Review of Basic probability – the contents of this review is from COB191 or equivalent. It is your responsibility to understand them and prepared yourself ready for the part of the COB291 that utilize your knowledge before.

The basic probability is the foundation for Decision Analysis, Decision Tree and Bayes' Theorem
Let us review what we have learned about the contingency table and basic probabilities.

Worryson, a potential cancer patient is considering going through further diagnosis. In the past, 1% of the patients have cancer (C), and 99% do not (NC). Before Worryson is going through further diagnosis, he did a study of patients with Diagnosed with Cancer (DC), Diagnosed without Cancer (DNC) or Diagnosed Inconclusive (DIN). In the past, 95% of the cancer patients (C) had received DC diagnosis and 0.152% of the patients without cancer (NC) received DC diagnosis, 97.455% of the patients without cancer (NC) received DNC diagnosis and 2% of cancer patients received DNC diagnosis. Worryson received a DC diagnosis, what is the probability that Worryson really has cancer? Construct the contingency table, and answer the questions below.

From the information given in the problem above, please fill out the tables below before you answer any question: The answers are given at the end of the questions.

	A	B	C	D
26		Cancer	NC	
27	Prior Probability			
28				
29	Conditional Probability Given P(Cancer) or P(NoCancer) or Prior Probabilities			
30	P(Row|C) or P(Row|NC)	Cancer	NC	
31	DC	P(DC|C)=	P(DC|NC)=	
32	DNC	P(DNC|C)=	P(DNC|NC)=	
33	DIN	P(DIN|C)=	P(DIN|NC)=	
34	ColTotal			
35				
36	Joint Probability or Contingency Table with Multiplication Rule			
37	P(Row&Col)	Cancer	NC	RowTotal
38	DC	P(DC&C)=	P(DC&NC)=	
39	DNC	P(DNC&C)=	P(DNC&NC)=	
40	DIN	P(DIN&C)=	P(DIN&NC)=	
41	ColTotal			
42				
43	Posterior Probability Given P(DC) or P(DNC) or P(DIN) or RowTotal with Conditional Prob. Rules			
44	P(Col|Row), i.e.,P(C|DC)	Cancer	NC	RowTotal
45	DC	P(C|DC)=	P(NC|DC)=	
46	DNC	P(C|DNC)=	P(NC|DNC)=	
47	DIN	P(C|DIN)=	P(NC|DIN)=	

(Decision.xls/Cancer)

Please answer each of the following questions with the notations and numerical answers, such as, P(DC) = 0.011

1. What is the probability that Worryson has cancer?

2. What is the probability that Worryson is diagnosed with cancer?

3. What is the probability that Worryson is diagnosed without cancer, but actually he does have cancer?

4. What is the probability that Worryson is either diagnosed with cancer or does have cancer?

5. What is the probability that Worryson is diagnosed with cancer given he does have cancer?

6. What is the probability that Worryson has cancer given he is diagnosed with cancer?

7. What is the probability that Worryson is diagnosed without cancer given he has no cancer?

8. What is the probability that Worryson has no cancer given he is diagnosed without cancer?

Circuit City is considering marketing new iPot. In the past, 40% of the iPots introduced are successful, and 60% not. Before introducing the iPots to the marketplace, the firm conducts a study either favorable or unfavorable. In the past, 80% of the successful iPots had received a favorable rating and 30% of the unsuccessful iPots received favorable ratings. For the new iPot under consideration, the firm has issued a favorable report, what is the probability that the new iPot will be successful? Construct the contingency table, and find P(S), P(F), P(F|S), P(NF|S), P(F ∩ S), P(F ∪ S), and P(S|F).

	A	B	C	D
24		Success	NS	
25	Prior Probability			
26				
27	Conditional Probability of Study Given Success or NS			
28		Success	NS	
29	Favorable			
30	Not Favorable			
31	ColSum			
32				
33	Joint Probability of Study & Success or NS			
34		Success	NS	RowSum
35	Favorable			
36	Not Favorable			
37	ColSum			
38				
39	Posterior Probability of Success or NS given Study			
40		Success	NS	RowSum
41	Favorable			
42	Not Favorable			

(Decision.xls/iPotPrior)

Probability Definitions and Rules

Definitions:

- Event: any possible outcome of a variable (being a cancer patient or diagnosed with cancer)
- Simple event A: an event with a single characteristic (being a cancer patient)
- Probability of event A: $0 \leq P(A) \leq 1$
- Joint event: an event with two or more characteristics (a person diagnosed with cancer and did have cancer)
- Complement of event A or $\bar{A}$: Event $\bar{A}$ includes all of events not in A. Being a cancer patient(C) and a non cancer patient (NC) are complementary events.
- Mutually exclusive events: events that cannot occur simultaneously or no overlap among events. Diagnosed with Cancer (DC) and Diagnosed without Cancer (DNC) are mutually exclusive events.
- Collectively exhaustive events: events that span or include all of the events in the sample space. Being a cancer patient and a non cancer patient (NC) are collectively exhaustive events. DC, DNC and DIN are also collectively exhaustive events.
- Independent events of A and B: the outcome of one event A does not affect the probability of the occurrence of another event B, that is, P(A) = P(A|B)
- Sample space S: the collection of all possible events.
- Marginal probability A: the probability of a simple event A designated as P(A). P(Cancer) $P(A) = P(A \cap B) + P(A \cap \bar{B})$. Sometimes, marginal probability is also called Prior probability.

- Joint Probability A and B: the probability of more than one event occurring at the same time, designated as P(A and B), or P(A&B) or P(A∩B). P(DC ∩ Cancer)

- Conditional Probability (Likelihood)of A given B: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ or $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$
  - P(Cancer|DC) = P(Cancer∩ DC)/P(DC) and P(DC|Cancer) = P(Cancer∩ DC)/P(Cancer)

- Posterior probabilities: Conditional probability that is derived from sample information to improve upon its prior probabilities. P(Cancer|DC)
- Addition rule or probability of union events: $P(AUB) = P(A) + P(B) - P(A \cap B)$ i.e., P(Cancer ∪ DC) = P(Cancer) + P(DC) – P(Cancer ∩ DC) and For mutually exclusive events A and B:

  $P(AUB) = P(A) + P(B)$, i..e. P(DIN ∪ DC) = P(DIN) + P(DC)

- Multiplication rule: $P(A \cap B) = P(A \mid B)P(B) = P(B \mid A)P(A)$,
  - i.e., P(Cancer∩ DC) = P(Cancer|DC) P(DC) and For independent events A and B:

    $P(A \cap B) = P(A)P(B) = P(B)P(A)$
- Bayes' Theorem: It applies the marginal probability and multiplication rules to conditional probability

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})}$$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\bar{B})P(\bar{B})}$$

Solution for Cancer Example

	A	B	C	D
1	Cancer Example			
2		Cancer	NC	
3	Prior Probability	0.01	0.99	
4				
5	Prior or Conditional Probability (Given P(Cancer) or			
6		Cancer	NC	
7	P(DC|Col)	0.95000	0.00152	
8	P(DNC|Col)	0.02000	0.97455	
9	P(DIN|Col)	0.03000	0.02393	
10	ColTotal	1.00000	1.00000	
11				
12	Joint Probability or Contingency Table			
13		Cancer	NC	RowTotal
14	DC	0.00950	0.00150	0.01100
15	DNC	0.00020	0.96480	0.96500
16	DIN	0.00030	0.02369	0.02399
17	ColTotal	0.01000	0.99000	1.00000
18				
19	Posterior Probability (Given P(DC) or P(DNC) or P(DIN) or RowTotal)			
20		Cancer	NC	RowTotal
21	P(DC|Row)	0.86326	0.13674	1.00000
22	P(DNC|Row)	0.00021	0.99979	1.00000
23	P(DIN|Row)	0.01250	0.98750	

(Decision.xls/Cancer)

Excel@ Formulas/Show Formulas for Cancer Example

	A	B	C	D
1	Cancer Example			
2		Cancer	NC	
3	Prior Probability	0.01	0.99	
4				
5	Prior or Conditional Probability (Given P(Cancer) or P(NoCancer)			
6		Cancer	NC	
7	P(DC|Col)	0.95	0.00152	
8	P(DNC|Col)	0.02	0.97455	
9	P(DIN|Col)	=1-B7-B8	=1-C7-C8	
10	ColTotal	=SUM(B7:B9)	=SUM(C7:C9)	
11				
12	Joint Probability or Contingency Table			
13		Cancer	NC	RowTotal
14	DC	=$B$3*B7	=$C$3*C7	=SUM(B14:C14)
15	DNC	=$B$3*B8	=$C$3*C8	=SUM(B15:C15)
16	DIN	=$B$3*B9	=$C$3*C9	=SUM(B16:C16)
17	ColTotal	=SUM(B14:B16)	=SUM(C14:C16)	=SUM(B17:C17)
18				
19	Posterior Probability (Given P(DC) or P(DNC) or P(DIN) or RowTotal)			
20		Cancer	NC	RowTotal
21	P(DC|Row)	=B14/D14	=C14/D14	=SUM(B21:C21)
22	P(DNC|Row)	=B15/D15	=C15/D15	=SUM(B22:C22)
23	P(DIN|Row)	=B16/D16	=C16/D16	

Solution for iPot Example

	A	B	C	D
1	Decision Analysis Review of Probabilities			
2	iPot Example			
3		Success	NS	
4	Prior Probability	0.4	0.6	
5				
6	Conditional Probability of Study Given Success or NS			
7		Success	NS	
8	Favorable	0.8	0.3	
9	Not Favorable	0.2	0.7	
10	ColSum	1	1	
11				
12	Joint Probability of Study & Success or NS			
13		Success	NS	RowSum
14	Favorable	0.32	0.18	0.5
15	Not Favorable	0.08	0.42	0.5
16	ColSum	0.4	0.6	1
17				
18	Posterior Probability of Success or NS given Study			
19		Success	NS	RowSum
20	Favorable	0.64	0.36	1
21	Not Favorable	0.16	0.84	1

(Decision.xls/iPotPrior)

Excel@ Formulas/Show Formulas

	A	B	C	D
1	Decision Analysis Revi			
2	iPot Example			
3		Success	NS	
4	Prior Probability	0.4	=1-B4	
5				
6	Conditional Probability of Study Given Success or NS			
7		Success	NS	
8	Favorable	0.8	=1-C9	
9	Not Favorable	=1-B8	0.7	
10	ColSum	=SUM(B8:B9)	=SUM(C8:C9)	
11				
12	Joint Probability of Study & Success or NS			
13		Success	NS	RowSum
14	Favorable	=B4*B8	=C4*C8	=B14+C14
15	Not Favorable	=B4*B9	=C4*C9	=B15+C15
16	ColSum	=SUM(B14:B15)	=SUM(C14:C15)	=B16+C16
17				
18	Posterior Probability of Success or NS given Study			
19		Success	NS	RowSum
20	Favorable	=B14/D14	=C14/D14	=SUM(B20:C20)
21	Not Favorable	=B15/D15	=C15/D15	=SUM(B21:C21)