Characteristics of Queuing

- Arrival process: the mean arrival rate per time unit (hour) (λ) versus the mean inter-arrival time (1/λ). If 160 customers arrive for service at a bar in an eight hour day,
  - What is the arrival rate λ per hour?

    λ = 160 Customers/8 hours = 20 Customers/hour

  - What is the inter-arrival time $\frac{1}{\lambda}$ (hour)?

    $$\frac{1}{\lambda}\textbf{ hour} = \frac{1}{20\ Customers/hour} = \frac{1\ hour}{20\ Customers} = \frac{1}{20} hour/Customer =$$

    $$= \textbf{0.05 hour/Customer} = \left(\frac{0.05\ hour}{Customer}\right)\left(\frac{60\ minutes}{hour}\right) = \frac{3\ minutes}{Customer} =$$

    =3 minutes/Customer

Please note: the mean arrival rate λ and the inter-arrival time 1/λ should initially have the same time units, an hour, for example.

  - What is the arrival rate λ per 15 minutes?

    $$\lambda = \frac{20\ Customers/Hour}{4 Fifteen\ minutes/Hour} = \frac{5\ Customers}{Fifteen\ minutes}$$

  - What is the inter-arrival time $\frac{1}{\lambda}$?

    $$\frac{1}{\lambda} = \frac{1}{5\frac{Customers}{Fisteen\ minutes}} = \frac{Fifteen\ minutes}{5\ Customers} = 3\ minutes/customer$$

- Service process: the mean service rate per time unit (hour) (μ) versus mean service time (1/μ). If the bar can serve 240 customers in an eight hour day,
  - What is the service rate μ per hour?

    μ = 240 Customers/8 hours = 30 Customers/hour

  - What is the mean service time $\frac{1}{\mu}$ (hour)?

    $$\frac{1}{\mu}\textbf{ hour} = \frac{1}{30\ Customers/hour} = \frac{1\ hour}{30\ Customers} = \frac{1}{30} hour/Customer =$$

    $$= \left(\frac{1/30\ hour}{Customer}\right)\left(\frac{60\ minutes}{hour}\right) = \frac{2\ minutes}{Customer} =$$

    =2 minutes/Customer

What are Operating Characteristics of Queuing Theory?

λ = mean arrival rate (mean number of arrivals per time unit)

$1/\lambda$ = mean inter-arrival time for arrivals

μ = mean service rate (mean number of services per time unit)

$1/\mu$ = mean service time per customer or job

L_q = average queue length or number of units in line waiting for service

W_q = average waiting time a unit spent in queue before being served

$$L_q = \lambda W_q$$

- ✓ The average queue length is the arrival rate multiplies by the average time spent waiting in the queue.
- ✓ Jobs blocked and refused entry to the system are not counted in λ.

L = average number of units in the system (Lq in queue plus being served)

W = average time a unit spent in the system (in queue plus being served)

$$L = \lambda W$$

- ✓ The average queue length plus the one being served is the arrival rate multiplies by the average time spent waiting in the queue plus the time being served.
- ✓ Jobs blocked and refused entry to the system are not counted in λ.

s = number of parallel or equivalent servers in the system

ρ (Rho) or U = server utilization factor = the proportion of time the server is busy

P_w = Probability of an arriving unit to wait in the queue before being served

P_0 = Probability of no unit in the system (empty) (neither in queue nor being served)

P_n = Probability of having n units in the system (in queue plus being served)

Queuing (Waiting Line) Theory and Applications

Queuing Problems (Class hand out)

Suppose that customers arrive about every 3 minutes on average to JMU Bookstore according to a Poisson process. There is one counter open for service, with two employees working. One employee fixes a customer's order and another employee takes their money. It take an average of two minutes (exponentially distributed) to complete each customer order.

a. What is the average arrival rate to the window at JMU Bookstore?

$\lambda = 20$ per hr

b. What is the probability distribution of the number of arrivals to JMU Bookstore?

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!} \qquad x = 0, 1, 2, \cdots\cdots$$

c. What are the chances that no customers arrive in a 15-minutes period? $P(X=0) = \frac{\lambda^x e^{-\lambda}}{x!} = \frac{5^0 e^{-5}}{0!}$

$\lambda = 5 = \frac{20}{4}/hr$, $P(X=0|\lambda=5)$ $= e^{-5} = .0067$

$e = 2.71828$

$= \text{POISSON}(0, 5, \text{False})$ TRUE

d. What is the probability of 3 customers arriving to the window in a 15-minutes period?

$\lambda = 5/15$ mins

$$P(X=3|\lambda=5) = \frac{5^3 e^{-5}}{3!} = \frac{125}{6} e^{-5} = \cancel{.1395}\ .1404$$

0 | 1 | 2 | 3 | 4 | - - -

e. What is the probability of more than 2 customers arriving to the window in a 15-minute period?

$P(X>2) = P(X \geq 3) = 1 - P(X \leq 2) = 1 - \{P(X=0) + P(X=1) + P(X=2)\}$

$P(X=0) = 5^0 e^{-5}/0! = e^{-5} = 0.0067$ $\quad = 1 - \{0.0067 + 0.0034 + 0.0842\}$

$P(X=1) = 5^1 e^{-5}/1! = 5e^{-5} = 0.00337$ $\quad = 1 - 0.1247 = 0.8753$

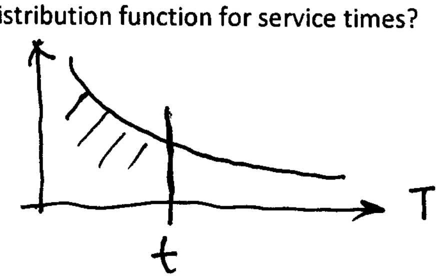
$P(X=2) = 5^2 e^{-5}/2! = 12.5 e^{-5} = 0.0842$ $\quad = 1 - \text{POISSON}(2, 5, \text{TRUE})$

f. What is the average service rate for preparing customer's orders?

$\mu = \frac{60}{2} = 30$ per hr

g. What is the probability density function and cumulative distribution function for service times?

$$P(T \leq t) = \cancel{De^{-t\frac{1}{\mu}}}$$
$$= 1 - e^{-t\mu}$$



h. What percentage of the orders will be prepared in exactly 2 minutes?

$$P(T=2)=\phi$$

i. What percentage of customer orders will take less than two minutes to prepare?

$$=\text{EXPONDIST}\left(\tfrac{2}{60}, 30, \text{TRUE}\right)$$

$$P(T<2) = \cancel{\ \ } = P(T < 2 \text{ mins}) = P\left(T < \tfrac{2}{60}\text{hr}\right)$$

$$= 1 - e^{-\frac{2}{60}30} = 1 - e^{-1} = .632$$

j. What are the chances it will take more than 3 minutes to prepare a customer's order?

$$= 1 - \text{EXPONDIST}\left(\tfrac{3}{60}, 30, \text{TRUE}\right)$$

$$P(T \geq 3 \text{ min}) = 1 - P(T \leq 3 \text{ min})$$

$$= 1 - 1 + e^{-3/60 \cdot 30} = \quad = 1 - P\left(T \leq \tfrac{3}{60} \text{ hr}\right) = 1 - \left(1 - e^{-30\frac{3}{60}}\right) = e^{-1.5} = 0.2231$$

k. What are the chances it will take between 2 and 3 minutes to prepare a customer's order?

$$P(2 \text{ min} \leq T \leq 3 \text{ min}) = P(T \leq 3 \text{ min}) - P(T \leq 2 \text{ min}) = \left(1 - e^{-30\frac{3}{60}}\right) - \left(1 - e^{-30\frac{2}{60}}\right)$$

$$= e^{-30\cdot 2/60} - e^{-30 \cdot 3/60} = e^{-1} - e^{-1.5} = 0.3679 - 0.2231 = 0.1448$$

$$= \text{EXPONDIST}(3/60, 30, \text{TRUE}) - \text{EXPONDIST}(2/60, 30, \text{TRUE})$$

$$= 0.7769 - 0.6321 = 0.1448$$

l. What percentage of the time are the employees busy at the window? M/M/1

$$\rho = u = \frac{\lambda}{\mu} = \frac{20}{30} = .667$$

m. What is the average number of customers waiting to order?

$$L_q = \frac{\lambda^2}{\mu(\mu-\lambda)} = \frac{20^2}{30(30-20)} = \frac{4}{3} = 1.333$$

n. What is the average number of customers at JMU Bookstore?

$$L = L_q + \frac{\lambda}{\mu} = \frac{\lambda}{\mu - \lambda} = \frac{20}{30-20} = \frac{2}{1} = 2$$

o. What is the average amount of time spent in line by customers at JMU Bookstore?

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L_q}{\lambda} = \frac{4/3}{20} = \frac{4}{60} = \frac{1}{15} \text{ hr} = \frac{60}{15} \text{ min} = 4 \text{ mins}$$

p. What is the probability that a customer will have to wait in line to get served at JMU Bookstore?

$$P_w = \rho = u = \frac{\lambda}{\mu} = P(n \geq 1) = 1 - P(n=0) = 1 - P_0 = 1 - \left(1 - \frac{\lambda}{\mu}\right)$$

0 | 1 | 2 | 3 | 4 | ....

$$\frac{20}{30} = \frac{2}{3} \qquad = \frac{20}{30} =$$

$$P_0 = 1 - \frac{2}{3} = \frac{1}{3}$$

q. How long, on average, does it take a customer to get served at JMU Bookstore?

$$W = \frac{1}{\mu - \lambda} = \frac{L}{\lambda} = \frac{1}{30-20} = \frac{1}{10} \text{ hr} = 6 \text{ mins}$$

r. What is the probability there are two customers at JMU Bookstore?

$$P_2 = P_0\left(\frac{\lambda}{\mu}\right)^2 = \left(\frac{1}{3}\right)\left(\frac{20}{30}\right)^2 = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 = \left(\frac{1}{3}\right)\left(\frac{4}{9}\right) = .1482$$

$$P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{20}{30} = \frac{30}{30} - \frac{20}{30} = \frac{10}{30} = \frac{1}{3}$$

s. What is the probability there are more than two customers waiting to order?

$n > 2$ in queue $\Rightarrow n \geq 3$ in queue + 1 being served or $n \geq 4$ in system

$P(n \geq 4) = 1 - P(n \leq 3) = 1 - (P_0 + P_1 + P_2 + P_3)$

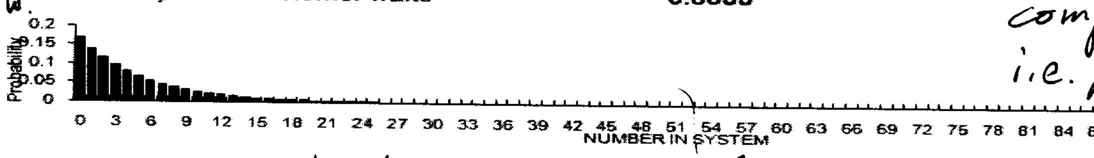
$= 1 - (.3333 + .2222 + .1482 + .0988)$

$= 1 - .8025$

$= .1975$

t. Suppose that business increases by 25% at the start of a semester, Can one counter handle the increased volume? Support your answer. How are customers' average waiting times affected?

	A	B	C	D	E	F	G	H
1	M/M/s							
2		Arrival rate			25		Assumes Poisson process for	
3		Service rate			30		arrivals and services.	
4		Number of servers			1			
5								
6	Utilization					83.33%		
7	P(0), probability that the system is empty					0.1667		
8	Lq, expected queue length					4.1667		
9	L, expected number in system					5.0000	120	
10	Wq, expected time in queue					0.1667	10	
11	W, expected total time in system					0.2000	12	
12	P_w Probability that a customer waits					0.8333		
13								
14								
15								
16								
17								
18								



note: for M/M/1 queue
1. $u = P_w$
2. P_0 & P_w are complementary
i.e. $P_0 + P_w = 1$.

As $\lambda \uparrow$ from 20 per hr to 25 per hr, $u = \frac{\lambda}{\mu} = 25/30 = 5/6 = 0.8333 < 1$, in steady state

$L_q = 4.1667$ $\quad L = 5$

$W_q = 0.1667$ hr $= 10$ mins. $\quad W = 0.2$ hr $= 12$ mins.

u. Study the results in the data table below. Compare the changes in Utilization and W as the arrival rate increases?

	A	B	C	D	E
1	Queue	30	Service Rate μ (# per hour)		
2	Arrival rate λ (# per hour)	Utilization	W (mins.)	W (hour)	L = λW
3	20	66.67%	6	0.10	2
4	21	70.00%	6.7	0.11	2.33
5	22	73.33%	7.5	0.13	2.75
6	23	76.67%	8.6	0.14	3.29
7	24	80.00%	10	0.17	4
8	*25*	*83.33%*	*12*	*0.20*	5
9	26	86.67%	15	0.25	6.5
10	27	90.00%	20	0.33	9
11	28	93.33%	30	0.50	14
12	29	96.67%	60	1.00	29
13	30	100.00%	#DIV/0!	#DIV/0!	#DIV/0!
14					
15		=A3/B1	=1/(B1-A3)*60		

2. Suppose that the management of JMU Bookstore estimates the average waiting cost for a customer to be ~~$0.40~~ $.60 per minute. The cost of operating a window, including employee wages, is approximately ~~$20~~ $30 per hour. What is the average total cost per hour at JMU Bookstore during none peak time when one window is open for service (assuming λ = 25 per hour)?

Employee pay = $30/hr

penalty Cost/hr = $.60 × 60 min = ~~$24~~ $36/hr

L = 5 $\quad$ Total cost = Employee pay/hr + Penalty cost

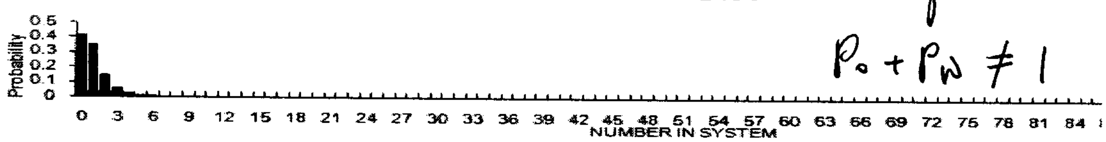
= $30/hr + $36/hr × (L = 5) = ~~$140~~ $210/hr

3. Suppose that the Bookstore opens a second (identical) window, with average ~~service~~ arrival rate $\lambda = 25$ per hour.

a. What is the approximate queuing model for JMU Bookstore and what assumptions are necessary to use this model?

	A	B	C	D	E	F	G	H
1		**M/M/s**						
2			**Arrival rate**		25		Assumes Poisson process for	
3			**Service rate**		30		arrivals and services.	
4			**Number of servers**		2			
5								
6		Utilization				**41.67%**		
7		P(0), probability that the system is empty				**0.4118**		
8		Lq, expected queue length				**0.1751**		
9		L, expected number in system				**1.0084**		
10		Wq, expected time in queue				**0.0070**		
11		W, expected total time in system				**0.0403**		
12		Probability that a customer waits				**0.2451**		

Note:
P_0 & P_W are Not complementary, i.e.
$P_0 + P_W \neq 1$



(Q.xlsx)

M/M/S Queue with assumptions:
poisson arrival, Exponential service, Single queue with s servers
Customers go to the first open window (FIFS)
$u = \lambda / s\mu \ll 1$ is steady state

b. On average, what percentage of the time are the employees busy at each window?

$$u = \frac{\lambda}{s\mu} = \frac{\lambda}{2\mu} = \frac{25}{2 \times 30} = \frac{5}{12} = 0.4167 < 1$$

c. What is the value of P_0 for this queuing system?

$S = 2$

$$P_0 = \frac{2\mu - \lambda}{2\mu + \lambda} = \frac{2(30) - 25}{2(30) + 25} = \frac{7}{17} = 0.4118$$

$$P(n \geq 1) = 1 - P_0 = 1 - .4118 = .5882$$

d. What is the average number of customers waiting to be served?

$Lq = .1751$

e. What is the average number of customers in Bookstore?

$L = 1.0084$

f. What is the average amount of time spent in line by customers at Bookstore?

$Wq = .007$ hr $= 0.4202$ min

g. What is the probability that a customer will have to wait in line to be served at Bookstore?

$$P_W = P_0 \left(\frac{\lambda^2}{\mu(2\mu - \lambda)} \right) = 0.4118 \left(\frac{25^2}{30(2 \times 30 - 25)} \right) = \frac{25}{102} = 0.2451$$

Once service started, how long on average, for a customer to finish the service $W - W_q = .0403 - .007 = 0.0333\ hr =$

h. How long, on average, does it take a customer to be served at Bookstore?

$$W = \frac{L}{\lambda} = \frac{1.0084}{25} = 0.0403\ hrs = 2.42\ mins.$$

i. What is the probability there are two customers at Bookstore?

$$P_2 = \frac{(\lambda/\mu)^2}{2!} P_0 = \frac{(25/30)^2}{2!} 0.4118 = 0.143 \quad \text{for } n \le s$$

j. What is the probability there are more than two customers waiting to be served?

$n > 2$ in queue, $n \geq 3$ in queue plus 2 in service

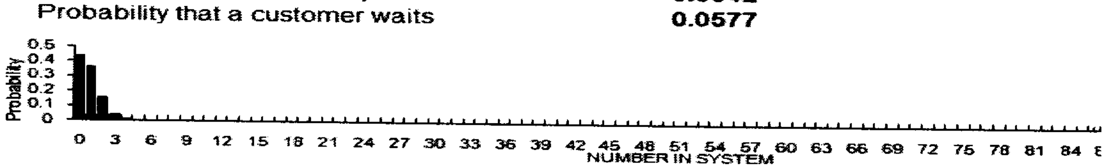
$$P(n > 4) = P(n \geq 5) = 1 - P(n \le 4) = 1 - (P_0 + P_1 + P_2 + P_3 + P_4)$$
$$= 1 - (0.41176 + 0.34314 + 0.14297 + 0.05957 + .02482)$$
$$= 1 - .98227 = .01773$$

4. What is the total cost per hour for JMU Bookstore to operate two windows? Is it more or less expensive than one?

$\$30 * 2 + \$36 * (L = 1.0084) = \cancel{\$76.30}$ $96.30   assume $30/hr $.60/min

5. Refer to the queuing model results provided by Q.xlsx for three windows. Is it cost effective for JMU Bookstore to open a third window?

	A	B	C	D	E	F	G	H
1		M/M/s						
2			Arrival rate		25		Assumes Poisson process for	
3			Service rate		30		arrivals and services.	
4			Number of servers		3			
5								
6		Utilization				27.78%		
7		P(0), probability that the system is empty				0.4321		
8		Lq, expected queue length				0.0222		
9		L, expected number in system				0.8555		
10		Wq, expected time in queue				0.0009		
11		W, expected total time in system				0.0342		
12		Probability that a customer waits				0.0577		



(Q.xlsx)

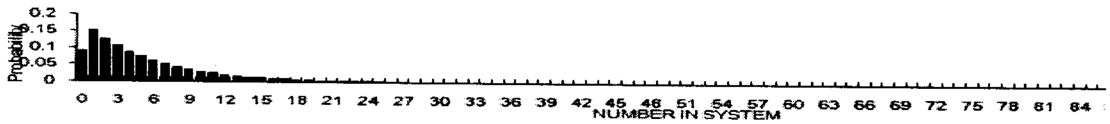
$\$30 * (S = 3) + \$36 * (L = .8555) = \$120.80$

6. Suppose that on a particular first day of a semester at JMU, businesses at Bookstore actually increases so that customers are arriving about every 1.2 minutes, on average. There are two windows open for service, and it still takes an average of 2 minutes (exponential distributed) to serve each customer.

a. What is a customer's average waiting time and the total cost per hour if there is one line and customers go to the first open window?

M/M/s	
Arrival rate	50
Service rate	30
Number of servers	2
Utilization	83.33%
P(0), probability that the system is empty	0.0909
Lq, expected queue length	3.7879
L, expected number in system	5.4545
Wq, expected time in queue	0.0758
W, expected total time in system	0.1091
Probability that a customer waits	0.7576

Assumes Poisson process for arrivals and services.



(Q.xlsx)

$W_q = .0758$ hrs

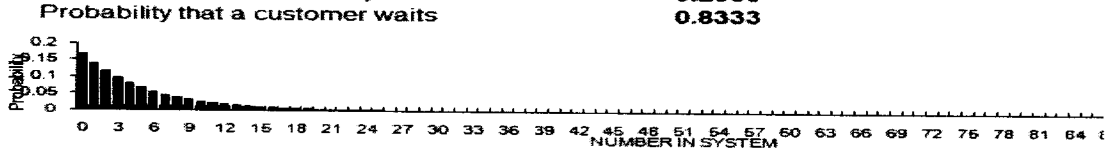
$W = .1091$ hrs $= 6.$ mins.

$\$30 \times 2 + \$36 \times (L = 5.4545) = \$256.36$

b. What is a customer's average waiting time and the total cost per hour if there is a separate line for each window, and we assume that approximately half of the customers join each line?

M/M/s		
Arrival rate	25	
Service rate	30	
Number of servers	1	
Utilization	83.33%	
P(0), probability that the system is empty	0.1667	
Lq, expected queue length	4.1667	
L, expected number in system	5.0000	120
Wq, expected time in queue	0.1667	10
W, expected total time in system	0.2000	
Probability that a customer waits	0.8333	

Assumes Poisson process for arrivals and services.



2 M/M/1 queues

$W_q = .1667$ $W = 0.2$ hr $= 12$ mins

$\$30 + \$36 \times (L = 5) = \$210$

Total Cost $= 2 \times \$210 = \$420$

7. Suppose that customers arrive to JMU Bookstore according to a Poisson distribution at an average rate of 25 per hour with one window open for service. Two pairs of employees rotate shifts at the window, and both pairs can fill customer orders in an average of two minutes. However, James and Sarah frequently chat with customers so that their customer service times are more variable than Ryan's and Heather's: the standard deviation of service times for James and Sarah is 2 minutes, while it's only 1 minute for Ryan and Heather.

a. What is the approximate queuing model for JMU Bookstore and what assumptions are necessary to use this model?

b. Compare the operating characteristics of the window at Bookstore when each pair of employees is working.

	A	B	C	D	E	F	G
1		**M/G/1**	**Ryan and Heather**				**average**
2							**service RATE**
3		**Arrival rate**			25		**30**
4		**Average service TIME**			0.03333		
5		**Standard dev. of service time**			0.01667		
6							
7							
8							
9		Utilization				**83.33%**	
10		P(0), probability that the system is empty				**0.1667**	
11		Lq, expected queue length				**2.6042**	
12		L, expected number in system				**3.4375**	
13		Wq, expected time in queue				**0.1042**	6.25
14		W, expected total time in system				**0.1375**	8.25

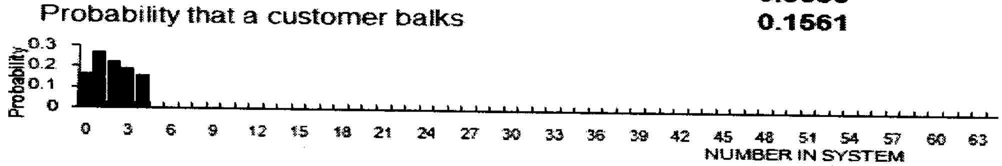
	A	B	C	D	E	F	G
1		**M/G/1**	**James and Sarah**				**average**
2							**service RATE**
3		**Arrival rate**			25		**30**
4		**Average service TIME**			0.03333		
5		**Standard dev. of service time**			0.03333		
6							
7							
8							
9		Utilization				**83.33%**	
10		P(0), probability that the system is empty				**0.1667**	
11		Lq, expected queue length				**4.1667**	
12		L, expected number in system				**5.0000**	
13		Wq, expected time in queue				**0.1667**	10
14		W, expected total time in system				**0.2000**	12

8. Suppose that customers arrive to JMU Bookstore according to a Poisson distribution at an average of 50 per hour with two windows open and 2 minutes service times, on average. There is currently room for a maximum of four customers to wait for being served, including those being served. Assume that customers will leave if there is no space in the queue.

   a. What is the appropriate queuing model for JMU Bookstore and what assumptions are necessary to use this model?

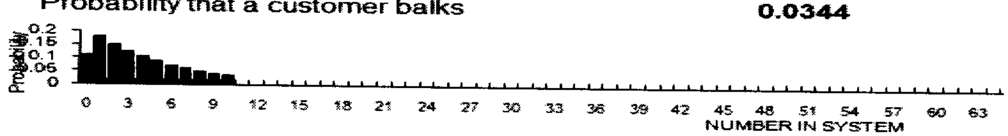
   b. Based on the operating characteristics shown below, what percentage of customers will be lost during a busy time?

	A	B	C	D	E	F	G
1		**M/M/s with Finite Queue**					
2		**Arrival rate**			50		
3		**Service rate**			30		
4		**Number of servers**			2		
5		**Maximum queue length**			2		
6		Utilization				**70.32%**	
7		P(0), probability that the system is empty				**0.1619**	
8		Lq, expected queue length				**0.4996**	
9		L, expected number in system				**1.9061**	
10		Wq, expected time in queue				**0.0118**	0.71048
11		W, expected total time in system				**0.0452**	2.71048
12		Probability that a customer waits				**0.5683**	
13		Probability that a customer balks				**0.1561**	



   c. How much improvement would there be if the Bookstore builds an extension so that it can accommodate up to a total of 10 customers?

	A	B	C	D	E	F	G
1		**M/M/s with Finite Queue**					
2		**Arrival rate**			50		
3		**Service rate**			30		
4		**Number of servers**			2		
5		**Maximum queue length**			8		
6		Utilization				**80.47%**	
7		P(0), probability that the system is empty				**0.1066**	
8		Lq, expected queue length				**2.0305**	
9		L, expected number in system				**3.6398**	
10		Wq, expected time in queue				**0.0421**	2.52342
11		W, expected total time in system				**0.0754**	4.52342
12		Probability that a customer waits				**0.7159**	
13		Probability that a customer balks				**0.0344**	

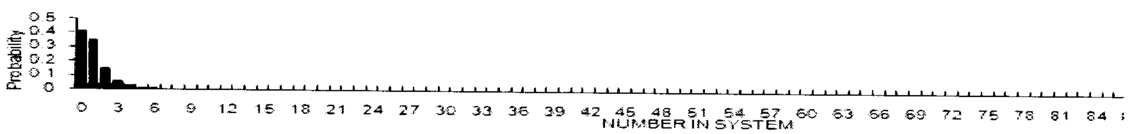


3. Suppose that the Bookstore opens a second (identical) window, with average service rate λ = 25 per hour.

M/M/s

Arrival rate	25	Assumes Poisson process for arrivals and services.
Service rate	30	
Number of servers	2	

Utilization	**41.67%**
P(0), probability that the system is empty	**0.4118**
Lq, expected queue length	**0.1751**
L, expected number in system	**1.0084**
Wq, expected time in queue	**0.0070**
W, expected total time in system	**0.0403**
Probability that a customer waits	**0.2451**



(Q.xlsx)

Note: 1) $u \neq P_w$

2) P_o *and* P_w *are NOT complementary any more, i.e.,* $P_o + P_w \neq 1$

a. On average, what percentage of the time are the employees busy at each window?

b. What is the value of P_0 for this queuing system?

s = 2

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu-\lambda}\right)\right]^{-1} = \left[\sum_{n=0}^{2-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^2}{2!}\left(\frac{2\mu}{2\mu-\lambda}\right)\right]^{-1} =$$

$$= \frac{1}{\left[\sum_{n=0}^{2-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^2}{2!}\left(\frac{2\mu}{2\mu-\lambda}\right)\right]} = \frac{1}{1 + \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2}{2}\left(\frac{2\mu}{2\mu-\lambda}\right)} =$$

$$P_0 = \frac{1}{1 + \frac{25}{30} + \frac{(25/30)^2}{2}\left(\frac{2(30)}{2(30)-25}\right)} = 0.4118$$

$$\sum_{n=0}^{2-1} \frac{(\lambda/\mu)^n}{n!} = \frac{(\lambda/\mu)^0}{0!} + \frac{(\lambda/\mu)^1}{1!} = 1 + \frac{\lambda}{\mu}$$

$$\left(\frac{\lambda}{\mu}\right)^0 = 1 \; and \; 0! = 1$$

$$P_0 = \frac{2\mu-\lambda}{2\mu+\lambda} = \frac{2(30)-25}{2(30)+25} = \frac{35}{85} = \frac{7}{17} = 0.4118$$

Or from Q.xlsx

P_0 = probability of no customer in system

(Work out the procedure in class)

c2. What is the chance that there is at least on customer in the system?

$$P(n \geq 1) = 1 - P_0 = 1 - 0.4118 = 0.5882$$