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COB291, Management Science, Test 1. Fall 2008

Decision Analysis

1. DukeFan Inc. develops a predictive model to identify buyers and non buyers for its products. Among the 100 customers studied from their database, 30 of them are buyers. The predictive model will put each customer in the study into one of the three groups: identified as buyer, identified as non buyer and identified as inconclusive. When the predictive model is applied to buyers, it successfully identifies buyers 75% of times, 11% of times as inconclusive and the rest as non buyers. When the predictive model is applied to non buyers, it successfully identifies non buyers 85% of times, and 7% of time as buyers.

1). (10 pts.) Use the information provided in the problem above to fill the tables below with prior probability, conditional probability, joint probability and posterior probability, each with 3-decimal points of accuracy. (must show details with equations and appropriate notations)

	A	B	C	D	E	F
1			Buyers	Non Buyers		
2		Prior Probability				
3						
4						
5		Conditional	Buyers	Non Buyers		
6						
7						
8						
9						
10						
11		Joint	Buyers	Non Buyers		
12						
13						
14						
15						
16						
17		Posterior	Buyers	Non Buyers		
18						
19						
20						
21						
22						

2). (3 pt) Based on your results above, what is the probability that a randomly selected customer is a non buyer? (must show details with equations and appropriate notations)

3). (4 pts.) Based on your results above, among the buyers, what is the probability that a randomly selected customer is identified as non buyer? (must show details with equations and appropriate notations)

4). (4 pts) Based on your results above, what is the probability that a randomly selected customer is either a buyer or is identified as buyer? (must show details with equations and appropriate notations)

5). (4 pts.) Based on your results above, among those identified as non buyers, what is the probability that a randomly selected customer is actually a buyer? (must show details with equations and appropriate notations)

--------- **END of DukeFan Problem** -------

DecisionT1F08.pdf

Duke Fan Inc.

	Actual Buyers	NonBuyers
Prior Probability (Simple or marginal prob.)	$30/100 = .30$	

$P(\text{Buyers}) = 30/100 = .30$

$P(\text{NonBuyers}) = 1 - P(\text{Buyers}) = 1 - .30 = .70$

Probability of complementary Events A versus $\bar{A}$, $P(A) + P(\bar{A}) = 1$

or $P(A) = 1 - P(\bar{A})$

$P(\text{NonBuyers}) = 1 - P(\overline{\text{NonBuyers}}) = .70$

so, $P(\overline{\text{NonBuyers}}) = 1 - .70 = .30 = P(\text{Buyers})$

or Events Buyers and NonBuyers are complementary events.

Events of Buyers and NonBuyers are

Collectively exhaustive (No leftover)

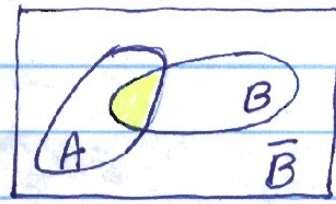
$P(\text{Buyers}) + P(\text{NonBuyers}) = .30 + .70 = 1.00$

Events of Buyers and NonBuyers are

Mutually Exclusive (No Overlap)

One cannot be a Buyer and a NonBuyer at the same time

Marginal Probability



$$P(A) = P(A \cap B) + P(A \cap \overline{B})$$

$$P(\text{Id Buyers}) = P(\text{Id Buyers} \cap \text{Buyers}) + $$
$$+ P(\text{Id Buyers} \cap \text{NonBuyers})$$
$$=$$

Conditional Probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$$P(\text{Id Buyers}|\text{Buyers}) = \frac{P(\text{Id Buyers} \cap \text{Buyers})}{P(\text{Buyers})} = .75$$

So: $P(\text{Id Buyers} \cap \text{Buyers}) = P(\text{Id Buyers}|\text{Buyers})P(\text{Buyers}) = .75 * .30$
$= .225$

Table for Conditional prob.	Buyers	NonBuyers
Id Buyers	.75	.07
Id NonBuyers		.85
Id Inconclusive	.11	
	1.00	1.00

$$P(\text{Id NonBuyers}|\text{NonBuyers}) = \frac{P(\text{Id NonBuyer} \cap \text{NonBuyers})}{P(\text{NonBuyers})} = .85$$

$$P(\text{Id NonBuyers} \cap \text{NonBuyers}) = P(\text{Id NonBuyers}|\text{NonBuyers})P(\text{NonBuyers})$$
$$= .85 * .70 = .595$$

Note, in Joint probability, $\cap$, and, & are interchangable.

$$P(A \cap B) = P(A \text{ and } B) = P(A \& B)$$

However: $P(A \cap B) \neq P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$P(\text{Id Inconclusive} \mid \text{Buyers}) = \frac{P(\text{Id Inc} \cap \text{Buy})}{P(\text{Buyers})} = .11$$

$$P(\text{Id Inc} \cap \text{Buyers}) = P(\text{Id Inc} \mid \text{Buy})\, P(\text{Buyers}) = .11 * .30 = .033$$

$$P(\text{Id NonBuyer} \mid \text{Buyers}) = \frac{P(\text{Id NonBuyer} \cap \text{Buyer})}{P(\text{Buyers})} = 1 - .75 - .11 = .14$$

$$P(\text{Id NonBuyer} \cap \text{Buyer}) = P(\text{Id NonBuyer} \mid \text{Buyer})\, P(\text{Buyer})$$
$$= .14 * .30 = .042$$

$$P(\text{Id Buyers} \mid \text{NonBuyer}) = \frac{P(\text{Id Buyer} \cap \text{NonBuyer})}{P(\text{NonBuyer})} = .07$$

$$P(\text{Id Buyer} \cap \text{NonBuyer}) = P(\text{Id Buyer} \mid \text{NonBuyer})\, P(\text{Non Buyer})$$
$$= .07 * .70 = .049$$

$$P(\text{Id Inconclusive} \mid \text{NonBuyer}) = \frac{P(\text{Id Inc} \cap \text{NonBuyer})}{P(\text{NonBuyer})} = 1 - .07 - .85$$
$$= .08$$

$$P(\text{Id Inc} \cap \text{NonBuyer}) = P(\text{Id Inc} \mid \text{NonBuyer})\, P(\text{NonBuyer})$$
$$= .08 * .70 = .056$$

Multiplication Rule:

from: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(A \mid B)\, P(B)$

When events A and B and independent, $P(A \mid B) = P(A)$

So: $P(A \cap B) = P(A \mid B)\, P(B) = P(A)\, P(B)$

Joint Probability $(A \cap B)$

	Buyers	NonBuyers	Col Sum
Id Buyers	.225	.049	.274
Id NonBuyers	.042	.595	.637
Id Inconclusive	.033	.056	.089
Row Sum	P(Buyer) = .30	P(NonBuyer) = .70	1.00

P(Id NonBuyer $\cap$ NonBuyr) = .595

Union Probability

$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

If Events A and B are mutually exclusive, $P(A \cap B) = \phi$
then. $P(A \cup B) = P(A) + P(B)$

$P(\text{Id Buyers} \cup \text{Buyers}) = P(\text{Id Buyers}) + P(\text{Buyers}) - P(\text{Id Buyers} \cap \text{Buyers})$
$= .274 + .30 - .225 = .349$

$P(\text{Id Buyers} \cup \text{Id NonB}) = P(\text{Id Buyers}) + P(\text{Id NonBuyers})$
$= .274 + .637 = .911$

Because: $P(\text{Id Buyers} \cap \text{Id NonBuyers}) = \phi$

Baye's Theorem & Posterior Prob: $P(B) = P(B \cap A) + P(B \cap \bar{A})$

apply marginal probability Rule $P(A) = P(A \cap B) + P(A \cap \bar{B})$

and Multiplication Rule $P(A \cap B) = P(A)\,P(B|A)$

$$= P(B)\,P(A|B)$$

to conditional probability $P(A|B) = P(A \cap B)/P(B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)\,P(B|A)}{P(A)P(B|A) + P(\bar{A})(P(B|\bar{A})}$$

$$P(A \cap B) = P(A)\,P(B|A)$$

$$P(B) = \underbrace{P(B \cap A)}_{P(A)\,P(B|A)} + \underbrace{P(B \cap \bar{A})}_{P(\bar{A})\,P(B|\bar{A})}$$

$$P(Buy|Id\,Buy) = \frac{P(Buy \cap Id\,Buy)}{P(Id\,Buy)}$$

$$= \frac{P(Buyer)P(Id\,Buyer|Buyer)}{P(Buyer)P(Id\,Buyer|Buyer) + P(NonBuy)P(Id\,Buyr|NonBuy)}$$

$$= \frac{.30 * .75}{.30 * .75 + .70 * .07} = \frac{.225}{.225 + .049} = \frac{.225}{.274}$$

$$= .821$$

$$P(Non\,Buy|Id\,Buyer) = \frac{P(Non\,Buy \cap Id\,Buyer)}{P(Id\,Buyer)}$$

$$= \frac{P(Non\,Buyer)\,P(Id\,Buyer|NonBuyer)}{P(NonBuy)P(Id\,Buy|NonBuy) + P(Buy)\,P(Id\,Buy|Buy)}$$

$$= \frac{.70 * .07}{.274} = \frac{.049}{.274} = .179$$

$P(Buyer|Id\,Buyer) + P(Non\,Buyer|Id\,Buyer) = .821 + .179 =$

$= 1.00$

$$P(\text{Buyer} \mid \text{Id NonBuy}) = \frac{P(\text{Buyer} \cap \text{Id NonBuyer})}{P(\text{Id NonBuyer})}$$

$$= \frac{P(\text{Buyer})P(\text{Id NonBuy} \mid \text{Buyer})}{P(\text{Buy})P(\text{Id NonBuy} \mid \text{Buy}) + P(\text{NonBuy})P(\text{Id NonBuy} \mid \text{NonBuy})}$$

$$= \frac{.30 * .14}{.30 * .14 + .70 * .85} = \frac{.042}{.042 + .595} = \frac{.042}{.637} = .066$$

$$P(\text{NonBuyer} \mid \text{Id NonBuy}) = \frac{P(\text{NonBuyer} \cap \text{Id NonBuyer})}{P(\text{Id NonBuyer})}$$

$$= \frac{P(\text{NonBuyer})P(\text{Id NonBuyer} \mid \text{NonBuy})}{P(\text{NonBuy})P(\text{Id NonBuyer} \mid \text{NonBuyer}) + P(\text{Buy})P(\text{Id NonBuy} \mid \text{Buy})}$$

$$= \frac{.70 * .85}{.70 * .85 + .30 * .14} = \frac{.595}{.637} = .934$$

$$P(\text{Buy} \mid \text{Id NonBuy}) + P(\text{NonBuy} \mid \text{Id NonBuy}) = .066 + .934 = 1.00$$

$$P(\text{Buyer} \mid \text{Id Incon}) = \frac{P(\text{Buyer} \cap \text{Id Inc})}{P(\text{Id Incon})}$$

$$= \frac{P(\text{Buy})P(\text{Id Inc} \mid \text{Buy})}{P(\text{Buy})P(\text{Id Inc} \mid \text{Buy}) + P(\text{NonBuy})P(\text{Id Inc} \mid \text{NonBuy})}$$

$$= \frac{.30 * .11}{.30 * .11 + .70 * .08} = \frac{.033}{.033 + .056} = \frac{.033}{.089} = .371$$

$$P(\text{NonBuyer} \mid \text{Id Inc}) = \frac{P(\text{NonBuyer} \cap \text{Id Inc})}{P(\text{Id Inc})}$$

$$= \frac{P(\text{NonBuy})P(\text{Id Inc} \mid \text{NonBuy})}{P(\text{NonBuy})P(\text{Id Inc} \mid \text{NonBuy}) + P(\text{Buy})(P(\text{Id Inc} \mid \text{Buy})}$$

$$= \frac{.7 * .08}{.7 * .08 + .30 * .11} = \frac{.056}{.089} = .629$$

$$P(\text{Buyer} \mid \text{Id Inc}) + P(\text{NonBuyer} \mid \text{Id Inc}) = .629 + .371 = 1.00$$

2. A vendor at Bluestone Stadium must determine whether to sell ice cream or soft drinks at Saturday's game. The vendor believes that the profit made will depend on the weather. For a cool weather, she can make \$50 if she sells soft drinks, and she can make \$30 if she sells ice cream. For a warm weather, she can make \$60 if she sells soft drinks, and she can make \$90 if she sells ice cream. Based on her past experience at this time of year, the vendor estimates the probability of warm weather as 0.60.

I). With the information given in the problem above, you may use either a payoff table or a decision tree when you answer the following questions. A portion of Excel spreadsheet is provided for your convenience. You must show details with appropriate equations and terms and keep 3 decimal points.

a). (5 pts) What is the payoff table for the problem? You may use the blank table on the next page.

b). (6 pts) Construct the decision tree to find expected moneitary values (EMV) and max EMV for the problem. You may use the structure of the tree on the next page.

c). (10 pts.) What is the maximum or optimal expected profit (EMV) = ________________? (You must show details)

d). (4 pts.) What is the optimal strategy for the vendor and why?

Bluestone Stadium

Sell < Ice cream / Soft drinks

Weather < Cool / Warm

Payoff Table

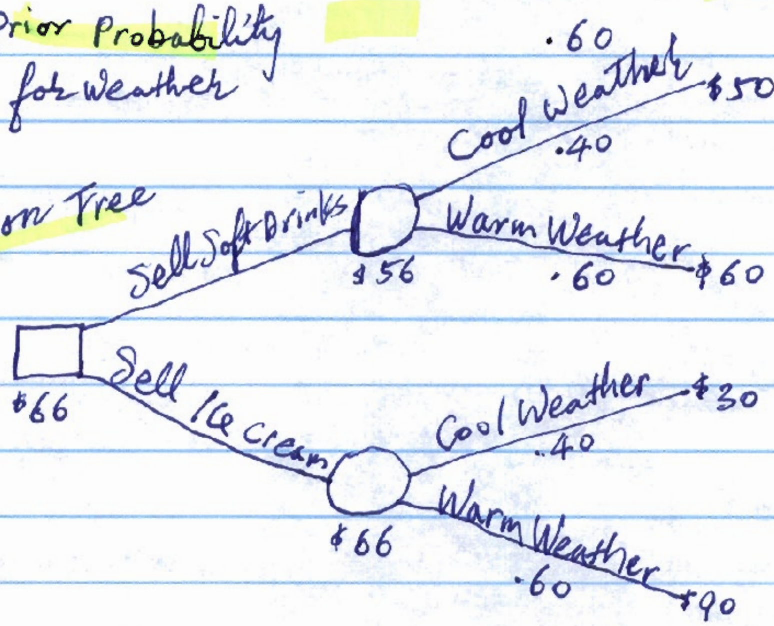
Alternative Course of Action	Events: Weather — Cool	Weather — Warm	Expected Monetary Value (EMV)
Sell Ice Cream	\$30	\$90	$30 \times .4 + 90 \times .6 = \$66$
Sell Soft Drinks	\$50	\$60	$50 \times .4 + 60 \times .6 = \$56$

payoffs

Max EMV = Max (66, 56) = \$66

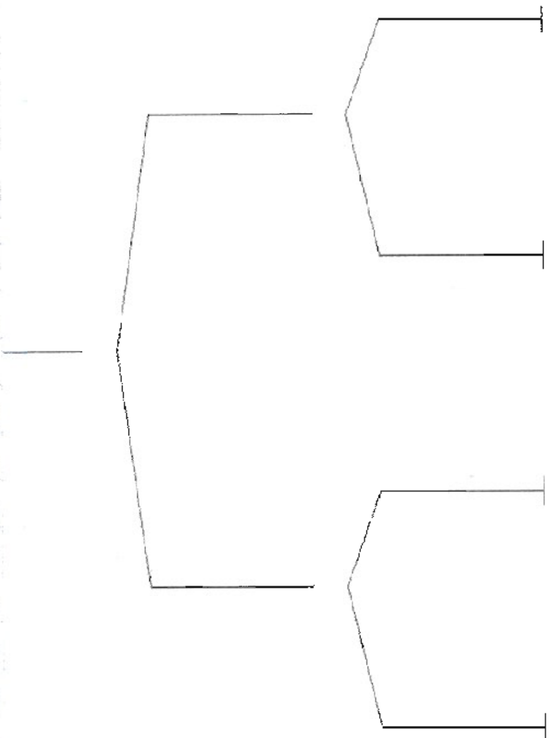
Prior Probability for Weather

Decision Tree



What is the optimal strategy ?

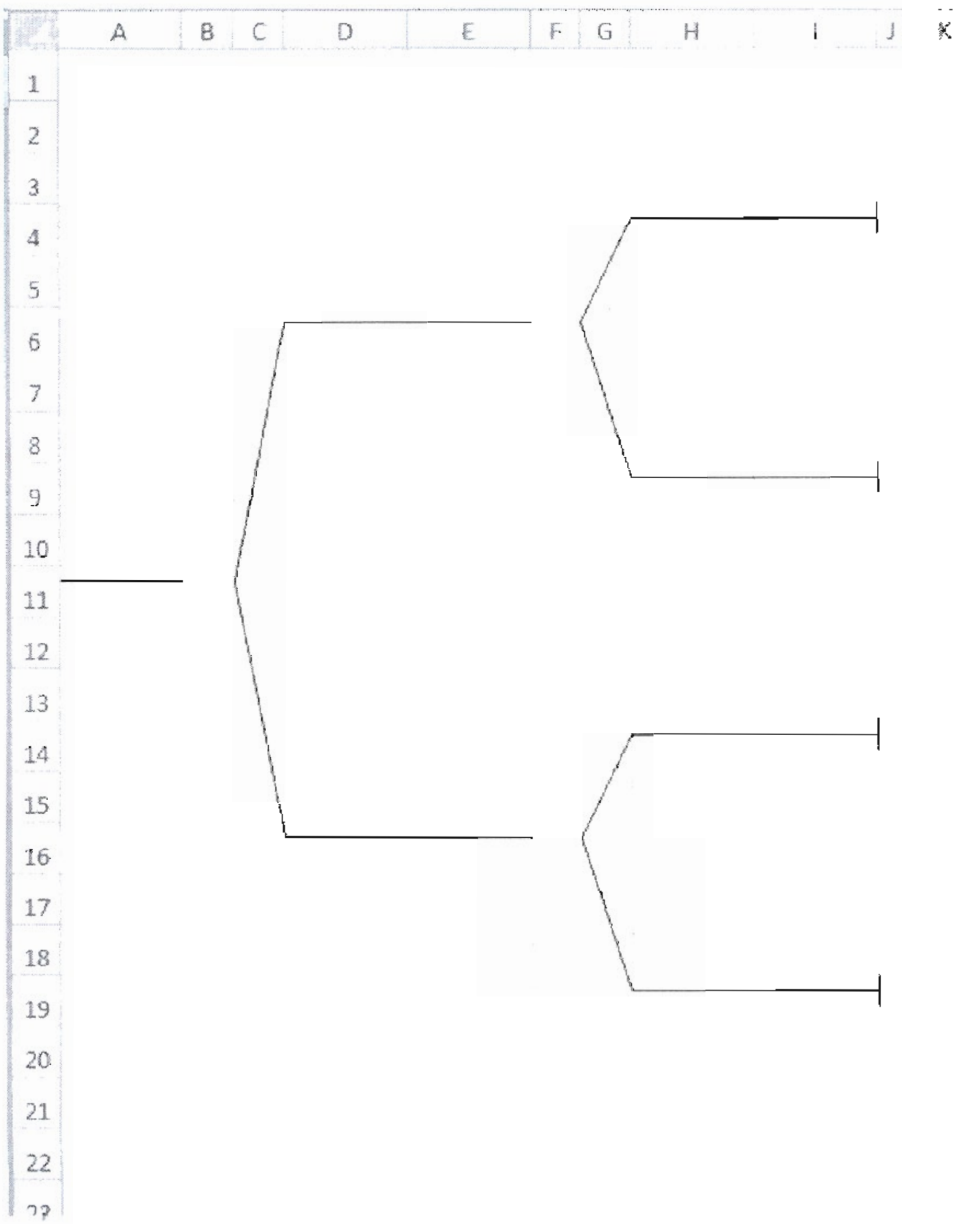
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1													Cool Weather	Warm Weather		



(2). What is EVwPI and EVPI?

a). (8 pts) Construct the decision tree to find the expected value with perfect information (EVwPI) for the problem.

b). (3 pts.) What is the expected value of perfect information (EVPI) for the problem? (You must show details)



Expected Value with Perfect Information (EVwPI)

$= \max(30, 50) * P(\text{Cool}) + \max(90, 60) * P(\text{Warm})$

$= 50 * .40 + 90 * .60 = \$74$

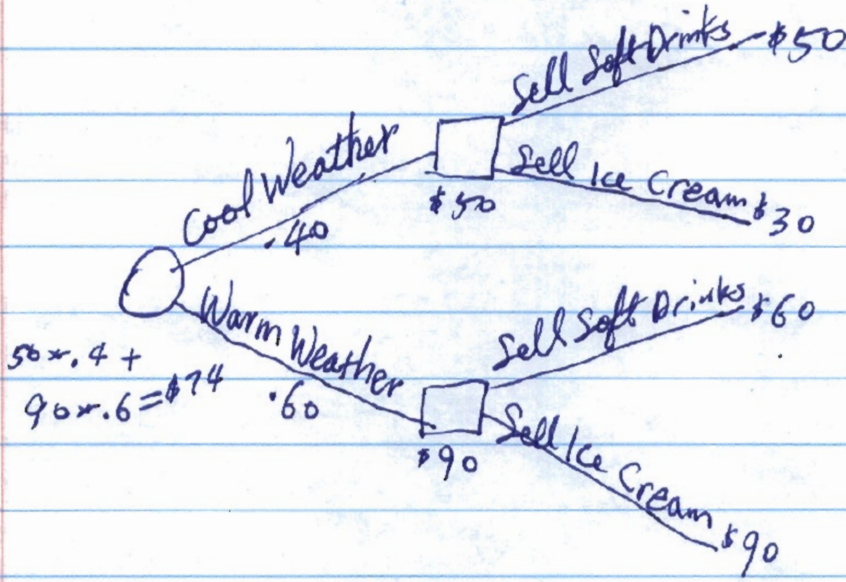
Expected Value of Perfect Information (EVPI)

$= EVwPI - EVwiPI$

$\qquad\qquad\llcorner = \max EMV$

$= \$74 - \$66 = \$8$

Use Decision Tree to get EVwPI



(EOL)

Expected Opportunity Loss or Regret Table

Events

	Cool Weather	Warm Weather	EOL
Sell Soft Drinks	Max(50,30)−50=∅	max(90,60)−60=30	18
Sell Ice Cream	Max(50,30)−30=20	Max(90,60)−90=∅	$8
Prior Prob.	.40	.60	Min(EOL)=EVPI

Value of Weather Report

cost for a weather report : \$5

Conditional probability $P(A \mid B) = P(A \cap B)/P(B)$

	Cool Weather	Warm Weather
A predict Cool Weather	.80	.30
Predict Warm Weather	.20	.70
Col sum	1.00	1.00
Prior Probability $P(B)$	.40	.60

Joint Probability $P(A \cap B) = P(B)\,P(A \mid B)$

	Cool Weather	Warm Weather	Row Sum
Predict Cool Weather	$.40 * .8 = .32$	$.60 * .30 = .18$	.50 ← P(Pred. Cool)
Predict Warm Weather	$.40 * .2 = .08$	$.60 * .70 = .42$	.50 ← P(Pred. Warm)
Col sum	.40 ← P(Cool)	.60 ← P(Warm)	

$P(\text{pred. Cool} \cap \text{Cool}) = P(\text{Cool})\,P(\text{pred Cool} \mid \text{Cool})$

$= .40 * .80 = .32$

Posterior Probability $P(\text{Cool} \mid \text{pred Cool})$, $P(\text{Warm} \mid \text{pred. Warm})$

	Cool	Warm	Row Sum
Pred. Cool	$.32/.50 = .64$	$.18/.50 = .36$	1.00
Pred. Warm	$.08/.50 = .16$	$.42/.50 = .84$	1.00

$$P(\text{Cool} \mid \text{pred. Cool}) = \frac{P(\text{Cool} \cap \text{pred Cool})}{P(\text{pred Cool})} = \frac{.32}{.50} = .64$$

$$= \frac{P(\text{Cool})\,P(\text{pred Cool} \mid \text{Cool})}{P(\text{Cool})\,P(\text{Pred Cool} \mid \text{Cool}) + P(\text{Warm})\,P(\text{Pred Cool} \mid \text{Warm})}$$

$$= \frac{.40 * .80}{.40 * .80 + .60 * .30} = \frac{.32}{.50} = .64$$

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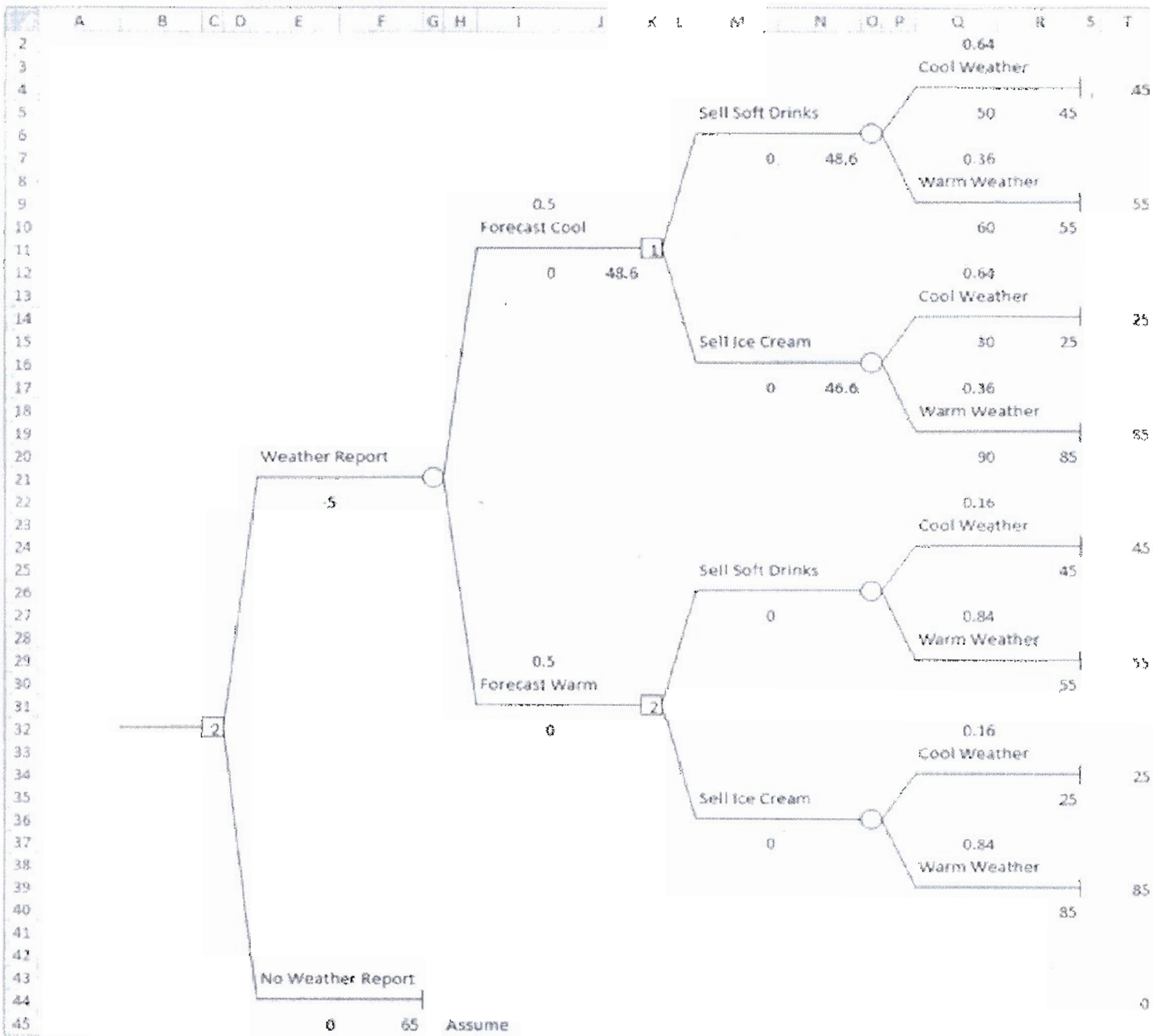
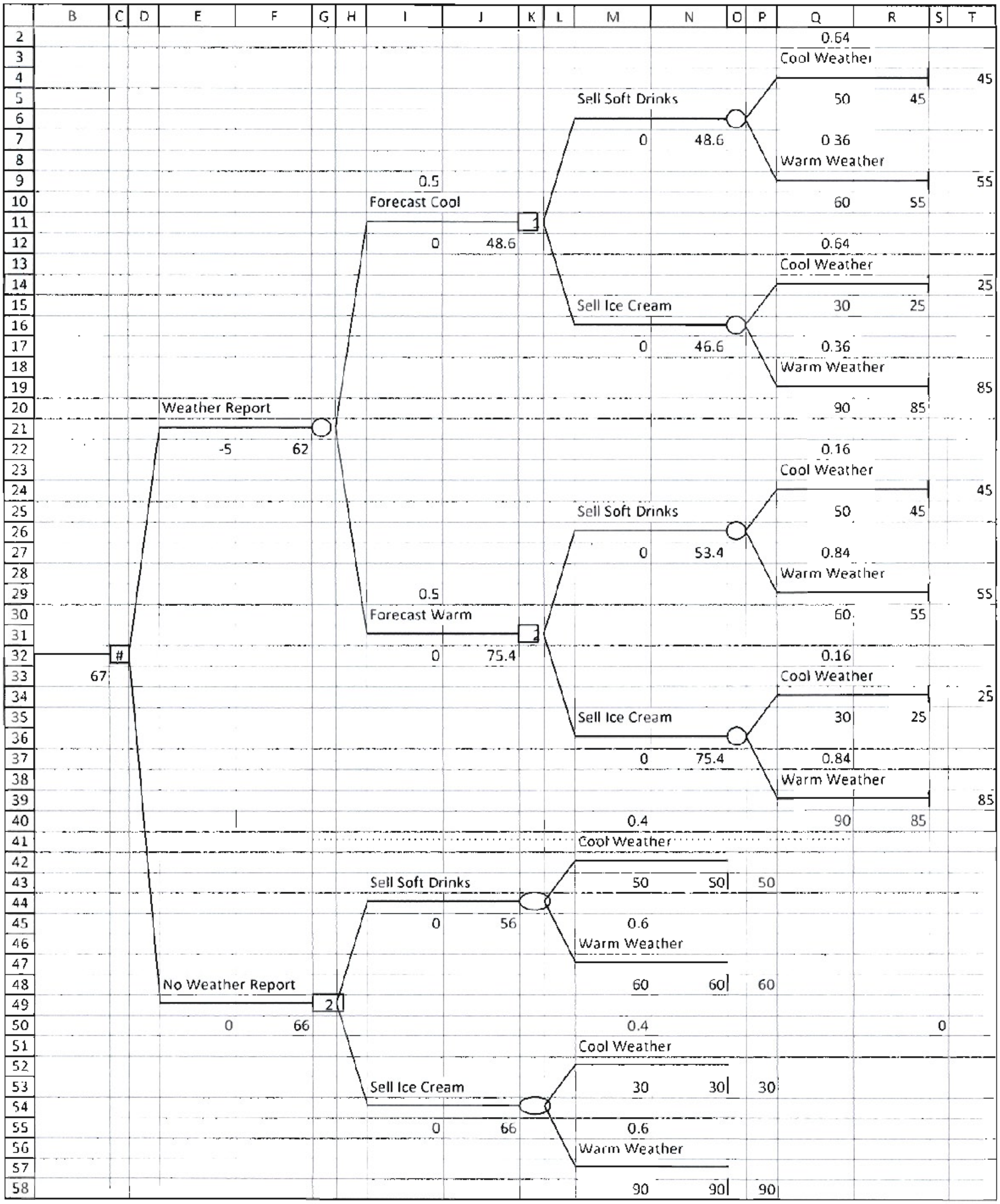
3). Given the expanded decision tree for expected value with weather information (EVwSI) below, some of the entries on the tree are taken out. Assume she pays $5 for a reliable weather report.

a). (10 pts) Compute EVwSI and EVSI for the problem. You must show details.

b). (3 pts) Compute the efficiency of the sample information and discuss your findings. You must show details.

c). (3 pts) What choice would you recommend to the vendor for the whole situation (with or without weather forecast) and why?





B C D E F G H I J K L M N O P Q R S T
0.64
Cool Weather
45
Sell Soft Drinks
50 45
0 48.6
0.36
Warm Weather
55
0.5
Forecast Cool
60 55
0 48.6
0.64
Cool Weather
25
Sell Ice Cream
30 25
0 46.6
0.36
Warm Weather
85
Weather Report
90 85
-5 62
0.16
Cool Weather
45
Sell Soft Drinks
50 45
0 53.4
0.84
Warm Weather
55
0.5
Forecast Warm
60 55
0 75.4
#
0.16
67
Cool Weather
25
Sell Ice Cream
30 25
0 75.4
0.84
Warm Weather
85
0.4
90 85
Cool Weather
Sell Soft Drinks
50 50 50
0 56
0.6
Warm Weather
No Weather Report
60 60 60
2
0 66
0.4
0
Cool Weather
Sell Ice Cream
30 30 30
0 66
0.6
Warm Weather
90 90 90