things as halls of mirrors, the Moebius strip, and the famous Maurits Cornelis Escher drawing of “Hands Drawing” (Figure 2), which shows two hands drawing each other with pencils and which has been mentioned and reproduced by many others who discuss reflexivity. We shall discuss this drawing further below.

 The second broad category is *Substantive reflexivity*, which in turn has two sub-categories: *systemic reflexivity* and *reflexive social construction*. The first is more associated with political scientists (Beck et al., 1994) and such ideas as “reflexive modernization” that deals with interactions between public opinion and policy making, especially when this involves experts intervening in this supposedly reflexive process. The second seems more to involve “self-reflection” and emphasizes how consensual beliefs and concerted practices give rise to objective social institutions, including specifically economic markets (Berger and Luckmann, 1966). These socially constructed facts are real (such as prices), but their reality is said to depend upon and be continually sustained by reflexive subscription to that very reality.

 The third broad category is *Methodological reflexivity*, which in turn has four sub-categories: *Philosophical self-reflection*, *Methodological self-consciousness*, *Methodological self-criticism*, and *Methodological self-congratulation*. The first of these Lynch attributes to Descartes (1968 [1637]), whom he identifies as the father of Enlightenment self-knowledge gained by philosophical introspection. Methodological self-consciousness is “mundane” and involves getting students to recognize their own relations to what they study and how these can lead to distorting biases (Hammersly and Atkinson, 1983). Extending this leads to Methodological self-criticism, attributed to Merton (1938) and Popper (1963), Soros’s main teacher and influence. This emphasizes the systematic self-criticism that supposedly real sciences engage in continuously to rid themselves of false or flawed theories. This in turn in the hands of functionalist sociologists of science (Bloor, 1976; Merton, 1978) led to Methodological self-congratulation, where these sociologists praised themselves for following the practices of “mature” sciences, making the sociology of science “self-exemplifying.” Arguably the original studies of reflexivity in economics by Hands (1994a,b; 1998) fit somewhat into this broad category of Methodological reflexivity as do the first discussions of it by Davis and Klaes (2003).

 Lynch’s fourth broad category is *Meta-theoretical reflexivity*, seen broadly as involving the ironic detachment of the intellectual who “steps back” (Mannheim, 1936), and which has three sub-categories: *Reflexive objectification*, *Standpoint reflexivity*, and *Breaking frame*. Reflexive objectification involves trying to be “hyper-objective” in stepping back as advocated by Bourdieu and Wacquant (1992). Standpoint reflexivity involves more rigorous self-criticism of one’s own standpoints in the effort to step back towards objectivity (Harding, 1996). Finally Breaking frame invokes illusionism and theatrical frames to understand social standpoints (Goffman, 1974).

 The fifth broad category is *Interpretative reflexivity*, which has just two sub-categories: *Hermeneutical reflexivity* and *Radical referential reflexivity*. The former emphasizes a circle between texts and their interpretations that expands to theorize “the constitution of society” (Giddens, 1993). The second focuses on *representation* and challenges the possibility of objectification, searching for deep preconceptions, with this supposedly especially important in science and technology studies (Woolgar, 1991). Lynch spends much of his later discussion on this type of reflexivity, viewing it as disruptive and “problematical” in its challenge to more conventional modes of self-reflection and methodological moves to objectivity.

 The final category is *Ethnomethodological reflexivity*, no sub-categories, which was associated with a broader influential movement in sociology beginning in the 1960s (Garfinkel, 1967). It involves both methodology and substance and emphasizes the importance of accounts of social phenomena. It warns against attempting to decontextualize such accounts.

Lynch summarizes all of these by noting that they all involve some sort of “reflexive turning back,” with many identifying reflexivity with the more radical programs of attacking objective sociology, even as Lynch notes that many of them attempt to make sociology more scientific through rigorous self-reflection and understanding. In any case, these largely sociological approaches provided the framework for the first discussions of reflexivity by Hands (1994a,b; 1998) and by Davis and Klaes (2003).

 **Davis on Reflexivity Before Soros**

As noted above, the first excursion by John B. Davis (Davis and Klaes, 2003) focused largely on the sorts of methodological issues that had long had the attention of the sociologists discussed above, even citing some of them, including Mannheim, Popper, Giddens, Bloor and Woolgar.[[2]](#footnote-2) Davis and Klaes developed a three-tiered approach to reflexivity and then used this to analyze the work of four economists: D. Wade Hands (1994a,b; 1998), Esther-Mirjam Sent (1998, 1999), Uskali Mäki (1998, 1999), and Philip Mirowski (2002). The three tiers of reflexivity for Davis and Klaes are *Immanent*, *Epistemic*, and *Transcendent*.

The paper opens by noting that reflexivity comes from the Latin *reflectere*, which means “bending back.” This can apply to light reflecting off a mirror or a person engaging in internal thought or self-reflection. They also note a set theoretical definition such that a relation involves every element of a set mapping to itself. In linguistics, a reflexive pronoun refers back to the subject of a sentence. They also note recursive functions and close the paragraph by mentioning the ubiquitous “Drawing Hands” by Maurits C. Escher, although without reproducing it or discussing it further.

Rather than Escher’s print, they focus on possibly the most famous painting in Spain, *Las Meňinas* (“The Ladies in Waiting” or “The Family of King Philip IV,”), painted in 1656 by Diego Velásquez da Silva. This somewhat mysterious painting (see Figure 1) has front and center the royal Infanta, who was later Holy Roman Empress, the young daughter of the king and queen of Spain, with some attendants around her, including a dwarf and a dog, with her governess and a male servant behind this group . On the left-hand side is Velasquez himself, painting something, although we only see the back of part of the canvas, not what is on it, a matter of much speculation. On the right is a male figure leaving the room, apparently the queen’s chamberlain. On the wall in back is a mirror with two somewhat murky figures reflected in it, with the traditional identification of these as being the king and queen, possibly located outside the painting, and also quite likely the subjects of the unseen painting in the painting that Velasquez shows us him painting. This certainly sets up various possibilities for self-referencing reflexivities of various sorts, and that is what Davis and Klaes do, using the painting to describe their three types., while drawing heavily on interpretations of the painting by Foucault (1966).

Their lowest tier, immanent reflexivity, is also the one that contains the core concept of reflexivity, namely self-referencing. The key point is that this painting is about painting, the crucial self-referencing. They note later in the paper this level involves the possibility of “malign” logical paradoxes involving self-referencing such as the Liar’s Paradox: “All Cretans are liars, and I am a Cretan,” which leads to an endless do-loop, one of those reflections ad infinitum discussed by Hofstadter, and which Bertrand Russell tried to eliminate from mathematics by developing his theory of hierarchical types. Lynch had also noted “malign” radical reflexivity associated with Woolgar and associates as well as a more “benign” sort associated with taking account of one’s own biases and paradigms.

It is a curious point that Davis and Klaes miss an even potentially deeper form of self-referencing that might be implied in the painting. While most observers think that in the painting Velásquez is painting the king and queen who are reflected in the mirror on the back wall, and others simply say there is uncertainty here, another possibility is that the painting Velásquez is painting is the painting itself, the painting he is in. If somehow we could see that painting, then it would contain a potentially infinite set of paintings of paintings within paintings, all of each other, getting smaller and smaller. This would be a full-blown self-referencing of the sort Hofstadter makes so much of and is also the sort implied by the Liar’s Paradox. Such an interpretation of the painting was first made by André Gide in 1893 (Gide, 1972) when he coined the term, *mise en abyme*, which refers to images that contain images of themselves, with implied infinite sequences.[[3]](#footnote-3) But Davis and Klaes do not go there. Their immanent reflexivity only self-references in that this painting is about painting, at least partly.

Epistemic reflexivity enters in because of Velásquez having himself in the painting, a higher form of self-referencing. For Davis and Klaes it is crucial that the painter has made it explicit that it is himself in the painting. This supposedly brings to light the “foundations of the painting as a representation, attending to its perspective and point of origin.” The emphasis on representation links this type of reflexivity to the radical referential reflexivity of Woolgar and company, with its possibility for a “malign” interpretation. Indeed, this matter of malign versus benign reflexivity is a central theme of this paper, which has as its title, “Reflexivity: curse or cure?” The malign “threatens to undermine itself,” while the benign “has a self-reinforcing character” (Davis and Klaes, 2003, p. 333).

Transcendent reflexivity involves stepping outside the painting. So for the painting this is possibly the royal couple who appear reflected in the mirror on the back wall, but are implicitly outside looking at the painting and putting it in its social and historical place in the world of art, with all observers of the painting also potentially part of this as well.

 They then proceed to apply this framework to the four thinkers they have selected, with their focus fundamentally on methodological issues. They focus on Hands (2001) who focuses on reflexivity and the economics of scientific knowledge. The crucial immanent reflexivity involves an argued Quinean turning, in which ultimately economic methodology derives from itself through other sciences. They argue that the others covered in this paper are also caught up in the debate over this matter. The shift to epistemic reflexivity arises with the commentary on this by Hands himself. The transcendent enters in with the commentary by others on these arguments and Hands’s responses.

 Regarding Sent (1998), the immanent is seen in her study of the Sargent’s role and positions in the development of the rational expectations hypothesis. She sees him setting up a *symmetry* between economists forecasting agent behavior and those agents themselves. This become epistemic when she considers her own analysis of Sargent’s work as a symmetry between the two of them, with both of them having an ability to reconstruct their own views. The transcendent appears as others comment on all this, with Sent (as with Hands) losing control as outsiders enter in, although in the end this results in a call for pluralism.

 Mäki’s (1999) immanent reflexivity involves considering Coase’s call for an “economics of economics” that would itself obey free market principles, bringing about a possibly desirable “Consistency Supposition” where one’s object-economics matches one’s meta-economics. The epistemic level arises from Mäki’s own meta-analysis of Coase forcing him to consider his own analysis. However, for him the transcendent level is more complicated in that it is not just how others interpret or comment on this, but the fact that Mäki himself helped organize a petition in 1992 that advocated a free market approach to economic methodology, albeit with some not signing who supported this position while some who did sign not obviously being likely supporters of such a position. Davis and Klaes argue that this shows that as one moves from the immanent into the epistemic and then the transcendent, “how we respond to reflexivity ends up depending on a variety of additional factors that explain how reflexive relationships are socially embedded” (Davis and Klaes, 2003, p. 344).

 The case of Mirowski (2002) involves his analysis of “the Colonel’s dilemma” regarding the study of using game theory in the Cold War at the RAND Corporation in the early 1950s. The immanent reflexivity arises from using game theory to study using game theory, which came about from realizing an enigma involved that looks like a Liar’s Paradox Hall of Mirrors. On the one hand, to successfully use game theory in war requires that the enemy knows one is doing that, while on the other one does not want an enemy to know one is using randomized strategies involving bluffing and so forth. According to Mirowski, this de facto paradox put a “chilling effect” on the study of game theory for war purposes at RAND. Of course, the immanent reflexivity here is the use of game theory to study the use of game theory. The epistemic part comes from considering Mirowski’s own analysis of this: if it is caught in a sort of paradox, then is he not also? However, in this case the transcendent has another aspect than for the other three cases due to the fact that Mirowski was discussing matters long kept secret and known only to a few. Thus as outsiders come in through reading *Machine Dreams*, effectively a new history is created.

 In some respects this final case of Mirowski most clearly provides the bottom line for Davis and Klaes. The problem of potentially destructive malignity with reflexivity largely lies at the immanent level, where the basic self-referencing opens the door to Liar’s Paradox kinds of dilemmas that can lead to essentially a nihilistic outcome for methodological researchers. It is by moving on to the epistemic and especially the transcendent levels that one turns this destructive malignity into potential benignity.

 As a conclusion and a segue into our next sections, it should be noted that this hierarchy of increasingly benign “self-reinforcing” reflexivities does not relate particularly to a complexity vision of reflexivity. When we contemplate emergence complexly arising from reflexivity in a system, this is usually thought of as an endogenous process arising from the dynamics or structure of the system itself. But in this triadic formulation, that is not what is happening. The epistemic and transcendent levels do not emerge endogenously from the immanent level. Rather they are exogenously imposed by bringing in extra parties, first the principal agent and then out and out outsiders. The saving of reflexivity from its malignant cursedness due to the paradoxes inherent in the self-referencing at the immanent level essentially involves calling in a methodological cavalry of exogenous outsiders to save the day and make reflexivity a benign cure rather than a malignant curse.

 **Soros and the Turn to Reflexive Complexity**

 John B. Davis began discussing complexity in economics as early as 2006 (Davis, 2006), but this was not in connection with reflexivity, rather in connection with debates triggered by Colander et al. (2004) over the relationship between neoclassical and heterodox economics, with Colander et al. arguing that complexity was becoming a major influence on modern economics. It would be after the Great Recession, and particularly increased attention paid to the ideas on reflexivity of George Soros (1987), with his ideas seen as helpful in explaining the financial crash that led to the Great Recession that Davis would begin linking the two ideas. This increase in attention culminated in a special issue in 2013 of the *Journal of Economic Methodology* that featured a paper by Soros (2013), who accepted that reflexivity was connected to complexity, although the argument was probably pushed more strongly by others in the issue (that came out of a symposium funded by Soros), including especially Beinhocker (2013), Caldwell (2013),[[4]](#footnote-4) Hommes (2013), as well as Davis (2013), who from this point on has been fully on board with the idea of the two concepts being deeply linked as seen in Davis (forthcoming a,b).

 However, before getting into this further, let us very briefly review basic ideas of complexity, which have received much more attention in economics than reflexivity has (Rosser, 2004, 2009). More than 40 types of complexity have been identified, although many are variations of each other. Three broader categories have had the most influence in economics, and all three have links with reflexivity. One is *computational*, another is *dynamic*, and the third is *hierarchical*. Velupillai (2009) provides a good discussion of computational complexity. Very roughly the complexity of a computer program is correlated with how many steps it takes to solve it. A conventional view sees this as being in roughly four levels. The lowest involves linear systems that are not complex, whose solutions are quick and easy basically. The next level up is polynomial, or P, which take qualitatively longer than linear ones, but which are mostly able to be solved eventually by modern computers. It is a widely accepted but still unproven hypothesis that the P level is distinct from the non-P (NP) level above it involving more complex nonlinear systems that are not polynomial. While in principal solvable, many of these take so long that even now they are nearly impossible to solve in any remotely reasonable time with current computers.[[5]](#footnote-5) Finally there are computer programs that cannot be solved, that end up in infinite do-loops. Here see the clearest connection with reflexivity in that the kinds of ad infinitum hall of mirrors effects can underlie such outcomes, as these ultimately can arise from self-referencing paradoxes such as the Liar’s Paradox, with the earliest understandings of this in computer science ultimately referring to Gödel’s incompleteness theorem.

 Rosser (1999) identifies Richard Day (1994) as having provided a definition of dynamic complexity. A system is dynamically complex if it is nonlinear and endogenously generates dynamics that do not converge smoothly to an equilibrium point or a non-oscillating growth path (possibly negative). This has included or excluded two-period cycles, but we shall not delve into that minor issue here. Rosser (1999) identified four kinds of such dynamic complexity, the “four C’s”: cybernetics, catastrophes, chaos, and “small-tent” or agent-based complexity. The dynamics described by Soros in his reflexive view of financial markets is widely viewed as fully consistent with this form of complexity.

 Simon (1962) is widely seen as the seminal formulation of hierarchical complexity. While not absolutely necessary, he viewed the more interesting forms of this as involving a spontaneous or endogenous emergence of the higher levels in a system, such as we have observed in biological evolution, even though in some cases the emergence may have been triggered by some external pressure such as a climatic change for biological evolution. In any case, we can see elements of hierarchical complexity in the other two, with the hierarchies of computational complexity, and in dynamic complexity there are hierarchies of degrees of complexity. The possibility of emergence is also linked with the old “whole is greater than the sum of its parts” idea that dates back to Aristotle and that many see as a deep aspect of broader complexity. For now we shall hold off on how reflexivity relates to this type of complexity, although reminding that the hierarchy of immanent, epistemic, and transcendent reflexivity formulated by Davis and Klaes (2003) was essentially exogenously imposed rather than endogenously and spontaneously emergent.

 So let us return to the version of reflexivity in Soros (2013), which draws on much of his earlier work, heavily influenced by Popper (1959), although partly in reaction. Thus where Popper thought that one can have certainty in science through falsifiability that at least some propositions are not true, Soros decided after studying economics and becoming disillusioned by the rational agents of conventional theory, that when thinking agents are in a system there may be no definite standard of truth because of reflexivity. The very actions and thinking of the agents can change the system, which in turn can change their thinking and actions.[[6]](#footnote-6) He saw this reflexive interaction as a basically two-way street: people perceive and think with their *cognitive function* while then they act on external reality using their *manipulative function*. The results of these then feed back through the cognitive function in an ongoing dynamic process that is nonlinear and self-referential. He recognizes that self-referencing leads to the logical paradoxes, but that in actual dynamic situations these simply imply indeterminacy and radical Knight-Keynes uncertainty. He invokes the beauty contest of Chapter 12 in Keynes’s *General Theory* and links this to Merton’s (1948) self-fulfilling prophecy idea. Thus he describes boom-bust cycles in financial markets as agents creating market conditions that they react to. They may even be believing economic theories, but their own actions undermine those theories, and in actual financial markets, there are no independent market fundamentals or long-run equilibrium. He specifically applies this to the housing bubble and decline that led to the Great Recession. There is simply an ongoing interactive dynamic process, and he recognizes that this is consistent with the now large literature on nonlinear complex dynamics in financial markets (Hommes, 2013).

 In his piece for the symposium, Davis (2013) mostly repeats these arguments while noting that they imply fat tails in financial market returns. He also notes connections to behavioral economics and the critiques of the efficient market hypothesis.

 Davis seems to have largely continued to hold to this essentially Sorosian view of reflexive complexity, but has applied it and expanded it further. Thus, long a student of the philosophical underpinnings of Keynes’s thought (Davis, 1994), he (Davis, forthcoming a) revisits philosophical aspects of Keynes’s thought in light of reflexive complexity. Unsurprisingly, like Soros, he sees the beauty contest of Keynes as a central place where Keynes profoundly foreshadows reflexive complexity. It also shows the deep link with uncertainty, as what is going on in the beauty contest involves agents trying to figure out what the other agents are thinking, with them realizing the other agents are trying to figure out what they are thinking. Rather than resolving to some nice game theoretic probabilistic equilibrium, the agents are potentially caught in an infinite regress that can go to higher and higher levels of thinking about thinking about thinking about what each are thinking about, as in the Holmes-Moriarty problem (Koppl and Rosser, 2002). This is Hall of Mirrors reflexivity at its clearest.

 Somewhat further afield, Davis (forthcoming b) considers dynamics of Veblenian cumulative causation as studied by Geoffrey Hodgson (1998, 2010) in critiquing methodological individualism. He argues that the self-reinforcing positive feedback of cumulative causation is ultimately a reflexive process involving reflexive agents. Given that Rosser and Rosser (2017) have tied cumulative causation to complexity, this suggests that this is yet another example of reflexive complexity.

 **“Drawing Hands” versus the “Print Gallery”**

 We have noted already that M.C. Escher’s “Drawing Hands” (*Tekenen* in original Dutch) from 1948 (See Figure 2) has been repeatedly cited by people discussing reflexivity as an image of it, including that Douglas Hofstadter (1979, 2006) has been among those, citing it as an image of a *strange loop* that he considers to be the foundation of consciousness. In his 1979 book he also discusses another Escher print, “Print Gallery,” (*Prentententoonstelling* in original Dutch) from 1956 (see Figure 3) as another example of a strange loop, but he does not discuss it in his 2006 book, *I am a Strange Loop*, whereas “Drawing Hands” makes multiple appearances in that book. Nevertheless, I shall argue here that if we wish to see an artistic image that truly represents *reflexive complexity*, it is “Print Gallery” that does so more profoundly and effectively.

 It is not that Hofstadter dismisses “Print Gallery” as trivial or unimportant, far from it. In his 1979, *Gödel, Escher, Bach*, he notes that it has three forms of “in-ness.” One is that the gallery is in the town depicted in the picture on the wall the man is looking at, which he labels “inclusion.” The second is that the town is in the picture on the wall that the man is looking at, which he labels ”depiction.” Finally he argues that the picture is mentally in the person, which he labels “representation.” However, we can also put these together to see that the man is in the print gallery that he is looking at on the wall, thus proving the sort of indirectness Hofstadter repeatedly argues is the key to a strange loop. Indeed, it has this character of hierarchical emergence that we identify with complexity in a way that one does not find in “Drawing Hands” as one moves from man to picture to city to print gallery back to man contained in the print gallery.[[7]](#footnote-7)

“Drawing Hands” is largely at two levels, a print about drawing, even if it implies an infinite do-loop or recursion as one hand draws the other drawing the first, ad infinitum. One can even argue that it is not really all that reflexive. Rather than a going back and forth, it might be depicting a simultaneous solution of a pair of systems, much like with the basic partial equilibrium of supply and demand in economics. The two are jointly determining each other simultaneously with no such reflecting back and forth. Of course in the economy it is precisely that we see such an echoing back and forth between agents and parts that raises the reflexivity question.

We do not know why Hofstadter has come to focus more on the possibly simpler “Drawing Hands” that seems both less complex and less indirect in its “strange loopiness” than “Print Gallery,” although it is possibly because the former is much more well-known than the latter and has indeed been discussed by so many people dealing with reflexivity and related issues. But there is another possibility. In 1979 Hofstadter made an inaccurate speculation regarding “Print Gallery.” This involves the hole in its center (which happens to contain Escher’s signature). The question arises if the hole must be there for its grid/metric bending[[8]](#footnote-8) that brings about the multiple embeddings Hofstadter noticed to happen. Hofstadter speculated that it must be.

However, in 2003, de Smil and Lenstra demonstrated that it was possible to eliminate the hole through a rotation across the complex number plane. The result of this was to eliminate the bending grid, leaving an image that has the Droste effect (or mise en abyme effect),with one looking at the city and seeing in it a print gallery in which a man is looking at the picture of the city that contains the print gallery with another image of the man and the picture, and so on, much like the Droste cocoa jar with an image of itself on it (and a similar such image on a US food container, a Land ‘o Lakes butter packets that showed a Native Indian woman holding an image of the packet with her on it holding an image of the packet). So, indeed, buried in “Print Gallery” is a more conventional infinite set of self-referencing images, once one unravels it appropriately.[[9]](#footnote-9) But then Hofstadter could counter that it is no longer really the “Print Gallery” in this deconstructed form, and indeed in that form it loses the special indirect and self-feeding hierarchy effect that implies complexity and emergence in this print in a way that one may not see in any other work of art there is. It is truly unique.

 **On the Foundations of Reflexive Complexity**

 We now see the possibility of linking our major schools of complexity through the subtle strange loopiness involved in indirect self-referencing at the heart of a deeper form of reflexivity. The indirect self-referencing at the heart of Gödel’s incompleteness theorem is deeply linked to computational complexity in that it leads to the infinite do loops of the highest level of computational complexity in which a program never stops. The way out of incompleteness involves in effect what Davis and Klaes invoked: moving to a higher hierarchical level in which an exogenous agent or program determines what is true or false, although this opens the door to incoherence (Landini et al, forthcoming). The indirect self-referencing opens the door to dynamic complexity in its implications for market dynamics, with this also linking to hierarchical complexity as new levels of hierarchy can be generated. Let us consider briefly how this comes out of the fundamental Gödel (1931) theorem.

 The Gödel theorem is really two theorems. The first one is the incompleteness one: any consistent formal system in which elementary arithmetic[[10]](#footnote-10) can be carried out is incomplete; there are statements in the language of the formal system that can neither be proved nor disproved within the formal system. The second one addresses the problem of consistency[[11]](#footnote-11): for any consistent formal system in which elementary arithmetic can be carried out, the consistency of the formal system cannot be proved within the formal system itself. So, coherence implies incompleteness, but any attempt to overcome incompleteness by moving to a higher level involves one being unable to prove the consistency of this higher level system, with both parts of this failing due to paradoxes of (reflexive) self-referencing leading to paradoxes.

 Hofstadter (2006) provides an excellent discussion of the nature of the indirectness involved in proving the main part of the theorem, which involves the use of “Gödel numbers.” These are numbers assigned to logical statements, and their use can lead to the creation of self-referencing paradoxical statements even within a system especially designed to avoid such self-referencing statements. The system that Gödel subjected this treatment to eventually generates a statement equivalent to “This sentence is unprovable” was the logical system developed by Whitehead and Russell (1910-1913) specifically to provide a consistent formal foundation for mathematics without logical paradoxes. Russell in particular was much concerned about the possibility of paradoxes in set theory, such as that involving self-referencing sets. The classic problem was “Does the set of all sets that do not contain themselves contain itself?” A famous simple version of this involves “Who shaves the barber in a town where the barber only shaves those who do not shave themselves?” Both of these involve similar endless do-loops arising from their self-referencing. Whitehead and Russell attempted to eliminate these annoyances by developing the theory of types that established hierarchies of sets in ways to avoid having them refer to themselves. But then Gödel pulled his trick of establishing his numbers, which he applied to the system of Whitehead and Russell so as through indirection to generate a self-referencing statement that involved a paradox unresolvable within the system. It is rather like how the hole Escher put in the middle of his “Print Gallery” allowed for the man to look at a print on a wall in a gallery of a city that contains the gallery in which he is standing looking at it.

 Thus it is not surprising that the problem of self-referencing has lain at the core of much of the thinking about reflexivity from an early point, and that this thinking took on a sharper edge when various figures thought about Gödel’s theorem, or even earlier about the paradoxes considered by Russell.[[12]](#footnote-12) Linking this to understanding to complexity provides a foundation for a reflexive complexity that encompasses all the major forms of complexity.

 **Conclusions**

 We have reflected on reflexivity and its relation to complexity by considering the history of the development of the idea along with a consideration of its underpinnings and foundations. Ultimately involving some sort of self-referencing through reflecting off an other, with that other possibly something one is inside of, such as a community or profession, it links to complexity in its various main forms of computational, dynamic, and hierarchical in various ways, potentially showing how these can be linked to each other.

 The initial explicit discussion of reflexivity occurred mostly among sociologists, with a substantial number of varieties of it considered. Many of these involved having social scientists more broadly take seriously their own roles in their disciplines and how their interactions with what they study impact them and in turn impact what they study. This self-study eventually led to radical critiques of doing social science as the inability to escape from the reflexive self-aware loops led to nihilism.

 Following methodological work on reflexivity by Hands and others, Davis and Klaes (2003) extol the bringing in of outsiders or higher level considerations when self-referencing threatens to generate “malignant” paradoxes. They saw these self-referencing paradoxes as arising for immanent reflexivity in which there is a direct self-referencing, such as the painting in the Velásquez painting *Las Meňinas*. The next level up of epistemic reflexivity involves the agent contemplating their own role in the system, as with Velásquez inserting himself into the painting. Finally at the highest level is transcendent reflexivity when other parties enter into the contemplation or discussion of the system, as symbolized by the images of the king and queen in a mirror on the wall in the painting, suggesting that they are outside looking in at it. This calling upon higher level outsiders to overcome malignant paradoxes to produce a benign reflexivity echoes somewhat the move to a higher level implied by Gödel’s Theorem in order to resolve incompleteness in a system, even if doing so generates a possible inconsistency.

Following the 2008 financial crash, ideas of reflexivity proposed by Soros (2013) became influential, which drew more explicitly on self-referencing and how the interactions of heterogeneous agents trying to figure out what the other agents are thinking about what they are thinking can lead to complex, emergent, self-organized and self-fulfilling prophetic dynamic outcomes in markets that can manifest themselves as speculative bubbles and crashes. Soros and others (Davis, 1994, 2013, forthcoming a) clearly link these ideas to those of Keynes (1936) in his *General Theory*, Chapter 12, in which he poses the beauty contest in which the winner is not the person who selects the most beautiful woman but best guesses the selections made by the other participants in the game, with this possibly involving an infinite regress of endless levels of thinking about what others are thinking about each other’s guesses.

As argued by Hofstadter (1979, 2006) a deep aspect of how emergence of higher levels occurs implying both dynamic and hierarchical complexity may involve indirect self-referencing that also implies computational complexity, as well as a deep form of reflexivity. While many observers of reflexivity have cited M.C. Escher’s “Drawing Hands” as a picture of such self-referencing, it may be that his “Print Gallery” provides a deeper picture of this more important indirect self-referencing that may generate the more profound form of reflexive complexity that involves all the major forms of complexity.

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 Figure 1: *Las Meňinas*, Diego Velásquez da Silva, 1656



 Figure 2: “Drawing Hands,” Maurits Cornelis Escher, 1948



 Figure 3: “Print Gallery,” Maurits Cornelis Escher, 1956

1. Another group drawing on Keynes to emphasize the importance of self-fulfilling prophecies and how they can arise from essentially random origins is the *sunspot equilibrium* approach (Azariadis, 1981; Farmer, 1999), who see the possibility of multiple such equilibria. [↑](#footnote-ref-1)
2. They also cite some not mentioned by Lynch, including anthropologist Clifford Geertz (1973) on people understanding their own positions in studying societies as well as Alvin Goldman (2001), a radical ally of Woolgar who may be more challenging of how to do the Scientific Study of Knowledge (SSK) than even Woolgar. [↑](#footnote-ref-2)
3. Gide took this term from medieval heraldry, where sometimes a heraldic image would have an image of itself within itself. It is usually translated in English as “into the abyss.” In Dutch discussions it has been called the *Droste effect*, after a 1904 cocoa company’s container that has an image of itself on it. An early painting showing it is Giotto’s *Stefaneschi Tryptich* from 1320, which depicts Cardinal Stefaneschi kneeling while presenting a copy of the tryptich to St. Peter. See Snow (2016) for a much deeper discussion of Gide’s analysis. A modified version of *Las Meňinas* that shows such a Droste effectimage of itself with implied multiple such embedded images on the rear wall above the mirror is at <https://www.flick.com/photos/centrasian/5549818324/in/photostream> . [↑](#footnote-ref-3)
4. Caldwell’s paper focused on Hayek, who had also known Popper as had Soros. Soros had long ignored Hayek, but Caldwell argues for his ideas being deeply tied to complexity (Hayek, 1952, 1967) and compatible with Soros’s view of reflexivity, with Soros largely accepting this argument in his paper in the special issue. [↑](#footnote-ref-4)
5. Whether or not P = NP is one of the leading unsolved problems in mathematics. It appears that it was first recognized by John Nash in a long-classified letter to the US National Security Agency in 1955. He thought that they did not equal each other but stated he could not prove it. He identified it as important for cryptology, which is why his letter would become classified for many decades until only shortly before Nash died. [↑](#footnote-ref-5)
6. A variation of this is the idea of “performativity,” wherein a theoretical idea such as the Black-Scholes options formula can become a crucial part of market dynamics when agents begin to believe in it and act on it (MacKenzie, 2006). [↑](#footnote-ref-6)
7. Another curious tidbit about “Print Gallery” is that most of the prints one sees on the walls aside from the one of the city with the print gallery curiously embedded in it, are ones by Escher himself, suggesting that if this is a mental image by the man, that man might well be Escher himself, another subtle form of reflexivity in it. [↑](#footnote-ref-7)
8. In a letter to his son, Escher reported being partly inspired by the “bending” of space in Einstein’s general relativity theory in creating “Print Gallery.” [↑](#footnote-ref-8)
9. An alternative outcome is to leave the bending but to fill in the hole with an infinitely descending set of images of the print in the hole. These can be seen on the website escherdroste.math.leidenuniv.nl/index.php?menu=im . [↑](#footnote-ref-9)
10. By “elementary arithmetic” is meant that which can be derived from Peano’s axiom set assuming standard logic of the Zermelo-Frankel type with the Axiom of Choice (ZFC). [↑](#footnote-ref-10)
11. It should be noted that in his original theorem Gödel was only able to prove incompleteness for a limited form of ω-consistency. A proof for a more general form of consistency was provided by Rosser (1936) who used the “Rosser Sentence” (or “trick”): “If this sentence is provable, then there is a shorter proof of its negation.” This has led some to refer to the combined theorem as the “Gödel-Rosser Theorem.” [↑](#footnote-ref-11)
12. Without directly addressing either reflexivity or complexity, some who saw Gödel’s theorem as implying deep limits for economic analysis include Simon (1955) who saw computational limits from this as one of the bounds on rationality, with both Albin (1982) and Winrich (1984) raising similar such issues involving self-referencing statements in economics that imply unsolvable infinite regresses. As already noted, the beauty contest of Keynes (1936, Chapter 12) also had the possibility of infinite regress through a clearly reflexive setup involving indirect self-referencing. [↑](#footnote-ref-12)