*27 ADDITIONAL SAMPLE QUESTIONS FOR COB 291, EXAM 1*

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*These questions are offered as additional study aids, but without answers. I will be happy to discuss any of these questions the Friday question session.*



**Questions 1-8 refer to the graphical solution to an LP problem, shown to the left.** The objective function for this problem is 10 - x - y. As in class, solid lines are constraint lines, and the dotted line is an OFL.

1. Shade the feasible region on this graph, if one exists.
2. Circle an optimal solution point on this graph, if one exists.
3. Sketch the optimal OFL on the graph, if one exists.
4. How many constraints does the program have?
5. How many redundant constraints does the program have?
6. How many of the program’s constraints are binding on the optimal solution?
7. How many equality constraints does the program have?
8. Is the program above a MAX or a MIN?



**Questions 9 -13 refer to the (partial) graphical LP solution shown to the left.** The graph shows all program constraint lines, but does not show any OFLs. The optimal point is shown, circled.

**Note that this program has only two constraint lines.**

1. Provide arrows on the graph to show the “good side” of each constraint line.
2. Shade the feasible region in this problem.
3. Does this program have an unbounded feasible region?
4. Find the coordinates of the optimal point.

x = y =

1. Is this program infeasible? Is this program unbounded?
2. Suppose that a linear program has two decision variables, and so can be solved graphically. We focus on a particular constraint, and the relationship its constraint line has with the optimal solution point of the program. Which of the following statements is not equivalent to all of the rest?
3. The constraint line passes through the optimal solution point.
4. The constraint is binding on the optimal solution.
5. When the coordinates of the optimal point are substituted into the constraint, the two sides of the constraint are equal.
6. The optimal solution satisfies the constraint.
7. In the optimal solution to the program, the slack in the constraint is zero.
8. Any linear program written with an auxiliary variable
9. will include at least one equality constraint.
10. can be written without auxiliary variables.
11. will have at least two variables.
12. all of a) - c) are true
13. none of a) - c) are true.
14. A certain linear program with objective MAX P = 3x + 2y has its optimal point at the intersection of the lines 2x + 3y = 5 and 3x + 4y = 6. Then the optimal value of P is
15. 0 b) 5 c) 11 d) 13 e) P cannot be determined from the information given.
16. Inequality constraints
    1. appear in every linear program which includes auxiliary variables.
    2. appear in graphical solutions as straight lines with no “good” sides.
    3. are used to represent “conservation constraints”.
    4. always have zero slack in an optimal solution.
    5. none of the above is true.
17. Consider the following problem:

A company has available up to $17,500 per week with which to pay the salaries of a number of new employees. The firm has decided it must hire between 30 and 40 applicants. In the past, the company has been rather lax in meeting governmental quotas on the hiring or minorities and women, and so the new hirings must conform to the following requirements.

At least 35% of the newly hired employees must be women. Further, at least 20% of the new employees must be representative of minorities. Although federal regulations do not mandate it, management also insists that at least 15 (not 15%, but 15!) of the new employees be men.

To recruit quality personnel, management as determined that it is necessary to adopt the following pay scale. Minority males will be paid $500/week. Minority females will be paid $510/week. White males will be paid $485/week. White females will be paid $497/week. Applicants of all categories are in unlimited supply, except for minority women. There are only 5 of them.

The firm wishes minimize the total weekly salary outlay for its new employees. Assuming all applicants are equally qualified, how many of each of the four classifications of potential employees (white males, minority females, etc.) should the firm hire?

Formulate but do not solve the linear program which corresponds to this problem. **Be sure to define your decision variables.**

1. Suppose we have a LP with exactly two decision variables, x and y, and whose only constraints are the nonnegativity constraints on x and y. For each of the following statements, write T if the statement must be true, F if the statement must be false, and ? if the statement's veracity cannot be determined from the information given. You need not support your answers.

The program is infeasible.

The program is unbounded.

The optimal solution to the program occurs at x = 0, y = 0.

The program has at least one redundant constraint.

No constraint is binding on the solution x = 0, y = 2.

1. Suppose Gringo Newtrich, president of Home Speaker Systems, places this requirement upon his company's spending: "Our expenditures for this year will not exceed our expenditures from last year." In good English, understandable to the layperson, explain clearly what the slack in this constraint physically represents.
2. Solve the following problem graphically, using the 10 step approach developed in class. In particular, indicate coordinates for all constraint intercepts, feasible region, original and optimal OFLs, optimal point, and optimal values of x, y, and C.

Minimize C = 2x + 4y

subject to 4x < y, 20y - 30x > 60, 12x + 12y = 60, x > 0 and y > 0.

1. A farmer grows grain (wheat and/or corn), all of which he then feeds to his pigs. A pig, in a year, will eat 20 bushels of wheat, or on 36 bushels of corn, or on some combination of the two--like 10 bushels of wheat and 18 bushels of corn, or 5 bushel of wheat and 27 bushels of corn, etc.

Let W = # of bushels of wheat grown, C = # of bushels of corn grown, and P = # of pigs raised. Then the constraint(s) which represent the relationship among pigs, corn, and wheat is (are): Circle the appropriate letter.

a) P = 20 W + 36 C b) P = W/20 + C/36 c) P = 20 W d) 20 P = W

P = 36 C 36 P = C

e) None of these

1. A manufacturing firm is limited by 4 (nontrivial) constraints: available time, available budget, available resources, and the need to fulfill a certain quota. Their goal is to maximize the profit they make from selling their two products. They represent their situation by a linear program in order to decide how much of each of their two products to make.

There is a unique optimal solution for their problem--make 4000 units of product A and 100 units of product B. As it turns out, in this solution the quota and budget constraints are binding, while the resource and time constraints are nonbinding. Further, the time constraint turns out to be a redundant constraint.

In light of this information, mark each of the following statements with a T if the statement is true, an F if the statement is false.

The firm might make more money if it had additional time available.

If the firm's required quota were lowered, profit might fall.

If the profit per unit of each of the firm's products dropped

below zero, the quota constraint would become nonbinding.

If the resource constraint were removed from the linear program, the

optimal solution would still be the same.

If the time constraint were removed from the linear program, the

optimal solution would still be the same.

1. Write an infeasible linear program with at most two nontrivial constraints. What makes your program infeasible?
2. You are authorized by your firm to invest up to $10,000,000 in the stock market, and are to make your investments so as to maximize the expected value of the stock that you hold at the end of the year. (No stock is sold until after the end of the year.) Your possible investments are shown below:

Stock Expected Yield Risk Maximum Allowable Purchase

American Iron 8% Med $1,000,000

Iranian Oil 12% High $3,000,000

American Oil 10% High $2,000,000

Japanese VCRs 7% Low $4,000,000

U.S. Oil 6% Low $3,000,000

At the instruction of your superiors, at most half of your investment can be in high risk ventures. Further, at least 60% of the money must be invested in the United States. Of the money invested in the United States, at least half of this must be invested in oil. Formulate the linear program which will allow you to determine your wisest investment strategy in light of this information.

1. Suppose I have solved a formulation problem graphically, and find the optimal value of the objective function. Now I change one of the program's constraints so that the feasible region grows; it now contains all of the old feasible region, and more. [The program's objective function and its other constraints remain unchanged.] What effect will this change have on the optimal objective function value (OFV) of the program?

a) The optimal OFV will improve.

b) The optimal OFV will worsen.

c) The optimal OFV will become no worse than it was before.

d) The optimal OFV will become no better than it was before.

e) We cannot conclude any of the above without more information.

1. You have been commissioned to design a lift system for the skiers at a local ski resort that is currently under construction. The owners of the resort want you to design a lift system that will raise the skiers from the base of the mountain to the maximum possible elevation, subject to a number of limitations. After looking over the site and considering the desires of the owners, you decide on a two-stage lift system: a drag line, followed by a chair-lift.

For the first stage, skiers at the bottom of the mountain hold onto a drag line that pulls them up a gentle, smooth slope. The grade of this slope is such that the skier’s elevation increases by one foot for every three feet of distance that he or she covers along the slope. The drag line moves at a constant speed of 20 feet per second, and pulls the skier a total distance of **X** feet along the slope before depositing him or her at the boarding point of the second stage, the chair-lift. Experience with similar systems has shown that it takes 30 seconds for a skier to make the transfer from a drag line to a chair-lift.



The chairs on the chair lift move at a constant

10 feet per second, only half as fast as the drag line.

The angle of ascent for the chair-lift is greater, however. Specifically, a chair gains one foot of altitude for

every two feet that it moves along the lift system.

The total length of the chair-lift system is **Y** feet. Government regulations prevent the chair-lift system from having an overall length of more than 1000 feet.

Your design, as described above, is almost complete, but you must still decide how long the drag line will be and how long the chair-lift system will be. In making these decisions, you must keep in mind two additional pieces of information. First, the total cost of construction cannot exceed $54000. The drag line will cost $90 **per yard** to construct, while the chair-lift system will cost $120 **per yard**.

Second, the total time required to get a skier from the bottom of the mountain to the top of the chair-lift cannot exceed 2.5 minutes. Given this information, write the linear program whose solution would give the optimal values of **X** and **Y**. You will need only these variables in your program, but if you use others, be sure to properly define them.