*Note: My answers appear in red. When you see yellow highlighting like this, it means I’ve included a comment. Place your cursor over the comment symbol to read the comment. Placing the cursor over a correct answer will tell you it's correct. Placing the cursor over an incorrect answer will tell you why it's incorrect. For this to work, you have to change your View setting (in the Word ribbon) to DRAFT--Scott*

A. Evelyn Flaumel, a baker, will be selling sweetcakes (cookies and/or brownies) at an Arts Festival this Saturday, and wishes to maximize her profit from doing so. She has only $100 with which to cover the cost of making these sweetcakes, though if she wishes, she need not spend it all. It costs her 12 cents to make a cookie and 20 cents to make a brownie. She sells her wares for 50 cents per cookie and 75 cents per brownie. Her previous experience tells her that at these prices, the natural demand for cookies is 500 and the natural demand for brownies is 100. She can increase these demands by giving away free samples. A brownie can be broken into 8 samples, while a cookie can be broken into 4 samples. Every two cookie samples given away will result in a demand for one more cookie, and every two brownie samples given away will result in a demand for one more brownie. So, for example, if Evelyn decided to use one of her cookies for samples, the four samples she would get from it would increase her cookie demand by two cookies, from 500 up to 502. **You may assume that Evelyn will use all of the sweetcakes she makes—either she sells them, or uses them as free samples.**

We’ll use these decision variables:

# BS = # of brownies sold CS = # of cookies sold

# FB = # of brownies broken up into samples FC = # of cookies broken up into samples

Note that FB (“free brownies”) and FC (“free cookies”) are not the number of samples!

The measurable quantity formulation of this program is given below, in *italics*. Use the coefficient rule (and the constant rule) to fill in each of the blanks in the LP formulation. Be sure to fill in all of the blanks. **Any empty blank will be treated as a 0.** The right hand side of the first constraint is done for you, as a demonstration; it simply shows that the number of dollars available is 100.

Maximize 0.55 BS + 0.38 CS + -0.2 FB + -0.12 FC + 0 Comment

*# of dollars of profit made*

subject to

0.2\_BS + 0.12 CS +0.2 FB + 0.12 FC + \_0\_ < \_0\_BS + \_0\_ CS + \_0\_ FB + \_0\_ FC + 100

*# of dollars spent < # of dollars available*

\_0\_BS + \_1 CS + \_0\_ FB + \_0\_ FC + \_0\_ < \_0\_BS + \_0\_ CS + \_0\_ FB + \_2\_ FC + \_500

*# of cookies sold* < *# of cookies demanded*

\_1\_BS + \_0\_ CS + \_0\_ FB + \_0\_ FC + \_0\_ < \_0\_BS + \_0\_ CS + \_4\_ FB + \_0\_ FC + \_100

*# of brownies sold* < *# of brownies demanded*

*and nonnegativity on all variables*

Comment Comment

1. Use the 10 step approach developed in class and in LPPE to find the optimal solution to the linear program given below. Be sure to label the intercept coordinate for all axis intercepts, to shade the feasible region, and to include both an original and the optimal OFL. Circle the optimal point, and state the optimal values of x, y and P. **Note that there is no nonnegativity constraint on y**.

MAXIMIZE P = 3x +3y subject to

1. y > -1
2. 2x > 5y
3. 6x +2y > 6
4. y > x - 4

and x > 0.

Y axis

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

#4

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

optimal ofl #5

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

optimal point

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

+----+----+----+----+----+----+----+----+----+----+----+----+----+----+----+

#3 X axis

+ #2 + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

original ofl

+ + + + + + + + + + + + + + + +

+ + + + + + + + + + + + + + + +

2x = 5y and y = x – 4, so 2x = 5(x – 4) = 5x – 20, or 3x = 20, or

x = 20/3

y = x – 4 = 20/3 – 4 = 8/3

P = 3(20/3) + 3(8/3) = 28

Note: Constraint arrows not shown on this solution. Note that y measures height, so y > -1 is a horizontal line!

1. Questions 1-5 refer to the formulation problem below. Choose the best answer for each question.

Two cities, Canton and Dodd City, receive their water from two nearby reservoirs, called simply Reservoir A and Reservoir B. While both cities are able to receive water from either (or both) of these reservoirs, the Regional Water Authority (RWA) wishes to determine an optimal water distribution plan. For the RWA, an optimal plan is one which drains the minimum total number of gallons from the two reservoirs combined, yet still meets the water needs of the two cities.

CHANGE VIEW FROM DRAFT TO PRINT LAYOUT TO SEE GRAPHIC, THEN CHANGE IT BACK!



The four canals that carry water from the reservoirs to the cities have limited capacity. Each canal can carry at most 500,000 gallons per day (as measured at the canal entrance). The problem is complicated somewhat by evaporation and seepage, for not all of the water leaving a reservoir arrives at its destination city. Further, there is a limit to the number of gallons of water per day that each reservoir can provide. The details of this information (and the daily water needs of the cities) are shown below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Reservoir | Max Discharge  (gallons/day) | City | Water Demand  (gallons/day) | Canal | Evaporation/Seepage  Rate |
| A | 1,000,000 | Canton | 600,000 | Royal | 5% |
| Davis | 4% |
| B | 500,000 | Dodd City | 500,000 | Queen’s | 6% |
| Clinton | 3% |

So, for example, if 100 gallons of water is sent from Reservoir A toward Canton via the Royal Canal, 5 of those gallons will be lost to evaporation and seepage, and only 95 gallons will actually reach Canton.

1. Which of the following would be a sensible decision variable to use in this problem?
2. the number of gallons of water demanded each day by Canton.
3. the number of gallons of water per day entering the Queen’s Canal.
4. the number of gallons of water in Reservoir A before the canals are opened.
5. the number of dollars of cost generated by transferring 1 gallon to Canton via the Davis Canal.
6. the maximum number of gallons of water which the Clinton canal can carry per day.
7. The objective for this problem is
8. minimize the cost of supplying the two cities with water.
9. maximize the total number of gallons of water the two cities receive.
10. minimize the total number of gallons released from the reservoirs.
11. minimize the amount of water demanded by the two cities each day.
12. minimize the average % of seepage for all water transferred via canals.
13. Recall the underlined sentence in the problem above. In the linear program representing this problem, this phrase would be expressed as
14. a term in the objective function
15. a limited resource constraint
16. a quota constraint
17. an equality constraint
18. four limited resource constraints
19. Suppose my program includes variables that let me compute the total number of gallons drained from Reservoir A per day. Knowing this would allow me to compute exactly the amount of available water still available for discharge from Reservoir A. Let LEFT\_A = # of gallons of water in Reservoir A still available for discharge. Then
20. LEFT\_A cannot be used as a variable in this problem.
21. LEFT\_A must be used as a variable in this problem.
22. LEFT\_A would be an auxiliary variable in this problem.
23. LEFT\_A = 0.
24. LEFT\_A = 400,000.
25. Which of the following measurable quantity representations is most likely to represent of **one** of the program’s constraints?
26. *(# of gallons entering Davis canal) > (# of gallons leaving Davis canal)*
27. *(# of gallons entering each canal) < (# of gallons which each canal can hold)*
28. *(# of gallons drained from both reservoirs combined)* > *(total # of gallons of water demanded)*
29. *(# of gallons available from Reservoir A) > (# of gallons available from Reservoir B)*
30. *(# of gallons drained from Reservoir A)* < *(# of gallons which may be discharged from Reservoir A)*

Questions 6-8 refer to the graphical solution to some linear program formulation, given below. Remember that nonnegativity constraints count as constraints.



1. This program
2. is infeasible
3. is unbounded
4. has alternative optima
5. has exactly four constraints
6. none of these (a-d) are true.
7. The coordinates of the optimal point
8. are approximately x1 = 35, x2 = 10.
9. could be found by solving the equality forms of constraints 2 and 3 simultaneously.
10. would change if we made any change to constraint 2.
11. will satisfy all program constraints
12. would change if we made any change to the objective function.

*This problem set continues on the next page.*

1. This program
2. has exactly three constraints binding on the optimal solution.
3. has exactly two redundant constraints.
4. must be a MAX problem.
5. has two objective functions and exactly five constraints.
6. none of these statements (a-d) is true.
7. DELETED
8. Cab fare is $3 for the first mile, and $0.44 for every *quarter* mile thereafter. Assume that you are taking a cab for a distance of M miles, and **assume that M > 1**. Then the mathematical representation of the measurable quantity *# of $ required for cab fare* is
9. 3.44 M
10. 2.56 + .44 M
11. 1.24 + 1.76 M
12. 3 + .11 M
13. 3 + 1.76 M

## Note on #10: your usual way of finding the constant term won’t work, since the formula works only for

*M > 1. This about how you can find the correct constant.*