Arrival

Poisson Distribution
with $\lambda = 3/\text{hr}$

$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0,1,\cdots$$

<u>Probability of more than two and less than five to arrive in two hours?</u>

Queue to Wait

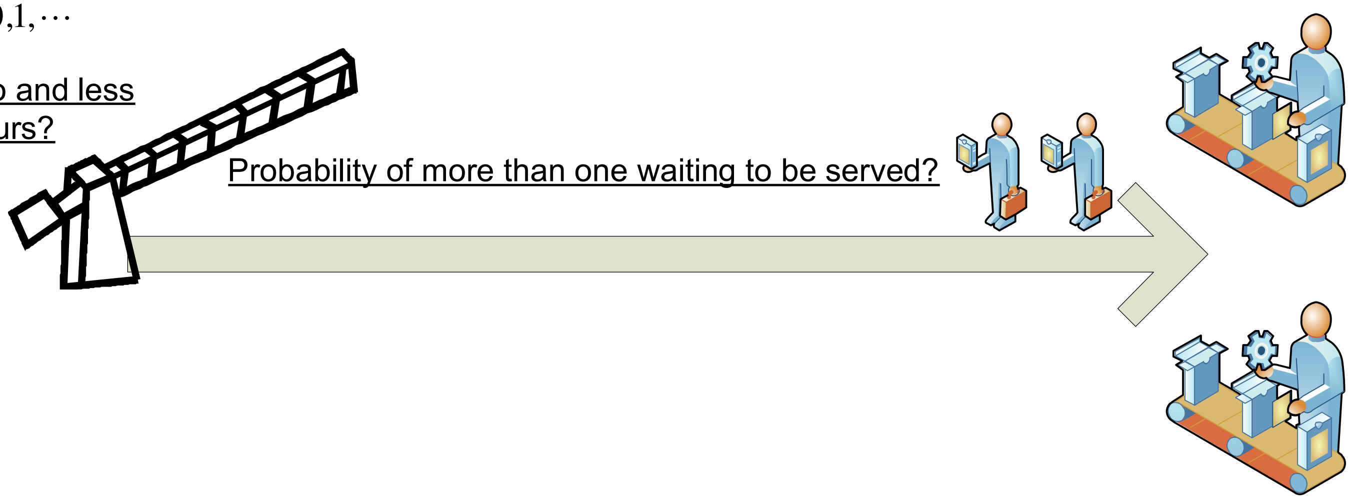
M/M/s
With $\lambda=3/\text{hr}$, $\mu=4/\text{hr}$ and s=2

If more than one waits, then n =?

<u>Probability of more than one waiting to be served?</u>

Servers

Exponential Distribution $P(T < t) = 1 - e^{-\mu t}$
With $\mu=4/\text{hr}$



<u>Probability of taking between five and fifteen minutes to serve one?</u>

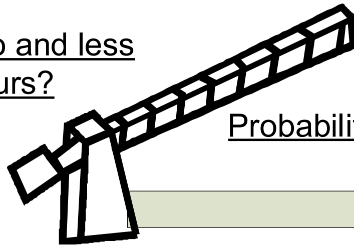
<u>What is the Total Hourly Operation Cost:</u>

Arrival

Poisson Distribution
with λ = 3/hr

$$P(X=x)=\frac{\lambda^x e^{-\lambda}}{x!},\ x=0,1,\cdots$$

Probability of more than two and less than five to arrive in two hours?



$$P(2<X\le 4)=$$
$$P(X\le 4)-P(X\le 2)=$$
$$P(X=3)+P(X=4)=$$
$$=\frac{6^3e^{-6}}{3!}+\frac{6^4e^{-6}}{4!}=$$
$$=0.08924+0.1385=0.2231$$
$$=POISSON(3,6,FALSE)+$$
$$+POISSON(4,6,FALSE)$$

	A	B	C	D	E	F
1		M/M/s				
2	a.		Arrival rate		3	
3	b.		Service rate		4	
4	c.		Number of servers		2	
5						
6		Utilization				37.50%
7		P(0), probability that the system is empty				0.4545
8		Lq, expected queue length				0.1227
9		L, expected number in system				0.8727
10		Wq, expected time in queue				0.0409
11		W, expected total time in system				0.2909
12		Probability that a customer waits				0.2045

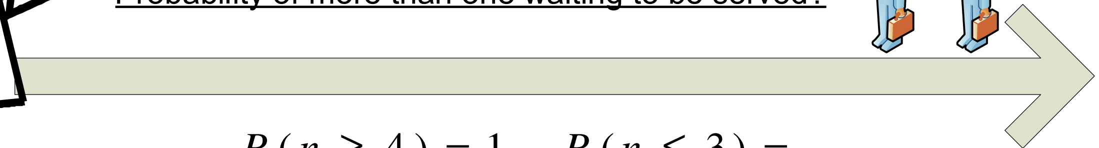
Queue to Wait

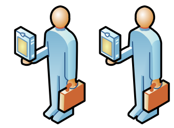
M/M/s
With λ=3/hr , μ=4/hr and s=2

If more than one waits, then n =?

n > 1 → n ≥ 2 in queue + s of 2 → n ≥ 4 in system

Probability of more than one waiting to be served?





$$P(n\ge 4)=1-P(n\le 3)=$$
$$=1-(P_0+P_1+P_2+P_3)$$

$$P_0=\frac{2\mu-\lambda}{2\mu+\lambda}$$

$$P_n=\begin{cases}\frac{(\lambda/\mu)^n}{n!}P_0, for\ n\le 2\\ \frac{(\lambda/\mu)^n}{2^{(n-1)}}P_0, for\ n>2\end{cases}$$

$$P_1=\frac{\lambda}{\mu}P_0$$

$$P_2=\frac{1}{2}\left(\frac{\lambda}{\mu}\right)^2P_0$$

$$P_3=\frac{(\lambda/\mu)^3}{2^{(3-1)}}$$

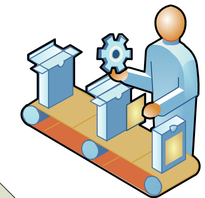
$$L_q=\lambda W_q=2\rho P_0\left(\frac{\rho}{1-\rho}\right)^2=\frac{P_0(\lambda/\mu)^3}{(2-\lambda/\mu)^2}$$

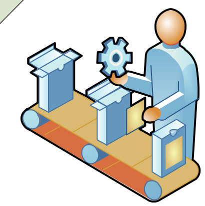
$$L=\lambda W=L_q+\frac{\lambda}{\mu}=L_q+2\rho$$

Servers

Exponential Distribution
With μ=4/hr

$$P(T<t)=1-e^{-\mu t}$$





Probability of taking between five and fifteen minutes to serve one?

$$P(5<T<15)=$$
$$=P(T<15)-P(T<5)=$$
$$=EXPONDIST(15/60,4,TRUE)-$$
$$-EXPONDIST(5/60,4,TRUE)=$$
$$=0.6321-0.2835=0.3487$$
$$=(1-e^{-4\times 15/60})-(1-e^{-4\times 5/60})=$$
$$=e^{-4\times 5/60}-e^{-4\times 15/60}=0.7165-0.3679=$$
$$=EXP(-4\times 5/60)-EXP(-4\times 15/60)=$$

What is the Total Hourly Operation Cost:

Hourly Pay x s Servers + L x Hourly Penalty Cost=

$100 x 2 + $1 x 60mins/hr x 0.8727 (L)=$252.36