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Market Dynamics And Stock Price Volatility

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Abstract. This paper presents a possible explanation for some of the empirical properties of asset returns within a heterogeneous-agents framework. The model turns out, even if we assume the input fundamental value follows an simple Gaussian distribution lacking both fat tails and volatility dependence, these features can show up in the time series of asset returns. In this model, the profit comparison and switching between heterogeneous play key roles, which build a connection between endogenous market and the emergence of stylized facts.

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1 Introduction

In the last decade with the availability of large data sets of high-frequency asset price series (including stock, foreign exchange and other asset price) and the application of computer-intensive methods for analyzing their properties, the research in empirical finance has enjoyed substantial development [1–5]. Pagan[6] and Cont[7] each provides an authoritative survey of these salient features that are common characteristics of all financial markets and classifies them as some "stylized facts" such as absence of autocorrelations and long-range dependence, heavy tails and conditional heavy tails, volatility clustering.

With the wealth of works in empirical research, to build a model to explain the stylized facts of the asset price volatility is still a competitive work. The goal is to have the simplest and most parsimonious description of the market and the most faithful representation of the observed market characteristics. Widely presented models

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are multi-agent models, which are based on interacting agents using different strategies corresponding to abstract or real market behavior, the examples include the "minority games" model[8–10],the percolation models[11, 12] and the spin models[13, 14], and other microscopic models[15–18]. A typical one based on both economical and physical approaches is Lux and Marchesi model(LM in the following)[19, 20],in which a relatively large number of parameters enables one to incorporate several aspects of real financial processes.

Following the route of LM, in this paper we present a model to explain some "stylized facts" of asset price volatility. We propose a simpler mechanism to account for the absence of autocorrelations and appearance of long-range dependence, heavy tail and power law in the extreme parts, and volatility clustering. In our model, there are two types of market participants: fundamentalist and Non-fundamentalist. This classification is similar to fundamentalist-chartist approach [20–22]. The fundamentalist traders who buy when the asset price is below the fundamental value and sell when it is above and noise traders who use moving average technical trading rules that can lead them to chase trends. The heterogeneous agents switch from one type of strategy to the other according to relative performance.

The first interesting contribution of our paper regards the social interaction among traders. In the LM model, there are two components that govern the transitions of the traders: the herding component and the profit comparison. In fact,to obtain a deviation from normality many other models take into account the herding behavior that determines (at least in part) the fat tail property of the distribution of returns[11–13, 17, 18]. In our model, the herding component is absent, and only the profit feedback plays a role. With the lack of the social interactions, the model still produces a realistic time series of returns. Another remarkable result is the implementation of the moving average as the main strategy of the noise traders. It allows a further simplification of the LM model since the optimistic pessimistic classification for the noise traders is no longer necessary. Furthermore, the moving average approach in effect captures the elements of herding. Herding moves back and forth between bubble and non-bubble dynamics and also between up and down bubbles within bubble dynamics. The moving average model essentially captures these elements. In short, in our model the fat tails behavior and the volatility clusters depend on two components: the one is the transitions of the traders governed by the profit comparison within effective memory, the other is the moving average trading rule used by chartist trader which tends to magnify the random shock to the market.

2 The Model

In our model, there exist two types of traders in the asset market. One is F-traders, who are fully rational and well informed; the other is N-traders, who are either less well-informed, irrational, or risk-loving [24]. Our model is based on stylized representations of these two types of market participants who use different strategies.

2.1 F-traders

F-traders may be referred as "fundamentalists" or "information traders". Let x be the current market price of a unit of asset and v be its fundamental value, which can be regarded as the present value of the rationally expected stream of future net earnings of a unit of the asset. We suppose that the F-traders know the fundamental value v by means of so-called "fundamental analysis" based on all available information about the asset. Obviously, the F-traders attempt to incorporate the most recent information into their estimates of fundamental value. As germane events may occur almost randomly, the fundamental value could be rather volatile.

In general, the current market price x diverges from the fundamental value v. The F-traders think this means temporary "false pricing" and believe x and v will converge in the long run. Furthermore, the F-traders' decision to buy or sell asset depends on the divergence x and v. If the spread is strongly positive, the opportunity for a capital gain and desire for the asset is great; while if it is strongly negative the risk of a capital loss and rejection of the asset is great. Therefore the F-traders' trading strategy is given by a simple excess demand function:

$$q^F = c^F(x_t - v_t)^3 \tag{1}$$

and

$$v_t = v_{t-1} + k\varepsilon_t. \tag{2}$$

where the nonnegative parameter c^F measures the F-traders' excess demand response to a price gap. The cubic formulation in function (1) is selected as a simple expression for the fact that greater spread induces more desire for trade[25]. Furthermore, This cubic formulation can block the market price to go far away from the fundamental value. ε_t is assumed to be standard Guassian white noise, which implies that v is a random walk.

2.2 N-traders

N-traders may be referred as "Non-fundamentalists" or "Noise traders". Here N-traders mainly correspond to chartists, who use relatively simple and low cost buy-sell rules, such as so-called "technical analysis". One of the most widely used technical rules is the moving average rule [26]. According to such a rule, buy and sell signals are generated by two moving averages of the level of the index: a long-period average and a short-period average. When the short-period moving average penetrates the long-period moving average, the N-traders think a trend to be initiated and capital gain or loss to be expected. This means the N-traders chase price up and down. Indeed, although they are generally deemed as irrational or poorly informed, DeLong et.al [27]demonstrated that noise traders can sometimes do better than all other market participants, especially when their behavior is driving the market outcomes.

Here we adopt one of the simplest rules: the short-period moving average is just the current market price and the long-period one is just an exponentially weighted moving average, which is also an adaptive expectation of the market price. Let x denote the market price and y denote the long-period moving average, the N-traders' trade strategy can also be given as a simplified expression by an excess demand function:

$$q^N = c^N(x_t - y_t) \tag{3}$$

and

$$y_t = \alpha x_{t-1} + (1-\alpha)y_{t-1}. \tag{4}$$

where the nonnegative parameter c^N measuring the N-traders' excess demand response to a price change, and the parameter α dominates the weight distribution for the long-period average.

2.3 Market Dynamics

Suppose that the total F-traders and N-traders equal one, and the F-traders' share is w, then the aggregate excess demand of the whole market is

$$q_t = w_t q^F + (1-w_t)q^N. \tag{5}$$

We suppose that there exists a market-maker who mediates the trading in the market. The market-maker helps to meet the excess demand and adjusts the next period market price depending on the excess demand. Generally, we can assume that the change in market price is determined by a continuous, monotonically increasing function of the aggregate excess demand. We model the dynamic adjustment of market price by the following difference equation:

$$x_t - x_{t-1} = bq_t = bc^F w_t(v_t - x_t)^3 + bc^N(1-w_t)(x_t - y_t). \tag{6}$$

with the nonnegative parameter b measuring price adjustment flexibility.

As the market price changes, the share of the two types of investors evolves. We assume that the type changes on the basis of the past relative performance of the two trade strategies. Let dz be the past relative return of the two trade strategies, we suppose that transition probability of a formerly N-trader switch to the F-trader group be π and vice versa be $1-\pi$, where

$$\pi = \frac{1}{1+e^{-\lambda dz}}. \tag{7}$$

with λ as a nonnegative parameter. Therefore the share of the two types of investors evolves according to following pattern:

$$w_{t+1} = \begin{cases} w_t + \delta & \text{with} \quad \pi \\ w_t - \delta & \text{with} \quad 1-\pi \end{cases}$$

with the nonnegative parameter $\delta < 1$ measuring type switch sensitivity. To avoid unreal values of the share, we let $w_{t+1} = 1$ if $w_{t+1} > 1$ and $w_{t+1} = 0$ if $w_{t+1} < 0$.

To define the past relative return of the two trade strategies, we define z^F and z^N as the so-called "effective memory of capital return" of the F-traders and N-traders respectively.

$$z_t^F = h(x_t - x_{t-1})q_{t-1}^F + (1-h)z_{t-1}^F \tag{8}$$

$$z_t^N = h(x_t - x_{t-1})q_{t-1}^N + (1-h)z_{t-1}^N. \tag{9}$$

with the nonnegative parameter $h < 1$ measuring the time horizon of the past performance evaluation. From the above recurrence formula we can see that "effective memory of capital return" is not real capital return, it just expresses the weighted average of past observations of capital return, and the exponent diminishing reflects memory law. So the past relative return is also the "effective memory" sense of capital return, which is defined as $dz = z_t^F - z_t^N$.

Here we assume transition between fundamentalists and chartists depends on comparison of "effective memory of capital return". This is similar to the LM model and other multi-agent models[8–10] but different from the other fundamentalists-chartists models[21–23], in which the share of different type of traders only depends on deviation of the market price and the fundamental value. Considering the "noise trader risk"[27], we think the return comparison assumption is more reasonable than the value deviation assumption.

3 Numerical Simulation of the Price Dynamics

The above section has presented a basic framework of the asset market model, and it is easy to examine its dynamical behavior by numerical simulation. This section we will give some simulation results and compare them with the "stylized facts".

3.1 Asset price

A simulated time series of asset price is presented by Figure 1. To explore dynamic property of asset prices, we do a Dickey-Fuller unit root test on the series to check if the price follows a unit root process. Table 1 gives results of the test and shows the test fails to reject the null hypothesis of a unit root in the asset price series at any of the reported significance levels. This means one is unable to reject the hypothesis that the asset prices follow a random walk or martingale process.

3.2 Asset return

We define the rate of asset return as $r_t = (x_t - x_{t-1})/x_{t-1}$. Figure 2 is trajectory of r_t corresponding to the realization of asset price showed in Figure 1. Figure 3 is the dynamics of share of the F-traders. Here we focus on the following characteristics of the trajectory:

Firstly, the phenomenon of volatility clustering and on-off intermittency shows up. The main feature of volatility bursts appears to be ubiquitous in our model and does not hinge on fine-tuning of the model parameters.

Secondly, from Figure 4 we can see that linear autocorrelation of asset return are insignificant but the autocorrelation function of absolute returns decays slowly as a function of the time lag. This means absence of short-range autocorrelations and existence of long-range dependence.

Thirdly, we turn to fat tail phenomenon. Figure 5 gives the distribution of the normalized returns (subtracting the mean and dividing for the standard deviation), the kurtosis statistics and Jarque Bera test indicate that there exists fat tail. The right inset of Figure 5 also gives the right 5% tail distribution of the returns, it can be looked upon as a nearly Pareto distribution. The tail index α in its distribution $F(x) = 1 - \alpha x^{-\alpha}$ can be estimated by Hill's method, and the Hill estimate is about 2.60. This result is close to the usual empirical finding of tail indices somewhere between 2 and 4.

Finally, we can see the on-off intermittency phenomena are strongly related with the population structure of the heterogeneous traders. This implies that it is the noise traders who enlarge the random shock and cause the volatility clusters. When the F-traders dominate the market, we can expect that the return keeps nearly Gaussian distribution.

3.3 Sensitivity Analysis

As an instance, above numerical simulation is limited to one set of parameters. To evaluate the robustness of the model, we have done more Monte Carlo simulations with different parameter sets. we find above characteristics of return series can exist at a wide range of parameters. Table 2 gives fat tail property of the return data, and the results are obtained from 5 different parameter sets and each set includes 100 samples. As computed with empirical data at daily frequency, the results (including kurtosis and tail index estimates) look very realistic.

4 Conclusion

We have studied the behavior of a model of asset market dynamics with two types of traders, one are fundamentalists who trade on the inferred market fundamental value, another are non- fundamentalists or noise traders who trade on the chase for guessed trend. The heterogeneous agents switch from one type to the other according to past relative performance which is based on their excess demand and the market price's movement. The asset market prices are determined by the aggregate excess demand of all traders. On this basic model, we investigate the relationship between random changes in fundamental value (as an "input" to the model system) and market price changes as outputs of the model system. The simulations turn out, even if we assume that the news arrival process follows a simple Gaussian distribution lacking both fat tails and volatility dependence, these features still show up in the time series of asset returns. These results suggest that these statistical properties appear as "emergent phenomena" from the market process itself and do not stem from movement of fundamental value. In fact, the profit feedback and switching between F-traders and N-traders play key roles, which build a connection between endogenous market and the emergence of stylized facts. This is different from the other approaches[11–13, 17–20]. Our result is very important to understand the efficient market hypothesis, which implies that random arrivals of new information lead to random walk of asset price. But our work shows even if random information can result in a random fundamental value, with the hands of the non-fundamentalists, the asset market price does not necessarily keep a random walk. This may support the argument that asset market is always not efficient but marginally efficient [28].

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Table 1. Augmented Dickey-Fuller Unit Root Test on Asset Price: The ADF Test Statistic is -2.471

1 % critical value	-3.434
5 % critical value	-2.862
10 % critical value	-2.567

Table 2. Fat tail property of the return data: kurtosis and tail index estimates (median from 100 samples and range of estimates in the parentheses)

set	kurtosis	2.5% tail	5% tail	10% tail
0	13.5 (8.1-40.1)	2.85 (2.39-3.63)	2.46 (1.99-2.83)	2.03 (1.74-2.31)
1	9.8 (5.9-331.0)	3.11 (2.37-3.75)	2.68 (2.16-3.15)	2.15 (1.74-2.49)
2	58.3 (8.2-170.6)	2.87 (2.33-3.55)	2.52 (2.14-3.08)	2.03 (1.66-2.29)
3	13.6 (8.1-42.8)	2.90 (2.32-3.61)	2.51 (2.09-3.08)	2.02 (1.71-2.34)
4	11.3 (7.6-27.8)	2.98 (2.48-3.58)	2.57 (2.15-3.03)	2.15 (1.69-2.46)

Note: Parameter sets are given as: Set 0: $b = 1, a = 0.02, g = 0.01, h = 0.01$; Set 1: $b = 0.8, a = 0.02, g = 0.01, h = 0.01$; Set 2: $b = 1, a = 0.02, g = 0.01, h = 0.02$; Set 3: $b = 1, a = 0.03, g = 0.01, h = 0.01$; Set 4: $b = 1, a = 0.02, g = 0.005, h = 0.01$.

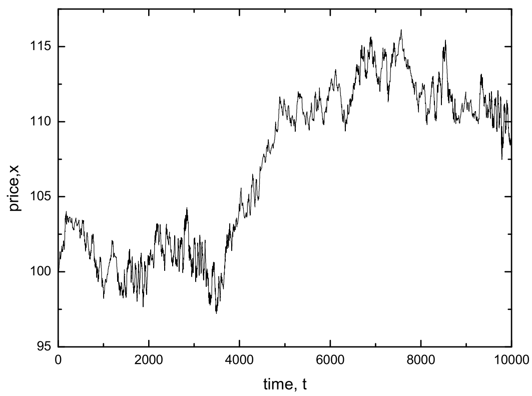

Fig. 1. The time series of asset prices Parameters: $b = 1, c^F = 2, c^N = 1, a = 0.02, g = 0.01, h = 0.01, k = 0.001, \lambda = 1, v = 100$.

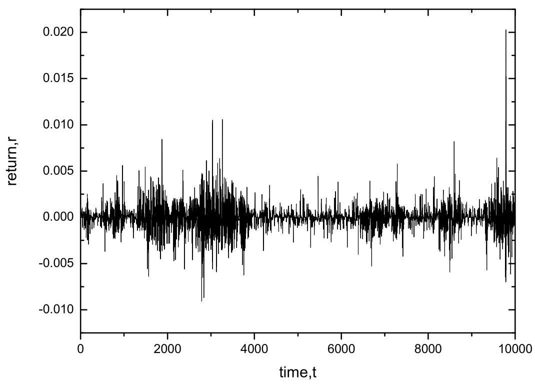

Fig. 2. The time series of returns.

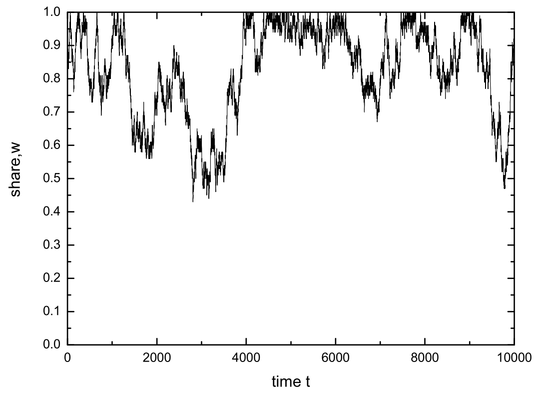

Fig. 3. The time series of F-traders' share.

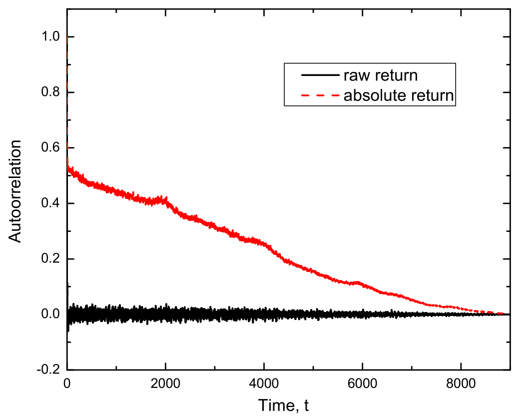

Fig. 4. The autocorrelations of raw and absolute returns. The upper dash line is for raw returns and the below solid line is for absolute returns.

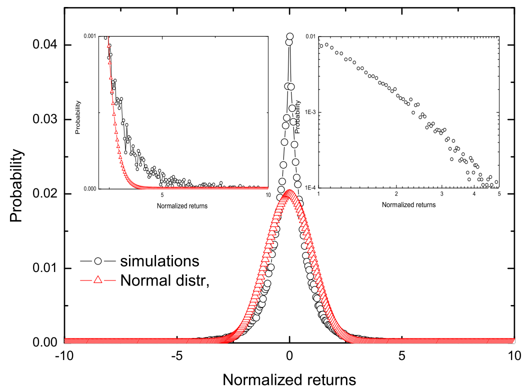

Fig. 5. The distribution of normalized returns. Where original returns' mean is -9.19E-7, std.dev is 1.30E-3, Kurtosis is 11.8, Jarque Bera is 3.26E5. The Hill estimate for the 5% tail returns is about 2.60.