Forecast 1 Problems are from Test 1, Fall 2008

1. The Table 1 below shows the number of cars Harrisonburg Toyota sold in the last few weeks. Answer the following questions based on the information given in Tables 1:

Table 1. Number of Cars Sold Over Time and Forecasts

	A	B	C	D	E	F	G	H
1	Week	Cars	SMA(3)	WMA(2)	EXP			Weights
2	1	19					W1	
3	2	14					W2	
4	3	12			18.500		Sum	
5	4	13			17.850			
6	5	15			17.365			
7	6	18			17.129		α value	.1
8	7	16			17.216			
9	8	15			17.094			weights
10	9	16			16.885		w_3	.2
11	10	13					w_2	.3
12	11						w_1	.5
13	12							
14							Sum =	=SUM(H10:H12)

(1) What is the three – period simple moving average or SMA(3 forecast for the number of cars to be sold in Week 12? (must show the equation and the process for your answer) What is the Excel formula for it?

(2) What is the two – period weighted moving average forecast for the number of cars to be sold in Week 12 using the weights 0.4 and 0.6 with the larger weight for the most recent week's sales data?

a. What is the equation to be used?

b. What is the Excel@ formula to be used for it in Cell D13?

c. What is the answer for it? (must show details by putting numbers in the equation and derive the answer)

(3) What is the Exponential smooth forecast for Week 12 using α value of 0.1?

a. What is the equation to be used?

b. What is the Excel@ formula to be used for it in Cell E13?

c. What is the answer for it? (must show details by putting numbers in the equation and derive the answer)

(4) How to compute the mean squared error (MSE) for the exponential smoothing forecasts developed for weeks 3 to 9 as given in Table 1 above?

a. What is the equation to be used?

b. What is the Excel@ formula to be used for it in Cell E14?

(5) Use the data of car sales and exponential forecasts in Table 1, assume the column F in Table 1 is used to compute the mean absolute percentage forecasting error (MAPE) from weeks 3 to 9, answer the following questions:

a. What is the equation to be used to compute MAPE?

b. What is the MAPE for weeks 8 and 9? (must show details and keep at least four decimal points)

Cars Sold

Week	Cars	SMA 3	SMA 4	SMA 2
1	19	└ Simple moving Average		
2	14			
⋮	⋮			
6	18			
7	16			
8	15			
9	16	$\frac{18+16+15}{3}=16.33$		$\frac{16+15}{2}=15.50$
10	13	$\frac{16+15+16}{3}=15.67$		$\frac{15+16}{2}=15.50$
11		$\frac{15+16+13}{3}=14.67$	$\frac{16+15+16+13}{4}=15$	$\frac{16+13}{2}=14.50$
12		$\frac{16+13+14.67}{3}=14.56$	$\frac{15+16+13+15}{4}=14.75$	$\frac{13+14.50}{2}=13.75$

Use Excel® to compute SMA 3

In C10 Fcst for Week 9: =AVERAGE(B7:B9)

In C11 ⋯ 10: =AVERAGE(B8:B10) OR COPY C10, PASTE in C11

$$F_t = \frac{A_{t-1} + A_{t-2} + \cdots + A_{t-k}}{k} \quad \text{or} \quad F_{10} = \frac{A_9 + A_8 + A_7}{3} = \frac{16+15+16}{3}$$

$$= \frac{1}{3}(16+15+16)$$

$$= \frac{1}{3}16 + \frac{1}{3}15 + \frac{1}{3}16 = 15.67$$

SMA3, SMA4 or SMA2, which one to use?

Cars Sold

Forecast accuracy as criteria:

The smaller, the forecast error, the better.

Week	Cars	SMA3	$A_t - F_t$ Error	$\lvert A_t - F_t \rvert$ Abs Error	$(A_t - F_t)^2$ Sqr Err	$\frac{\lvert A_t - F_t \rvert}{A_t} * 100\%$ Abs % Error
9	16	16.33	-.33	.33	.109	$\lvert\frac{16-16.33}{16}\rvert * 100\% = 2.063\%$
10	13	15.67	-2.67	2.67	7.129	$\lvert\frac{13-15.67}{13}\rvert * 100\% = 20.54\%$
		Sum:	-3.00	3.00	SSE (Sum of Sqrd Error) 7.238	22.603%
		Mean:	-1.50	1.50	3.619	11.302% MAPE
				MAD	MSE (mean sqrd Error)	
				RMSE	1.902	

Week	Cars	SMA2	Error	Abs Error	Sqr Err	Abs % Error
9	16	15.50	.5	.5	.25	$.5/16 = 3.13\%$
10	13	15.50	-2.50	2.50	6.25	$2.5/13 = 19.23\%$
		Sum:	-2.00	3.00	6.50	22.36%
		Mean:	-1.00	1.50	3.25	11.18%
				MAD	MSE	MAPE

SMA3 OR SMA2 ?

use Excel to compute Absolute Value: =ABS(Cell)

use Excel to compute Sum of squared Error (SSE)

=SUMXMY2(Cells of X, Cells of Y)

N SSE for SMA3, =SUMXMY2(B10:B12, C10:C12)

use Excel to compute Mean squared Error (MSE)

=SUMXMY2(X , Y)/COUNT(X)

w_3, w_2, w_1

Cars Sold — Weights for WMA 3 : .2, .3, .5

week	cars	WMA 3	WMA 2 (.4 & .6)
⋮	⋮		
6	18		
7	16		
8	15		
9	16	$15*.5+16*.3+18*.2$ $=15.90$	$15*.6+16*.4$ $=15.40$
10	13	$16*.5+15*.3+16*.2$ $=15.7$	$16*.6+15*.4$ $=15.60$
11		$13*.5+16*.3+15*.2$ $=14.3$	$13*.6+16*.4$ $=14.20$
12		$14.3*.5+13*.3+16*.2$ $=14.25$	$14.20*.6+13*.4$ $=13.72$

	weight
w_3	.2
w_2	.3
w_1	.5
Sum	1.00

= Sum (weights)

Equ for WMA

$$F_t = A_{t-1} * W_1 + A_{t-2} * W_2 + \cdots A_{t-k} * W_k$$

$$F_{10} = A_9 * W_1 + A_8 * W_2 + A_7 * W_3$$
$$= 16 * .5 + 15 * .3 + 16 * .2 = 15.7$$

Use Excel to compute WMA forecast:

in D11: =SUMPRODUCT(B8:B10, $H$10:$H$12)

Don't use: =B8 * $H$10 + ⋯ + ⋯ Due to higher chance of Error

Use Excel to compute MSE for WMA 3:

=SUMXMY2(B10:B11, D10:D11)/COUNT(B10:B11)

Use Excel to compute MSE for WMA 2 in Column F

=SUMXMY2(B10:B11, F10:F11)/COUNT(B10:B11)

Exp model in forecasting

Week	cars	Exp(.1)	Exp(.6)	EXP(.6)
1	19	19		19
2	14	19 + .1*(19-19)=		19 + .6*(19-19)
⋮				
8	15	17.094		
9	16	17.094 + .1*(15-17.094)=16.885		
10	13	16.885 + .1*(16-16.885)=16.797		
11		16.797 + .1*(13-16.797)=16.417		
12		16.417 + .1*(16.417-16.417)=16.417		

Equ for Exp model

$$F_t = F_{t-1} + \alpha(A_{t-1} - F_{t-1})$$

$$F_9 = F_8 + \alpha(A_8 - F_8) = 17.094 + .1*(15 - 17.094) = 16.885$$

Use Excel to compute Exp forecast for week 9:

In E10: =E9 + $H$7*(B9-E9)

Excel formula to compute Exp forecast for week 10:

in E11: =E10 + $H$7*(B10-E10)

Use Excel to compute MSE for Exp(.1) forecast:

=sumxmy2(B10:B11, E10:E11)/count(B10:B11)

$$MSE = \frac{SSE}{n} = \frac{\sum_{t=1}^{n}(A_t - F_t)^2}{n}$$

↳ matching rows

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| * 100\%$$

Forecast 1 Problem

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Week	Cars	SMA(3)	SMA(4)	SMA(2)		WMA(3)	WMA(2)		EXP(.1)	EXP(.6)			Weights	
2	1	19								19.000	19.000		w3	0.2	
3	2	14								19.000	19.000		w2	0.3	
4	3	12			16.50			16.000		18.500	16.000		w1	0.5	
5	4	13	15.000		13.00		14.000	12.800		17.850	13.600		Sum	1.0	=SUM(H2:H4)
6	5	15	13.000	14.500	12.50		12.900	12.600		17.365	13.240				
7	6	18	13.333	13.500	14.00		13.800	14.200		17.129	14.296		α value	0.1	0.6
8	7	16	15.333	14.500	16.50		16.100	16.800		17.216	16.518			Weights	
9	8	15	16.333	15.500	17.00		16.400	16.800		17.094	16.207		w2	0.4	
10	9	16	16.333	16.000	15.50		15.900	15.400		16.885	15.483		w1	0.6	
11	10	13	15.667	16.250	15.50		15.700	15.600		16.796	15.793		sum=	1	
12	11		14.667	15.000	14.50		14.300	14.200		16.417	14.117				
13	12		14.556	14.750	13.75		14.250	13.720		16.417	14.117				
14															
15			3.611	5.281	3.250		3.650	3.560		7.597	4.035				
16			MSE	MSE	MSE		MSE	MSE		MSE	MSE				

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Week	Cars	SMA(3)	SMA(4	SMA(2		WMA(3)	WMA(2)		EXP(.1)	EXP(.6			Weights	
2	1	19								=B2	=B2		w3	0.2	
3	2	14								=J2+$N$7*(B2-J2)	=K2+$		w2	0.3	
4	3	12			=AVER			=SUMPRC		=J3+$N$7*(B3-J3)	=K3+$		w1	0.5	
5	4	13	=AVERAGE(B2:B4)		=AVER		=SUMPRODUCT(B2:B4,$N$2:$N$4)	=SUMPRC		=J4+$N$7*(B4-J4)	=K4+$		Sum	=SUM(I	=SUM(H2:H4)
6	5	15	=AVERAGE(B3:B5)	=AVER	=AVER		=SUMPRODUCT(B3:B5,$N$2:$N$4)	=SUMPRC		=J5+$N$7*(B5-J5)	=K5+$				
7	6	18	=AVERAGE(B4:B6)	=AVER	=AVER		=SUMPRODUCT(B4:B6,$N$2:$N$4)	=SUMPRC		=J6+$N$7*(B6-J6)	=K6+$		α value	0.1	0.6
8	7	16	=AVERAGE(B5:B7)	=AVER	=AVER		=SUMPRODUCT(B5:B7,$N$2:$N$4)	=SUMPRC		=J7+$N$7*(B7-J7)	=K7+$			Weights	
9	8	15	=AVERAGE(B6:B8)	=AVER	=AVER		=SUMPRODUCT(B6:B8,$N$2:$N$4)	=SUMPRC		=J8+$N$7*(B8-J8)	=K8+$		w2	0.4	
10	9	16	=AVERAGE(B7:B9)	=AVER	=AVER		=SUMPRODUCT(B7:B9,$N$2:$N$4)	=SUMPRC		=J9+$N$7*(B9-J9)	=K9+$		w1	0.6	
11	10	13	=AVERAGE(B8:B10)	=AVER	=AVER		=SUMPRODUCT(B8:B10,$N$2:$N$4)	=SUMPRC		=J10+$N$7*(B10-J10)	=K10-		sum=	=SUM(I	
12	11		=AVERAGE(B9:B11)	=AVER	=AVER		=SUMPRODUCT(B9:B11,$N$2:$N$4)	=SUMPRC		=J11+$N$7*(B11-J11)	=K11-				
13	12		14.5555555555556	14.75	13.75		14.25	13.72		16.416587965	14.11				
14															
15			=SUMXMY2(B10:B11,(	=SUM	=SUM		=SUMXMY2(B10:B11,G10:G11)/COUNT(B10:B11)	=SUMXM		=SUMXMY2(B10:B11,J10	=SUM				
16			MSE	MSE	MSE		MSE	MSE		MSE	MSE				

	A	B	C	D	E	F	G	H	I	J	K	L
1	Week	Cars (At)	SMA(3) Ft	Err = At-Ft	|At-Ft| Abs Err	(At-Ft)^2 Sqr Err			|(At-Ft)/At|*100% Abs % Err			SMA(2)
2	9	16	16.333	-0.333	0.333	0.111			2.083%			15.50
3	10	13	15.667	-2.667	2.667	7.111			20.513%			15.50
4												
5			Sum=	-3.000	3.000	7.222	7.222	SSE	22.596%			
6			Mean=	-1.500	1.500	3.611	3.611	MSE	11.298%			
7					MAD	1.900		RMSE	MAPE			
8												
9												
10	Week	Cars (At)	SMA(2) Ft	Err = At-Ft	|At-Ft| Abs Err	(At-Ft)^2 Sqr Err			|(At-Ft)/At|*100% Abs % Err			
11	9	16	15.50	0.500	0.500	0.250			3.125%			
12	10	13	15.50	-2.500	2.500	6.250			19.231%			
13												
14			Sum=	-2.000	3.000	6.500	6.500	SSE	22.356%			
15			Mean=	-1.000	1.500	3.250	3.250	MSE	11.178%			
16					MAD	1.803		RMSE	MAPE			

	A	B	C	D	E	F	G	H	I
1	Week	ars (A	SMA(3) Ft	Err = At-Ft	|At-Ft| Abs Err	(At-Ft)^2 Sqr Err			t-Ft)/At|*100% Abs %
2	9	16	16.33333	=B2-C2	=ABS(D2)	=D2*D2			=E2/B2
3	10	13	15.66666	=B3-C3	=ABS(D3)	=D3*D3			=E3/B3
4									
5			Sum=	=SUM(D2:D3)	=SUM(E2:E3)	=SUM(F2:F3)	=SUMXMY2(B2:B3,C2:C3)	SSE	=SUM(I2:I3)
6			Mean=	=AVERAGE(D2:D3)	=AVERAGE(E2:E3)	=AVERAGE(F2:F3)	=SUMXMY2(B2:B3,C2:C3)/COUNT(B2:B3)	MSE	=AVERAGE(I2:I3)
7					MAD	=SQRT(F6)		RMSE	MAPE
8									
9									
10	Week	ars (A	SMA(2) Ft	Err = At-Ft	|At-Ft| Abs Err	(At-Ft)^2 Sqr Err			t-Ft)/At|*100% Abs %
11	9	16	15.5	=B11-C11	=ABS(D11)	=D11*D11			=E11/B11
12	10	13	15.5	=B12-C12	=ABS(D12)	=D12*D12			=E12/B12
13									
14			Sum=	=SUM(D11:D12)	=SUM(E11:E12)	=SUM(F11:F12)	=SUMXMY2(B11:B12,C11:C12)	SSE	=SUM(I11:I12)
15			Mean=	=AVERAGE(D11:D12)	=AVERAGE(E11:E12)	=AVERAGE(F11:F12)	=SUMXMY2(B11:B12,C11:C12)/COUNT(B11:B12)	MSE	=AVERAGE(I11:I12)
16					MAD	=SQRT(F15)		RMSE	MAPE

Forecast 2 Problem is from Test 1, Summer 2008

Requirements:

1) Manually derive the required answers;
2) Manually computer MSE for MA, WMA and EXP;
3) Use Excel to verify the answers;
4) Use Excel to find the optimal weights for WMA and the optimal smoothing constant α by minimizing corresponding MSE;
5) Compare the MSEs for WMA forecasting with current and optimal weights; and
6) Compare the MSEs for EXP forecasting with current and optimal smoothing constant α.

1. The Table 1 below shows the number of speeding tickets Harrisonburg Polices wrote during the last few days. Answer the following questions based on the information given in Tables 1, 2 and 3.

Table 1

	A	B	C	D	E	F	G	H
1	Day	Tickets	SMA (4)	WMA(3)	EXP		weights	
2	1	33			33.000		w3	0.1
3	2	25			33.000		w2	0.2
4	3	16			31.400		w1	0.7
5	4	24			28.320		sum	1
6	5	8	(1)	(5)	27.456			
7	6	6	(2)	(6)	(9)		alpha	0.2
8	7		(3)	(7)	(10)			
9	8		(4)	(8)	(11)			

Table 2

	A	B	C
11	Day	Tickets	WMA(3)
12	5	8	22.500
13	6	6	12.000
14			
15		MSE	(12)

Table 3

Methods	MSE
SMA (4)	211.156
WMA(3)	123.125
EXP	343.529

(1) What is the three – period weighted moving average forecast for the number of tickets for Day 7 using the weights 0.7, 0.2 and 0.1 with the largest weight for the most recent day's data?

a. What is the equation to be used?

b. What is the Excel@ formula to be used for it in Cell D8?

c. What is the answer for it? (put the numbers in the equation and derive the answer)

(2) What is the Exponential smooth forecast for Day 7 using α value of 0.2?

a. What is the equation to be used?

b. What is the Excel@ formula to be used for it in Cell E8?

c. What is the answer for it? (put the numbers in the equation and derive the answer)

(3) What is the MSE for the WMA(3) forecasts developed for Days 5 and 6 as given in Table 2 above?

a. What is the equation to be used?

b. What is the Excel@ formula to be used for it in Cell C15?

c. What is the answer for it? (put the numbers in the equation and derive the answer)

(4) Table 3 shows the Mean Squared Forecasting Errors (MSE) for the number of tickets data with three forecasting methods.

a. Which method would you recommend to be used in developing forecasts in the next few days and

b. why?

Solution to Forecast 2 Problem

SMA(4):

① $F_{Day\,5} = \frac{A_4 + A_3 + A_2 + A_1}{4} = (24 + 16 + 25 + 33)/4 = 98/4 = 24.50$

② $F_{Day\,6} = (8 + 24 + 16 + 25)/4 = 73/4 = 18.25$

③ $F_{Day\,7} = (6 + 8 + 24 + 16)/4 = 54/4 = 13.50$

④ $F_{Day\,8} = (13.50 + 6 + 8 + 24)/4 = 51.5/4 = 12.875$

WMA(3)

⑤ $F_{Day5} = 24*.7 + 16*.2 + 25*.1 = 22.50$
$= A_4 * W_1 + A_3 * W_2 + A_2 * W_3$

⑥ $F_{Day6} = 8*.7 + 24*.2 + 16*.1 = 12.00$

⑦ $F_{Day7} = 6*.7 + 8*.2 + 24*.1 = 8.20$

⑧ $F_{Day8} = 8.20*.7 + 6*.2 + 8*.1 = 7.74$

MSE for WMA(3):

Day	At Ticket	WMA(3)	$A_t - F_t$ Error	$(A_t - F_t)^2$ Sqd Error
5	8	22.5		
6	6	12.0		

SSE
MSE
RMSE

$$SSE = \sum_{i=1}^{n} (A_t - F_t)^2 \qquad MSE = \frac{SSE}{n} \qquad RMSE = \sqrt{MSE}$$

F7: Use Excel for WMA: =SUMPRODUCT(B5:B7, $H$2:$H$4)

MSE in C15: =SUMXMY2(B12:B13, C12:C13)/COUNT(B12:B13)
↳ Tickets ↳ WMA(3)

Exp ($\alpha = .2$) $F_{Day5} = 27.456$

⑨ $F_{Day6} = F_{Day5} + \alpha(A_5 - F_5)$

$= 27.456 + .2*(8 - 27.456) = 23.565$

23.565

⑩ $F_7 = F_6 + .2*(A_6 - F_6)$

$= 23.565 * .2 * (6 - 23.565) = 20.052$

$\Downarrow F_7$

⑪ $F_8 = F_7 + .2*(A_7 - F_7)$

$= 20.052 + .2 * (20.052 - 20.052) = 20.052$

use Excel for Exp:

F_{Day7} in E8: $= E7 + \$H\$7 * (B7 - E7)$

$L\ F_{Day6}$ $L\ \alpha$ $L\ F_{Day6}$ $L\ A_{Day6}$

Forecast 2 Problem Solutions

	A	B	C	D	E	F	G	H
1	Day	Tickets	SMA(4)	wma(3)	EXP			Weights
2	1	33			33		w3	0.1
3	2	25			33		w2	0.2
4	3	16			31.400		w1	0.7
5	4	24		19.500	28.320		Sum=	1
6	5	8	24.500	22.500	27.456			
7	6	6	18.250	12.000	23.565		alpha	0.2
8	7		13.500	8.200	20.052			
9	8		12.875	7.740	20.052			
10								
11		MSE	211.156	123.125	343.529			
12								
13	Day	Tickets	wma(3)	Error	Error^2			
14	5	8	22.50	-14.50	210.25			
15	6	6	12.00	-6.00	36			
16								
17			Sum		246.25			
18			MSE		123.125			

	A	B	C	D	E	F	G	H
1	Day	Tickets	SMA(4)	wma(3)	EXP			Weights
2	1	33			=B2		w3	0.1
3	2	25			=E2+$H$7*(B2-E2)		w2	0.2
4	3	16			=E3+$H$7*(B3-E3)		w1	0.7
5	4	24		=SUMPRODUCT(B2:B4,$H$2:$H$4)	=E4+$H$7*(B4-E4)		Sum=	=SUM(H2:H4)
6	5	8	=AVERAGE(B2:B5)	=SUMPRODUCT(B3:B5,$H$2:$H$4)	=E5+$H$7*(B5-E5)			
7	6	6	=AVERAGE(B3:B6)	=SUMPRODUCT(B4:B6,$H$2:$H$4)	=E6+$H$7*(B6-E6)		alpha	0.2
8	7		=AVERAGE(B4:B7)	=SUMPRODUCT(B5:B7,$H$2:$H$4)	=E7+$H$7*(B7-E7)			
9	8		12.875	7.74	20.05184			
10								
11		MSE	=SUMXMY2(B6:B7,C	=SUMXMY2(B6:B7,D6:D7)/COUNT(B6:B7)	=SUMXMY2(B6:B7,E			
12								
13	Day	Tickets	wma(3)	Error	Error^2			
14	5	8	22.5	=B14-C14	=D14*D14			
15	6	6	12	=B15-C15	=D15*D15			
16								
17			Sum		=SUM(E14:E15)			
18			MSE		=E17/2			

Use Excel Solver to find optimal weights in WMA

MIN: MSE (=SUMXMY2C ,)/COUNTC)

S.T. $W_1, W_2, \ldots W_K \geq 0$

$W_1, W_2, \ldots W_K \leq 1$

$SUM(W_1, W_2, \ldots, W_K) = 1$

Adjustable Cells, Where $W_1, W_2, \ldots, W_K$ are

Use Excel Solver to find optimal smoothing constant α in EXP.

MIN: MSE (=SUMXMY2C ,)/COUNTC)

S.T. $\alpha \geq 0$

$\alpha \leq 1$

Adjustable Cell: Where α is