Decision Analysis

1. DukeFan Inc. develops a predictive model to identify buyers and non-buyers for its products. Among the 100 customers studied from their database, 30 of them are buyers. The predictive model will put each customer in the study into one of the three groups: identified as buyer, identified as non-buyer and identified as inconclusive. When the predictive model is applied to buyers, it successfully identifies buyers 75% of the time, 11% of the time as inconclusive and the rest as non-buyers. When the predictive model is applied to non-buyers, it successfully identifies non-buyers 85% of the time, and 7% of the time as buyers.

1). (10 pts.) Use the information provided in the problem above to fill the tables below with prior probability, conditional probability, joint probability and posterior probability, each with 3-decimal points of accuracy. (must show details with equations and appropriate notations)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
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<tbody>
<tr>
<td>1</td>
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<td>Buyers</td>
<td>Non Buyers</td>
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<td>Prior Probability</td>
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<td>Buyers</td>
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<td>17</td>
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<td>Posterior</td>
<td>Buyers</td>
<td>Non Buyers</td>
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</tbody>
</table>
2. (3 pt) Based on your results above, what is the probability that a randomly selected customer is a non buyer? (must show details with equations and appropriate notations)

3. (4 pts) Based on your results above, among the buyers, what is the probability that a randomly selected customer is identified as non buyer? (must show details with equations and appropriate notations)

4. (4 pts) Based on your results above, what is the probability that a randomly selected customer is either a buyer or is identified as buyer? (must show details with equations and appropriate notations)

5. (4 pts) Based on your results above, among those identified as non buyers, what is the probability that a randomly selected customer is actually a buyer? (must show details with equations and appropriate notations)

-------- END of DukeFan Problem -------
Duke Fan Inc.  

Actual  

<table>
<thead>
<tr>
<th>Buyers</th>
<th>Non-Buyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30/100$ = $0.30$</td>
<td>$0.70$</td>
</tr>
</tbody>
</table>

**Prior Probability** (Simple or Marginal prob.)

$$P(\text{Buyers}) = \frac{30}{100} = 0.30$$
$$P(\text{Non-Buyers}) = 1 - P(\text{Buyers}) = 1 - 0.30 = 0.70$$

**Probability of Complementary Events $A$ versus $\bar{A}$:** $P(A) + P(\bar{A}) = 1$  

or $P(A) = 1 - P(\bar{A})$

$$P(\text{Non-Buyers}) = 1 - P(\text{Buyers}) = 0.70$$

So, $P(\text{Non-Buyers}) = 1 - 0.30 = 0.70 = P(\text{Buyers})$

or Events Buyers and Non-Buyers are complementary events.

Events of Buyers and Non-Buyers are

**Collectively Exhaustive (No Leftover)**

$$P(\text{Buyers}) + P(\text{Non-Buyers}) = 0.30 + 0.70 = 1.00$$

Events of Buyers and Non-Buyers are

**Mutually Exclusive (No Overlap)**

One cannot be a Buyer and a Non-Buyer at the same time.
Marginal Probability

\[ P(A) = P(A \cap B) + P(A \cap \overline{B}) \]

\[ P(\text{Id Buyers}) = P(\text{Id Buyers} \cap \text{Buyers}) + \]
\[ + P(\text{Id Buyers} \cap \text{NonBuyers}) \]

\[ = \]

Conditional Probability: \[ P(A | B) = \frac{P(A \cap B)}{P(B)} \]

\[ P(\text{Id Buyers} | \text{Buyers}) = \frac{P(\text{Id Buyers} \cap \text{Buyers})}{P(\text{Buyers})} = .75 \]

So: \[ P(\text{Id Buyers} \cap \text{Buyers}) = P(\text{Id Buyers} | \text{Buyers}) P(\text{Buyers}) = .75 \times .30 = .225 \]

<table>
<thead>
<tr>
<th>Table for Conditional prob.</th>
<th>Buyers</th>
<th>NonBuyers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Id Buyers</td>
<td>.75</td>
<td>.07</td>
</tr>
<tr>
<td>Id NonBuyers</td>
<td>.85</td>
<td></td>
</tr>
<tr>
<td>Id Inconclusive</td>
<td>.11</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

\[ P(\text{Id NonBuyers} | \text{NonBuyers}) = \frac{P(\text{Id NonBuyers} \cap \text{NonBuyers})}{P(\text{NonBuyers})} = .85 \]

\[ P(\text{Id NonBuyers} \cap \text{NonBuyers}) = P(\text{Id NonBuyers} | \text{NonBuyers}) P(\text{NonBuyers}) \]
\[ = .85 \times .70 = .595 \]

Note: Joint probability, \( A \), and \( B \) are interchangeable.

\[ P(A \cap B) = P(A \text{ and } B) = P(A \& B) \]

However: \[ P(A \cup B) \neq P(A \cup B) = P(A) + P(B) - P(A \cap B) \]
\[
\begin{align*}
\Pr(\text{Id Inconclusive} \mid \text{Buyers}) &= \frac{\Pr(\text{Id Inconclusive} \cap \text{Buyers})}{\Pr(\text{Buyers})} = .11 \\
\Pr(\text{Id Inconclusive} \cap \text{Buyers}) &= \Pr(\text{Id Inconclusive} \mid \text{Buyers}) \cdot \Pr(\text{Buyers}) = .11 \times .30 = .033 \\
\Pr(\text{Id NonBuyer} \mid \text{Buyers}) &= \frac{\Pr(\text{Id NonBuyer} \cap \text{Buyers})}{\Pr(\text{Buyers})} = 1 - .75 - .11 = .14 \\
\Pr(\text{Id NonBuyer} \cap \text{Buyers}) &= \Pr(\text{Id NonBuyer} \mid \text{Buyers}) \cdot \Pr(\text{Buyers}) \\
&= .14 \times .30 = .042 \\
\Pr(\text{Id Buyers} \mid \text{NonBuyer}) &= \frac{\Pr(\text{Id Buyers} \cap \text{NonBuyer})}{\Pr(\text{NonBuyer})} = .07 \\
\Pr(\text{Id Buyers} \cap \text{NonBuyer}) &= \Pr(\text{Id Buyers} \mid \text{NonBuyer}) \cdot \Pr(\text{NonBuyer}) \\
&= .07 \times .70 = .049 \\
\Pr(\text{Id Inconclusive} \cap \text{NonBuyer}) &= \frac{\Pr(\text{Id Inconclusive} \cap \text{NonBuyer})}{\Pr(\text{NonBuyer})} = 1 - .07 - .85 \\
&= .08 \\
\Pr(\text{Id Inconclusive} \cap \text{NonBuyer}) &= \Pr(\text{Id Inconclusive} \mid \text{NonBuyer}) \cdot \Pr(\text{NonBuyer}) \\
&= .08 \times .70 = .056
\end{align*}
\]

**Multiplication Rule:**

\[
\Pr(A \cap B) = \frac{\Pr(A \cap B)}{\Pr(B)}
\]

\[
\Pr(A \cap B) = \Pr(A \mid B) \cdot \Pr(B)
\]

When events \(A\) and \(B\) are independent, \(\Pr(A \mid B) = \Pr(A)\)

So,

\[
\Pr(A \cap B) = \Pr(A \mid B) \cdot \Pr(B) = \Pr(A) \cdot \Pr(B)
\]
Joint Probability \((A \cap B)\)

<table>
<thead>
<tr>
<th></th>
<th>Buyers</th>
<th>NonBuyers</th>
<th>Col Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Id Buyers</td>
<td>0.225</td>
<td>0.049</td>
<td>0.274</td>
</tr>
<tr>
<td>Id NonBuyers</td>
<td>0.042</td>
<td>0.595</td>
<td>0.637</td>
</tr>
<tr>
<td>Id inconclusive</td>
<td>0.033</td>
<td>0.056</td>
<td>0.089</td>
</tr>
<tr>
<td><strong>Row Sum</strong></td>
<td>0.30</td>
<td>0.170</td>
<td><strong>1.00</strong></td>
</tr>
</tbody>
</table>

P(Buyer) = 0.30  
P(NonBuyer) = 0.70

Union Probability

\[
P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)
\]

If Events \(A\) and \(B\) are mutually exclusive, \(P(A \cap B) = 0\), then,

\[
P(A \cup B) = P(A) + P(B)
\]

\[
P(\text{Id Buyers U Buyers}) = P(\text{Id Buyers}) + P(\text{Buyers}) - P(\text{Id Buyers \cap Buyers})
\]

\[
= 0.274 + 0.30 - 0.225 = 0.349
\]

\[
P(\text{Id Buyers U Id NonB}) = P(\text{Id Buyers}) + P(\text{Id NonBuyers})
\]

\[
= 0.274 + 0.637 = 0.911
\]

Because

\[
P(\text{Id Buyers \cap Id NonBuyers}) = 0
\]
Bayes' Theorem & Posterior Prob:

\[ P(B) = P(B \cap A) + P(B \cap \overline{A}) \]

Apply marginal probability rule:

\[ P(A) = P(A \cap B) + P(A \cap \overline{B}) \]

and multiplication rule:

\[ P(A \cap B) = P(A) \cdot P(B | A) \]

\[ = P(B) \cdot P(A | B) \]

to conditional probability:

\[ P(A | B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A) \cdot P(B | A)}{P(B) \cdot P(B | A) + P(A) \cdot P(B | \overline{A})} \]

\[ P(A \cap B) = P(A) \cdot P(B | A) \]

\[ P(B) = P(B \cap A) + P(B \cap \overline{A}) \]

\[ \frac{P(B \cap \overline{A})}{P(A) \cdot P(B | A)} \]

\[ P(B \cap \overline{A}) = \frac{P(B \cap \overline{A})}{P(A) \cdot P(B | A)} \]

\[ P(\text{Buy}|\text{IdBuy}) = \frac{P(\text{Buy}) \cdot P(\text{IdBuy}|\text{Buyer})}{P(\text{IdBuy})} \]

\[ = \frac{P(\text{Buyer}) \cdot P(\text{IdBuy}|\text{Buyer})}{P(\text{Buyer}) \cdot P(\text{IdBuy}|\text{Buyer}) + P(\text{NonBuy}) \cdot P(\text{IdBuy}|\text{NonBuy})} \]

\[ = \frac{.30 \times .75}{.30 \times .75 + .70 \times .07} = \frac{.225}{.225 + .049} = \frac{.225}{.274} = .821 \]

\[ P(\text{NonBuy}|\text{IdBuy}) = \frac{P(\text{NonBuy}) \cdot P(\text{IdBuy}|\text{NonBuy})}{P(\text{IdBuy})} \]

\[ = \frac{P(\text{NonBuy}) \cdot P(\text{IdBuy}|\text{NonBuy})}{P(\text{NonBuy}) \cdot P(\text{IdBuy}|\text{NonBuy}) + P(\text{Buy}) \cdot P(\text{IdBuy}|\text{Buy})} \]

\[ = \frac{.70 \times .07}{.274} = \frac{.049}{.274} = .179 \]

\[ P(\text{Buyer}|\text{IdBuy}) + P(\text{NonBuy}|\text{IdBuy}) = .821 + .179 = 1.00 \]
\[ P(\text{Buyer} \mid \text{Id NonBuy}) = \frac{P(\text{Buyer} \cap \text{Id NonBuy})}{P(\text{Id NonBuy})} \]
\[ = \frac{P(\text{Buyer})P(\text{Id NonBuy} \mid \text{Buyer})}{P(\text{Buyer})P(\text{Id NonBuy} \mid \text{Buyer}) + P(\text{NonBuy})P(\text{Id NonBuy} \mid \text{NonBuy})} \]
\[ = \frac{0.30 \times 0.14}{0.30 \times 0.14 + 0.70 \times 0.85} = \frac{0.042}{0.042 + 0.595} = 0.066 \]

\[ P(\text{NonBuy} \mid \text{Id NonBuy}) = \frac{P(\text{NonBuy} \cap \text{Id NonBuy})}{P(\text{Id NonBuy})} \]
\[ = \frac{P(\text{NonBuy})P(\text{Id NonBuy} \mid \text{NonBuy})}{P(\text{NonBuy})P(\text{Id NonBuy} \mid \text{NonBuy}) + P(\text{Buyer})P(\text{Id NonBuy} \mid \text{Buyer})} \]
\[ = \frac{0.70 \times 0.85}{0.70 \times 0.85 + 0.30 \times 0.14} = \frac{0.595}{0.637} = 0.934 \]

\[ P(\text{Buyer} \mid \text{Id Inc}) = \frac{P(\text{Buyer} \cap \text{Id Inc})}{P(\text{Id Inc})} \]
\[ = \frac{P(\text{Buyer})P(\text{Id Inc} \mid \text{Buyer})}{P(\text{Buyer})P(\text{Id Inc} \mid \text{Buyer}) + P(\text{NonBuy})P(\text{Id Inc} \mid \text{NonBuy})} \]
\[ = \frac{0.30 \times 0.11}{0.30 \times 0.11 + 0.70 \times 0.08} = \frac{0.033}{0.033 + 0.056} = \frac{0.033}{0.089} = 0.371 \]

\[ P(\text{NonBuy} \mid \text{Id Inc}) = \frac{P(\text{NonBuy} \cap \text{Id Inc})}{P(\text{Id Inc})} \]
\[ = \frac{P(\text{NonBuy})P(\text{Id Inc} \mid \text{NonBuy})}{P(\text{NonBuy})P(\text{Id Inc} \mid \text{NonBuy}) + P(\text{Buyer})P(\text{Id Inc} \mid \text{Buyer})} \]
\[ = \frac{0.7 \times 0.08}{0.7 \times 0.08 + 0.3 \times 0.11} = \frac{0.056}{0.089} = 0.629 \]

\[ P(\text{Buyer} \mid \text{Id Inc}) + P(\text{NonBuy} \mid \text{Id Inc}) = 0.629 + 0.371 = 1.00 \]
2. A vendor at Bluestone Stadium must determine whether to sell ice cream or soft drinks at Saturday's game. The vendor believes that the profit made will depend on the weather. For a cool weather, she can make $50 if she sells soft drinks, and she can make $30 if she sells ice cream. For a warm weather, she can make $60 if she sells soft drinks, and she can make $90 if she sells ice cream. Based on her past experience at this time of year, the vendor estimates the probability of warm weather as 0.60.

1). With the information given in the problem above, you may use either a payoff table or a decision tree when you answer the following questions. A portion of Excel spreadsheet is provided for your convenience. You must show details with appropriate equations and terms and keep 3 decimal points.

a). (5 pts) What is the payoff table for the problem? You may use the blank table on the next page.

b). (6 pts) Construct the decision tree to find expected monetary values (EMV) and max EMV for the problem. You may use the structure of the tree on the next page.

c). (10 pts) What is the maximum or optimal expected profit (EMV) = _________________? (You must show details)

d). (4 pts) What is the optimal strategy for the vendor and why?
Bluestone Stadium

Ice cream
Sell < Soft drinks

Weather < Warm
cool

Payoff Table
Alternative
Course 1

<table>
<thead>
<tr>
<th>Action</th>
<th>Cool</th>
<th>Warm</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell Ice Cream</td>
<td>$30</td>
<td>$90</td>
<td></td>
</tr>
<tr>
<td>Sell Soft Drinks</td>
<td>$50</td>
<td>$60</td>
<td></td>
</tr>
</tbody>
</table>

Expected Monetary Value

\[ \text{EMV} = \text{Max}(66, 56) = 66 \]

Prior Probability

for Weather

\[ \text{Cool Weather} = 0.40 \]

\[ \text{Warm Weather} = 0.60 \]

Decision Tree

What is the optimal strategy?
(2). What is EVwPI and EVPI?

a). (8 pts) Construct the decision tree to find the expected value with perfect information (EVwPI) for the problem.

b). (3 pts.) What is the expected value of perfect information (EVPI) for the problem? (You must show details)
Expected Value With Perfect Information (EVwPI)
\[ EVwPI = \max(30, 50) \cdot p(\text{Cool}) + \max(90, 60) \cdot p(\text{Warm}) \]
\[ = 50 \times .40 + 90 \times .60 = 74 \]

Expected Value of Perfect Information (EVPI)
\[ EVPI = EVwPI - EVwPI' \]
\[ = 74 - 66 = 8 \]

Use Decision Tree to get EVwPI

Expected Opportunity Loss or Regret Table

<table>
<thead>
<tr>
<th>Events</th>
<th>Cool Weather</th>
<th>Warm Weather</th>
<th>EOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sell Soft Drinks</td>
<td>Max(50, 30)-50 = 0</td>
<td>Max(90, 60)-60 = 30</td>
<td>18</td>
</tr>
<tr>
<td>Sell Ice Cream</td>
<td>Max(50, 30)-30 = 20</td>
<td>Max(90, 60)-90 = 0</td>
<td>8</td>
</tr>
<tr>
<td>Prior Prob.</td>
<td>.40</td>
<td>.60</td>
<td></td>
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</tbody>
</table>
Value of Weather Report

Cost for a weather report: $5

Conditional Probability

\[ P(A | B) = \frac{P(A \cap B)}{P(B)} \]

\begin{align*}
\text{Cool Weather} & \quad \text{Warm Weather} \\
\text{Predict Cool Weather} & \quad .80 \quad .30 \\
\text{Predict Warm Weather} & \quad .20 \quad .70 \\
\text{Col Sum} & \quad 1.00 \quad 1.00
\end{align*}

Prior Probability

\[ P(B) = .40 \quad .60 \]

Joint Probability

\[ P(A \cap B) = P(B) P(A | B) \]

\begin{align*}
\text{Predict Cool Weather} & \quad \frac{.40 \times .80 = .32}{.40} \quad \frac{.60 \times .30 = .18}{.60} \quad \text{Row Sum} = .50 \\
\text{Predict Warm Weather} & \quad \frac{.40 \times .20 = .08}{.40} \quad \frac{.60 \times .70 = .42}{.60} \quad \text{Row Sum} = .50 \\
\text{Col Sum} & \quad .40 \quad .60
\end{align*}

\[ P(\text{Pred. Cool}) = \frac{.40 \times .80 = .32}{.40 \times .80 + .60 \times .30} = .64 \]

Posterior Probability

\begin{align*}
\text{Pred. Cool} & \quad \frac{.32}{.50} = .64 \quad \frac{.18}{.50} = .36 \quad 1.00 \\
\text{Pred. Warm} & \quad \frac{.08}{.50} = .16 \quad \frac{.42}{.50} = .84 \quad 1.00
\end{align*}

\[ P(\text{Cool} | \text{Pred. Cool}) = \frac{P(\text{Cool} \cap \text{Pred. Cool})}{P(\text{Pred. Cool})} = \frac{.32}{.50} = .64 \]

\[ = \frac{P(\text{Cool}) P(\text{Pred. Cool} | \text{Cool})}{P(\text{Cool}) P(\text{Pred. Cool} | \text{Cool}) + P(\text{Warm}) P(\text{Pred. Cool} | \text{Warm})} \]

\[ = \frac{.40 \times .80}{.40 \times .80 + .60 \times .30} = .64 \]
3). Given the expanded decision tree for expected value with weather information (EVwSI) below, some of the entries on the tree are taken out. Assume she pays $5 for a reliable weather report.

a). (10 pts) Compute EVwSI and EVSI for the problem. You must show details.

b). (3 pts) Compute the efficiency of the sample information and discuss your findings. You must show details.

c). (3 pts) What choice would you recommend to the vendor for the whole situation (with or without weather forecast) and why?