CHAPTER 12

Simulation

CONTENTS

12.1 RISK ANALYSIS
PortaCom Project
What-If Analysis
Simulation
Simulation of the PortaCom Project

12.2 INVENTORY SIMULATION
Butler Inventory Simulation

12.3 WAITING LINE SIMULATION
Hammondsport Savings Bank ATM Waiting Line
Customer Arrival Times
Customer Service Times
Simulation Model
Hammondsport Savings Bank ATM Simulation
Simulation with Two ATMs
Simulation Results with Two ATMs

12.4 OTHER SIMULATION ISSUES
Computer Implementation
Verification and Validation
Advantages and Disadvantages of Using Simulation

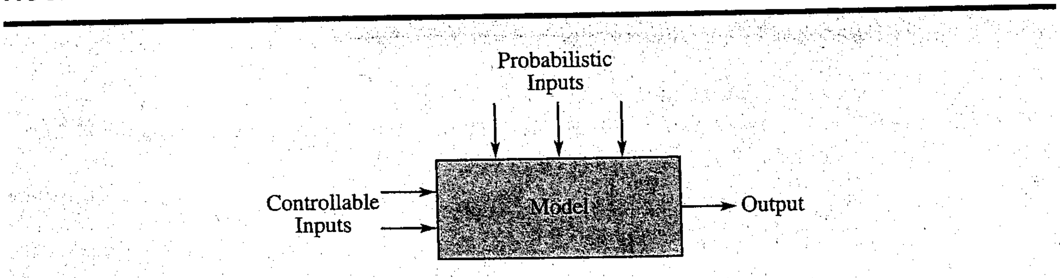
Simulation is one of the most widely used quantitative approaches to decision making. It is a method for learning about a real system by experimenting with a model that represents the system. The simulation model contains the mathematical expressions and logical relationships that describe how to compute the value of the outputs given the values of the inputs. Any simulation model has two inputs: controllable inputs and probabilistic inputs. Figure 12.1 shows a conceptual diagram of a simulation model.

In conducting a **simulation experiment,** an analyst selects the value, or values, for the **controllable inputs.** Then values for the **probabilistic inputs** are randomly generated. The simulation model uses the values of the controllable inputs and the values of the probabilistic inputs to compute the value, or values, of the output. By conducting a series of experiments using a variety of values for the controllable inputs, the analyst learns how values of the controllable inputs affect or change the output of the simulation model. After reviewing the simulation results, the analyst is often able to make decision recommendations for the controllable inputs that will provide the desired output for the real system.

Simulation has been successfully applied in a variety of applications. The following examples are typical.

1. *New product development.* The objective of this simulation is to determine the probability that a new product will be profitable. A model is developed relating profit (the output measure) to various probabilistic inputs such as demand, parts cost, and labor cost. The only controllable input is whether to introduce the product. A variety of possible values will be generated for the probabilistic inputs, and the resulting profit will be computed. We develop a simulation model for this type of application in Section 12.1.
2. *Airline overbooking.* The objective of this simulation is to determine the number of reservations an airline should accept for a particular flight. A simulation model is developed relating profit for the flight to a probabilistic input, the number of passengers with a reservation who show up and use their reservation, and a controllable input, the number of reservations accepted for the flight. For each selected value for the controllable input, a variety of possible values will be generated for the number of passengers who show up, and the resulting profit can be computed. Similar simulation models are applicable for hotel and car rental reservation systems.
3. *Inventory policy.* The objective of this simulation is to choose an inventory policy that will provide good customer service at a reasonable cost. A model is developed relating two output measures, total inventory cost and the service level, to probabilistic inputs, such as product demand and delivery lead time from vendors, and controllable inputs, such as the order quantity and the reorder point. For each setting of the controllable inputs, a variety of possible values would be generated for the probabilistic inputs, and the resulting cost and service levels would be computed.
4. *Traffic flow.* The objective of this simulation is to determine the effect of installing a left turn signal on the flow of traffic through a busy intersection. A model is developed

FIGURE 12.1 DIAGRAM OF A SIMULATION MODEL



relating waiting time for vehicles to get through the intersection to probabilistic inputs such as the number of vehicle arrivals and the fraction that want to make a left turn, and controllable inputs such as the length of time the left turn signal is on. For each setting of the controllable inputs, values would be generated for the probabilistic inputs, and the resulting vehicle waiting times would be computed.

5. *Waiting lines.* The objective of this simulation is to determine the waiting times for customers at a bank's automated teller machine (ATM). A model is developed relating customer waiting times to probabilistic inputs such as customer arrivals and service times, and a controllable input, the number of ATM machines installed. For each value of the controllable input (the number of ATM machines), a variety of values would be generated for the probabilistic inputs and the customer waiting times would be computed. The Management Science in Action, Call Center Design, describes how simulation of a waiting line system at a call center helped the company balance the service to its customers with the cost of agents providing the service.

Simulation is not an optimization technique. It is a method that can be used to describe or predict how a system will operate given certain choices for the controllable inputs and randomly generated values for the probabilistic inputs. Management scientists often use simulation to determine values for the controllable inputs that are likely to lead to desirable system outputs. In this sense, simulation can be an effective tool in designing a system to provide good performance.

MANAGEMENT SCIENCE IN ACTION

CALL CENTER DESIGN*

A call center is a place where large volumes of calls are made to or received from current or potential customers. More than 60,000 call centers operate in the United States. Saltzman and Mehrotra describe how a simulation model helped make a strategic change in the design of the technical support call center for a major software company. The application used a waiting line simulation model to balance the service to customers calling for assistance with the cost of agents providing the service.

Historically, the software company provided free phone-in technical support, but over time service requests grew to the point where 80% of the callers were waiting between 5 and 10 minutes and abandonment rates were too high. On some days 40% of the callers hung up before receiving service. This service level was unacceptable. As a result, management considered instituting a Rapid Program in which customers would pay a fee for service, but would be guaranteed to receive service within one minute, or the service would be free. Nonpaying customers would continue receiving service but without a guarantee of short service times.

A simulation model was developed to help understand the impact of this new program on the waiting line characteristics of the call center. Data available were used to develop the arrival distribution, the service time distribution, and the probability distribution for abandonment. The key design variables considered were the number of agents (channels) and the percentage of callers subscribing to the Rapid Program. The model was developed using the Arena simulation package.

The simulation results helped the company decide to go ahead with the Rapid Program. Undeı most of the scenarios considered, the simulatior model showed that 95% of the callers in the Rapic Program would receive service within one minut and that free service to the remaining customer could be maintained within acceptable limits. Withi nine months, 10% of the software company's cus tomers subscribed to the Rapid Program, generatin $2 million in incremental revenue. The compan viewed the simulation model as a vehicle for miti gating risk. The model helped evaluate the likel impact of the Rapid Program without experimen ing with actual customers.

*Based on Robert M. Saltzman and Vijay Mehrotr "A Call Center Uses Simulation to Drive Strateg Change," *Interfaces* (May/June 2001): 87–101.

In this chapter we begin by showing how simulation can be used to study the financial risks associated with the development of a new product. We continue with illustrations showing how simulation can be used to establish an effective inventory policy and how simulation can be used to design waiting line systems. Other issues, such as verifying the simulation program, validating the model, and selecting a simulation software package, are discussed in Section 12.4.

12.1 RISK ANALYSIS

Risk analysis is the process of predicting the outcome of a decision in the face of uncertainty. In this section, we describe a problem that involves considerable uncertainty: the development of a new product. We first show how risk analysis can be conducted without using simulation; then we show how a more comprehensive risk analysis can be conducted with the aid of simulation.

PortaCom Project

PortaCom manufactures personal computers and related equipment. PortaCom's product design group developed a prototype for a new high-quality portable printer. The new printer features an innovative design and has the potential to capture a significant share of the portable printer market. Preliminary marketing and financial analyses provided the following selling price, first-year administrative cost, and first-year advertising cost.

$$\begin{aligned} \text{Selling price} &= \$249 \text{ per unit} \\ \text{Administrative cost} &= \$400{,}000 \\ \text{Advertising cost} &= \$600{,}000 \end{aligned}$$

In the simulation model for the PortaCom project, the preceding values are constants and are referred to as **parameters** of the model.

The cost of direct labor, the cost of parts, and the first-year demand for the printer are not known with certainty and are considered probabilistic inputs. At this stage of the planning process, PortaCom's best estimates of these inputs are \$45 per unit for the direct labor cost, \$90 per unit for the parts cost, and 15,000 units for the first-year demand. PortaCom would like an analysis of the first-year profit potential for the printer. Because of PortaCom's tight cash flow situation, management is particularly concerned about the potential for a loss.

What-If Analysis

One approach to risk analysis is called **what-if analysis.** A what-if analysis involves generating values for the probabilistic inputs (direct labor cost, parts cost, and first-year demand) and computing the resulting value for the output (profit). With a selling price of \$249 per unit and administrative plus advertising costs equal to \$400,000 + \$600,000 = \$1,000,000, the PortaCom profit model is

$$\text{Profit} = (\$249 - \text{Direct labor cost per unit} - \text{Parts cost per unit})(\text{Demand}) - \$1{,}000{,}000$$

Letting

$$\begin{aligned} c_1 &= \text{direct labor cost per unit} \\ c_2 &= \text{parts cost per unit} \\ x &= \text{first-year demand} \end{aligned}$$

the profit model for the first year can be written as follows:

$$\text{Profit} = (249 - c_1 - c_2)x - 1{,}000{,}000 \tag{12.1}$$

The PortaCom profit model can be depicted as shown in Figure 12.2.

Recall that PortaCom's best estimates of the direct labor cost per unit, the parts cost per unit, and first-year demand are \$45, \$90, and 15,000 units, respectively. These values constitute the **base-case scenario** for PortaCom. Substituting these values into equation (12.1) yields the following profit projection:

$$\text{Profit} = (249 - 45 - 90)(15{,}000) - 1{,}000{,}000 = 710{,}000$$

Thus, the base-case scenario leads to an anticipated profit of \$710,000.

In risk analysis we are concerned with both the probability of a loss and the magnitude of a loss. Although the base-case scenario looks appealing, PortaCom might be interested in what happens if the estimates of the direct labor cost per unit, parts cost per unit, and first-year demand do not turn out to be as expected under the base-case scenario. For instance, suppose that PortaCom believes that direct labor costs could range from \$43 to \$47 per unit, parts cost could range from \$80 to \$100 per unit, and first-year demand could range from 1500 to 28,500 units. Using these ranges, what-if analysis can be used to evaluate a **worst-case scenario** and a **best-case scenario.**

The worst-case value for the direct labor cost is \$47 (the highest value), the worst-case value for the parts cost is \$100 (the highest value), and the worst-case value for demand is 1500 units (the lowest value). Thus, in the worst-case scenario, $c_1 = 47$, $c_2 = 100$, and $x = 1500$. Substituting these values into equation (12.1) leads to the following profit projection:

$$\text{Profit} = (249 - 47 - 100)(1500) - 1{,}000{,}000 = -847{,}000$$

The worst-case scenario leads to a projected loss of \$847,000.

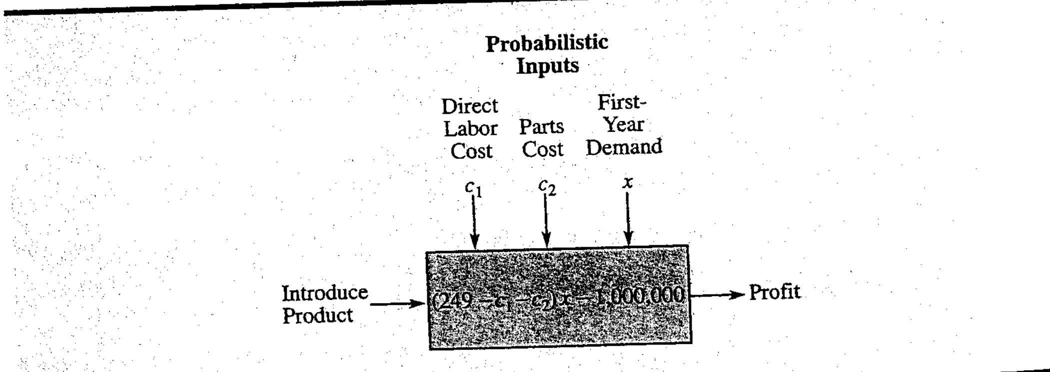
The best-case value for the direct labor cost is \$43 (the lowest value), the best-case value for the parts cost is \$80 (the lowest value), and the best-case value for demand is 28,500 units (the highest value). Substituting these values into equation (12.1) leads to the following profit projection:

$$\text{Profit} = (249 - 43 - 80)(28{,}500) - 1{,}000{,}000 = 2{,}591{,}000$$

Problem 2 will give you practice using what-if analysis.

The best-case scenario leads to a projected profit of \$2,591,000.

FIGURE 12.2 PORTACOM PROFIT MODEL



At this point the what-if analysis provides the conclusion that profits can range from a loss of $847,000 to a profit of $2,591,000 with a base-case profit of $710,000. Although the base-case profit of $710,000 is possible, the what-if analysis indicates that either a substantial loss or a substantial profit is possible. Other scenarios that PortaCom might want to consider can also be evaluated. However, the difficulty with what-if analysis is that it does not indicate the likelihood of the various profit or loss values. In particular, we do not know anything about the *probability* of a loss.

Simulation

Using simulation to perform risk analysis for the PortaCom project is like playing out many what-if scenarios by randomly generating values for the probabilistic inputs. The advantage of simulation is that it allows us to assess the probability of a profit and the probability of a loss.

Using the what-if approach to risk analysis, we selected values for the probabilistic inputs [direct labor cost per unit (c_1), parts cost per unit (c_2), and first-year demand (x)], and then computed the resulting profit. Applying simulation to the PortaCom project requires generating values for the probabilistic inputs that are representative of what we might observe in practice. To generate such values, we must know the probability distribution for each probabilistic input. Further analysis by PortaCom led to the following probability distributions for the direct labor cost per unit, the parts cost per unit, and first-year demand:

One advantage of simulation is the ability to use probability distributions that are unique to the system being studied.

Direct Labor Cost PortaCom believes that the direct labor cost will range from $43 to $47 per unit and is described by the discrete probability distribution shown in Table 12.1. Thus, we see a 0.1 probability that the direct labor cost will be $43 per unit, a 0.2 probability that the direct labor cost will be $44 per unit, and so on. The highest probability of 0.4 is associated with a direct labor cost of $45 per unit.

Parts Cost This cost depends upon the general economy, the overall demand for parts, and the pricing policy of PortaCom's parts suppliers. PortaCom believes that the parts cost will range from $80 to $100 per unit and is described by the uniform probability distribution shown in Figure 12.3. Costs per unit between $80 and $100 are equally likely.

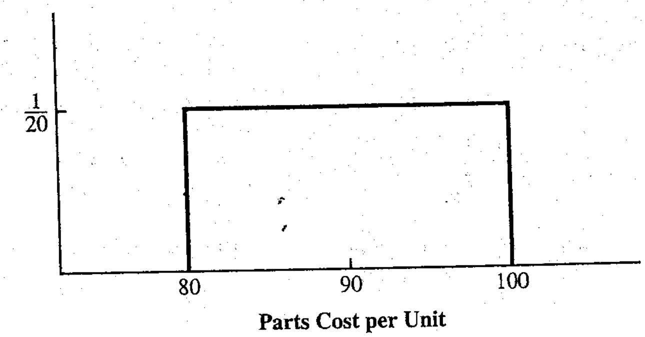
First-Year Demand PortaCom believes that first-year demand is described by the normal probability distribution shown in Figure 12.4. The mean or expected value of first-year demand is 15,000 units. The standard deviation of 4500 units describes the variability in the first-year demand.

To simulate the PortaCom project, we must generate values for the three probabilistic inputs and compute the resulting profit. Then we generate another set of values for the

TABLE 12.1 PROBABILITY DISTRIBUTION FOR DIRECT LABOR COST PER UNIT

Direct Labor Cost per Unit	Probability
$43	0.1
$44	0.2
$45	0.4
$46	0.2
$47	0.1

FIGURE 12.3 UNIFORM PROBABILITY DISTRIBUTION FOR THE PARTS COST PER UNIT



A flowchart provides a graphical representation that helps describe the logic of the simulation model.

probabilistic inputs, compute a second value for profit, and so on. We continue this process until we are satisfied that enough trials have been conducted to describe the probability distribution for profit. This process of generating probabilistic inputs and computing the value of the output is called *simulation*. The sequence of logical and mathematical operations required to conduct a simulation can be depicted with a flowchart. A flowchart for the PortaCom simulation is shown in Figure 12.5.

Following the logic described by the flowchart we see that the model parameters—selling price, administrative cost, and advertising cost—are \$249, \$400,000, and \$600,000, respectively. These values will remain fixed throughout the simulation.

The next three blocks depict the generation of values for the probabilistic inputs. First, a value for the direct labor cost (c_1) is generated. Then a value for the parts cost (c_2) is generated, followed by a value for the first-year demand (x). These probabilistic input values are combined using the profit model given by equation (12.1).

$$\text{Profit} = (249 - c_1 - c_2)x - 1{,}000{,}000$$

The computation of profit completes one trial of the simulation. We then return to the block where we generated the direct labor cost and begin another trial. This process is repeated until a satisfactory number of trials has been generated.

FIGURE 12.4 NORMAL PROBABILITY DISTRIBUTION OF FIRST-YEAR DEMAND

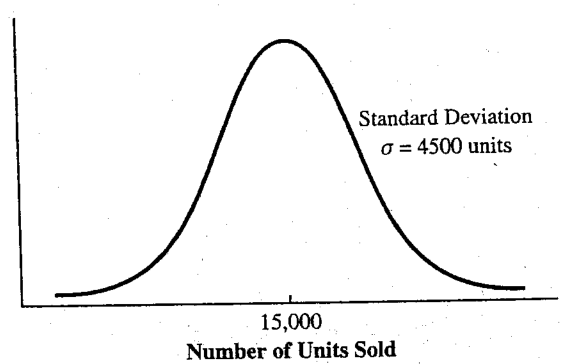
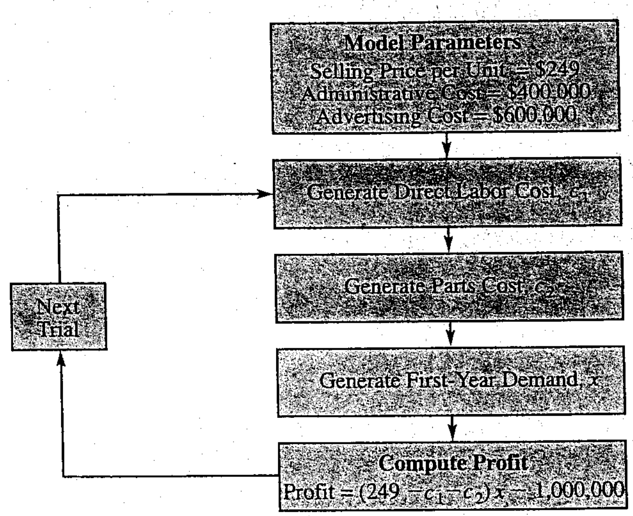


FIGURE 12.5 FLOWCHART FOR THE PORTACOM SIMULATION



At the end of the simulation, output measures of interest can be developed. For example, we will be interested in computing the average profit and the probability of a loss. For the output measures to be meaningful, the values of the probabilistic inputs must be representative of what is likely to happen when the PortaCom printer is introduced into the market. An essential part of the simulation procedure is the ability to generate representative values for the probabilistic inputs. We now discuss how to generate these values.

Random Numbers and Generating Probabilistic Input Values In the PortaCom simulation, representative values must be generated for the direct labor cost per unit (c_1), the parts cost per unit (c_2), and the first-year demand (x). Random numbers and the probability distributions associated with each probabilistic input are used to generate representative values. To illustrate how to generate these values, we need to introduce the concept of *computer-generated random numbers.*

Computer-generated random numbers[1] are randomly selected decimal numbers from 0 up to, but not including, 1. The computer-generated random numbers are equally likely and are uniformly distributed over the interval from 0 to 1. Computer-generated random numbers can be obtained using built-in functions available in computer simulation packages and spreadsheets. For instance, placing =RAND() in a cell of an Excel worksheet will result in a random number between 0 and 1 being placed into that cell.

Because random numbers are equally likely, management scientists can assign ranges of random numbers to corresponding values of probabilistic inputs so that the probability of any input value to the simulation model is identical to the probability of its occurrence in the real system.

Table 12.2 contains 500 random numbers generated using Excel. These numbers can be viewed as a random sample of 500 values from a uniform probability distribution over the interval from 0 to 1. Let us show how random numbers can be used to generate values for the PortaCom probability distributions. We begin by showing how to generate a value for the direct labor cost per unit. The approach described is applicable for generating values from any discrete probability distribution.

[1]Computer-generated random numbers are called *pseudorandom numbers.* Because they are generated through the use of mathematical formulas, they are not technically random. The difference between random numbers and pseudorandom numbers is primarily philosophical, and we use the term *random numbers* regardless of whether they are generated by a computer.

TABLE 12.2 500 COMPUTER-GENERATED RANDOM NUMBERS

0.6953	0.5247	0.1368	0.9850	0.7467	0.3813	0.5827	0.7893	0.7169	0.8166
0.0082	0.9925	0.6874	0.2122	0.6885	0.2159	0.4299	0.3467	0.2186	0.1033
0.6799	0.1241	0.3056	0.5590	0.0423	0.6515	0.2750	0.8156	0.2871	0.4680
0.8898	0.1514	0.1826	0.0004	0.5259	0.2425	0.8421	0.9248	0.9155	0.9518
0.6515	0.5027	0.9290	0.5177	0.3134	0.9177	0.2605	0.6668	0.1167	0.7870
0.3976	0.7790	0.0035	0.0064	0.0441	0.3437	0.1248	0.5442	0.9800	0.1857
0.0642	0.4086	0.6078	0.2044	0.0484	0.4691	0.7058	0.8552	0.5029	0.3288
0.0377	0.5250	0.7774	0.2390	0.9121	0.5345	0.8178	0.8443	0.4154	0.2526
0.5739	0.5181	0.0234	0.7305	0.0376	0.5169	0.5679	0.5495	0.7872	0.5321
0.5827	0.0341	0.7482	0.6351	0.9146	0.4700	0.7869	0.1337	0.0702	0.4219
0.0508	0.7905	0.2932	0.4971	0.0225	0.4466	0.5118	0.1200	0.0200	0.5445
0.4757	0.1399	0.5668	0.9569	0.7255	0.4650	0.4084	0.3701	0.9446	0.8064
0.6805	0.9931	0.4166	0.1091	0.7730	0.0691	0.9411	0.3468	0.0014	0.7379
0.2603	0.7507	0.6414	0.9907	0.2699	0.4571	0.9254	0.2371	0.8664	0.9553
0.8143	0.7625	0.1708	0.1900	0.2781	0.2830	0.6877	0.0488	0.8635	0.3155
0.5681	0.7854	0.5016	0.9403	0.1078	0.5255	0.8727	0.3815	0.5541	0.9833
0.1501	0.9363	0.3858	0.3545	0.5448	0.0643	0.3167	0.6732	0.6283	0.2631
0.8806	0.7989	0.7484	0.8083	0.2701	0.5039	0.9439	0.1027	0.9677	0.4597
0.4582	0.7590	0.4393	0.4704	0.6903	0.3732	0.6587	0.8675	0.2905	0.3058
0.0785	0.1467	0.3880	0.5274	0.8723	0.7517	0.9905	0.8904	0.8177	0.6660
0.1158	0.6635	0.4992	0.9070	0.2975	0.5686	0.8495	0.1652	0.2039	0.2553
0.2762	0.7018	0.6782	0.4013	0.2224	0.4672	0.5753	0.6219	0.6871	0.9255
0.9382	0.6411	0.7984	0.0608	0.5945	0.3977	0.4570	0.9924	0.8398	0.8361
0.5102	0.7021	0.4353	0.3398	0.8038	0.2260	0.1250	0.1884	0.3432	0.1192
0.2354	0.7410	0.7089	0.2579	0.1358	0.8446	0.1648	0.3889	0.5620	0.6555
0.9082	0.7906	0.7589	0.8870	0.1189	0.7125	0.6324	0.1096	0.5155	0.3449
0.6936	0.0702	0.9716	0.0374	0.0683	0.2397	0.7753	0.2029	0.1464	0.8000
0.4042	0.8158	0.3623	0.6614	0.7954	0.7516	0.6518	0.3638	0.3107	0.2718
0.9410	0.2201	0.6348	0.0367	0.0311	0.0688	0.2346	0.3927	0.7327	0.9994
0.0917	0.2504	0.2878	0.1735	0.3872	0.6816	0.2731	0.3846	0.6621	0.8983
0.8532	0.4869	0.2685	0.6349	0.9364	0.3451	0.4998	0.2842	0.0643	0.6656
0.8980	0.0455	0.8314	0.8189	0.6783	0.8086	0.1386	0.4442	0.9941	0.6812
0.8412	0.8792	0.2025	0.9320	0.7656	0.3815	0.5302	0.8744	0.4584	0.3585
0.5688	0.8633	0.5818	0.0692	0.2543	0.5453	0.9955	0.1237	0.7535	0.5993
0.5006	0.1215	0.8102	0.1026	0.9251	0.6851	0.1559	0.1214	0.2628	0.9374
0.5748	0.4164	0.3427	0.2809	0.8064	0.5855	0.2229	0.2805	0.9139	0.9013
0.1100	0.0873	0.9407	0.8747	0.0496	0.4380	0.5847	0.4183	0.5929	0.4863
0.5802	0.7747	0.1285	0.0074	0.6252	0.7747	0.0112	0.3958	0.3285	0.5389
0.1019	0.6628	0.8998	0.1334	0.2798	0.7351	0.7330	0.6723	0.6924	0.3963
0.9909	0.8991	0.2298	0.2603	0.6921	0.5573	0.8191	0.0384	0.2954	0.0636
0.6292	0.4923	0.0276	0.6734	0.6562	0.4231	0.1980	0.6551	0.3716	0.0507
0.9430	0.2579	0.7933	0.0945	0.3192	0.3195	0.7772	0.4672	0.7070	0.5925
0.9938	0.7098	0.7964	0.7952	0.8947	0.1214	0.8454	0.8294	0.5394	0.9413
0.4690	0.1395	0.0930	0.3189	0.6972	0.7291	0.8513	0.9256	0.7478	0.8124
0.2028	0.3774	0.0485	0.7718	0.9656	0.2444	0.0304	0.1395	0.1577	0.8625
0.6141	0.4131	0.2006	0.2329	0.6182	0.5151	0.6300	0.9311	0.3837	0.7828
0.2757	0.8479	0.7880	0.8492	0.6859	0.8947	0.6246	0.1574	0.4936	0.8077
0.0561	0.0126	0.6531	0.0378	0.4975	0.1133	0.3572	0.0071	0.4555	0.7563
0.1419	0.4308	0.8073	0.4681	0.0481	0.2918	0.2975	0.0685	0.6384	0.0812
0.3125	0.0053	0.9209	0.9768	0.3584	0.0390	0.2161	0.6333	0.4391	0.6991

TABLE 12.3 RANDOM NUMBER INTERVALS FOR GENERATING VALUES OF DIRECT LABOR COST PER UNIT

Direct Labor Cost per Unit	Probability	Interval of Random Numbers
\$43	0.1	0.0 but less than 0.1
\$44	0.2	0.1 but less than 0.3
\$45	0.4	0.3 but less than 0.7
\$46	0.2	0.7 but less than 0.9
\$47	0.1	0.9 but less than 1.0

An interval of random numbers is assigned to each possible value of the direct labor cost in such a fashion that the probability of generating a random number in the interval is equal to the probability of the corresponding direct labor cost. Table 12.3 shows how this process is done. The interval of random numbers greater than or equal to 0.0 but less than 0.1 is associated with a direct labor cost of \$43, the interval of random numbers greater than or equal to 0.1 but less than 0.3 is associated with a direct labor cost of \$44, and so on. With this assignment of random number intervals to the possible values of the direct labor cost, the probability of generating a random number in any interval is equal to the probability of obtaining the corresponding value for the direct labor cost. Thus, to select a value for the direct labor cost, we generate a random number between 0 and 1. If the random number is greater than or equal to 0.0 but less than 0.1, we set the direct labor cost equal to \$43. If the random number is greater than or equal to 0.1 but less than 0.3, we set the direct labor cost equal to \$44, and so on.

Try Problem 5 for an opportunity to establish intervals of random numbers and simulate demand from a discrete probability distribution.

Each trial of the simulation requires a value for the direct labor cost. Suppose that on the first trial the random number is 0.9109. From Table 12.3, the simulated value for the direct labor cost is \$47 per unit. Suppose that on the second trial the random number is 0.2841. From Table 12.3, the simulated value for the direct labor cost is \$44 per unit. Table 12.4 shows the results obtained for the first 10 simulation trials.

Each trial in the simulation requires a value of the direct labor cost, parts cost, and first-year demand. Let us now turn to the issue of generating values for the parts cost. The probability distribution for the parts cost per unit is the uniform distribution shown in Figure 12.3. Because this random variable has a different probability distribution than direct labor cost, we use random numbers in a slightly different way to generate values for parts cost. With

TABLE 12.4 RANDOM GENERATION OF 10 VALUES FOR THE DIRECT LABOR COST PER UNIT

Trial	Random Number	Direct Labor Cost (\$)
1	0.9109	47
2	0.2841	44
3	0.6531	45
4	0.0367	43
5	0.3451	45
6	0.2757	44
7	0.6859	45
8	0.6246	45
9	0.4936	45
10	0.8077	46

a uniform probability distribution, the following relationship between the random number and the associated value of the parts cost is used.

$$\text{Parts cost} = a + r(b - a) \tag{12.2}$$

where

r = random number between 0 and 1
a = smallest value for parts cost
b = largest value for parts cost

For PortaCom, the smallest value for the parts cost is \$80, and the largest value is \$100. Applying equation (12.2) with $a = 80$ and $b = 100$ leads to the following formula for generating the parts cost given a random number, r.

$$\text{Parts cost} = 80 + r(100 - 80) = 80 + r20 \tag{12.3}$$

Equation (12.3) generates a value for the parts cost. Suppose that a random number of 0.2680 is obtained. The value for the parts cost is

$$\text{Parts cost} = 80 + 0.2680(20) = 85.36 \text{ per unit}$$

Suppose that a random number of 0.5842 is generated on the next trial. The value for the parts cost is

$$\text{Parts cost} = 80 + 0.5842(20) = 91.68 \text{ per unit}$$

Spreadsheet packages such as Excel have built-in functions that make simulations based on probability distributions such as the normal probability distribution relatively easy.

With appropriate choices of a and b, equation (12.2) can be used to generate values for any uniform probability distribution. Table 12.5 shows the generation of 10 values for the parts cost per unit.

Finally, we need a random number procedure for generating the first-year demand. Because first-year demand is normally distributed with a mean of 15,000 units and a standard deviation of 4500 units (see Figure 12.4), we need a procedure for generating random values

TABLE 12.5 RANDOM GENERATION OF 10 VALUES FOR THE PARTS COST PER UNIT

Trial	Random Number	Parts Cost (\$)
1	0.2680	85.36
2	0.5842	91.68
3	0.6675	93.35
4	0.9280	98.56
5	0.4180	88.36
6	0.7342	94.68
7	0.4325	88.65
8	0.1186	82.37
9	0.6944	93.89
10	0.7869	95.74

from a normal probability distribution. Because of the mathematical complexity, a detailed discussion of the procedure for generating random values from a normal probability distribution is omitted. However, computer simulation packages and spreadsheets include a built-in function that provides randomly generated values from a normal probability distribution. In most cases the user only needs to provide the mean and standard deviation of the normal distribution. For example, using Excel the following formula can be placed into a cell to obtain a value for a probabilistic input that is normally distributed:

=NORMINV(RAND(),Mean,Standard Deviation)

Because the mean for the first-year demand in the PortaCom problem is 15,000 and the standard deviation is 4500, the Excel statement

=NORMINV(RAND(),15000,4500) (12.4)

will provide a normally distributed value for first-year demand. For example, if Excel's RAND() function generates the random number 0.7005, the Excel function shown in equation (12.4) will provide a first-year demand of 17,366 units. If RAND() generates the random number 0.3204, equation (12.4) will provide a first-year demand of 12,900. Table 12.6 shows the results for the first 10 randomly generated values for demand. Note that random numbers less than 0.5 generate first-year demand values below the mean and that random numbers greater than 0.5 generate first-year demand values greater than the mean.

Running the Simulation Model Running the simulation model means implementing the sequence of logical and mathematical operations described in the flowchart in Figure 12.5. The model parameters are \$249 per unit for the selling price, \$400,000 for the administrative cost, and \$600,000 for the advertising cost. Each trial in the simulation involves randomly generating values for the probabilistic inputs (direct labor cost, parts cost, and first-year demand) and computing profit. The simulation is complete when a satisfactory number of trials have been conducted.

Let us compute the profit for the first trial assuming the following probabilistic inputs:

Direct labor cost: $c_1 = 47$
Parts cost: $c_2 = 85.36$
First-year demand: $x = 17{,}366$

TABLE 12.6 RANDOM GENERATION OF 10 VALUES FOR FIRST-YEAR DEMAND

Trial	Random Number	Demand
1	0.7005	17,366
2	0.3204	12,900
3	0.8968	20,686
4	0.1804	10,888
5	0.4346	14,259
6	0.9605	22,904
7	0.5646	15,732
8	0.7334	17,804
9	0.0216	5,902
10	0.3218	12,918

TABLE 12.7 PORTACOM SIMULATION RESULTS FOR 10 TRIALS

Trial	Direct Labor Cost per Unit ($)	Parts Cost per Unit ($)	Units Sold	Profit ($)
1	47	85.36	17,366	1,025,570
2	44	91.68	12,900	461,828
3	45	93.35	20,686	1,288,906
4	43	98.56	10,888	169,807
5	45	88.36	14,259	648,911
6	44	94.68	22,904	1,526,769
7	45	88.65	15,732	814,686
8	45	82.37	17,804	1,165,501
9	45	93.89	5,902	−350,131
10	46	95.74	12,918	385,585
Total	449	912.64	151,359	7,137,432
Average	$44.90	$91.26	15,136	$713,743

Referring to the flowchart in Figure 12.5, we see that the profit obtained is

$$\begin{aligned} \text{Profit} &= (249 - c_1 - c_2)x - 1{,}000{,}000 \\ &= (249 - 47 - 85.36)17{,}366 - 1{,}000{,}000 = 1{,}025{,}570 \end{aligned}$$

The first row of Table 12.7 shows the result of this trial of the PortaCom simulation.

The simulated profit for the PortaCom printer if the direct labor cost is $47 per unit, the parts cost is $85.36 per unit, and first-year demand is 17,366 units is $1,025,570. Of course, one simulation trial does not provide a complete understanding of the possible profit and loss. Because other values are possible for the probabilistic inputs, we can benefit from additional simulation trials.

Suppose that on a second simulation trial, random numbers of 0.2841, 0.5842, and 0.3204 are generated for the direct labor cost, the parts cost, and first-year demand, respectively. These random numbers will provide the probabilistic inputs of $44 for the direct labor cost, $91.68 for the parts cost, and 12,900 for first-year demand. These values provide a simulated profit of $461,828 on the second simulation trial (see the second row of Table 12.7).

Repetition of the simulation process with different values for the probabilistic inputs is an essential part of any simulation. Through the repeated trials, management will begin to understand what might happen when the product is introduced into the real world. We have shown the results of 10 simulation trials in Table 12.7. For these 10 cases, we find a profit as high as $1,526,769 for the 6th trial and a loss of $350,131 for the 9th trial. Thus, we see both the possibility of a profit and of a loss. Averages for the 10 trials are presented at the bottom of the table. We see that the average profit for the 10 trials is $713,743. The probability of a loss is 0.10, because one of the 10 trials (the 9th) resulted in a loss. We note also that the average values for labor cost, parts cost, and first-year demand are fairly close to their means of $45, $90, and 15,000, respectively.

Simulation of the PortaCom Project

Using an Excel worksheet, we simulated the PortaCom project 500 times. The worksheet used to carry out the simulation is shown in Figure 12.6. Note that the simulation results for trials 6 through 495 have been hidden so that the results can be shown in a reasonably sized

FIGURE 12.6 EXCEL WORKSHEET SIMULATION FOR THE PORTACOM PROJECT

EXCELfile
PortaCom

	A	B	C	D	E	F
1	**PortaCom Risk Analysis**					
2						
3	Selling Price per Unit		$249			
4	Administrative Cost		$400,000			
5	Advertising Cost		$600,000			
6						
7	**Direct Labor Cost**			**Parts Cost (Uniform Distribution)**		
8	Lower	Upper		Smallest Value	$80	
9	Random No.	Random No.	Cost per Unit	Largest Value	$100	
10	0.0	0.1	$43			
11	0.1	0.3	$44			
12	0.3	0.7	$45	**Demand (Normal Distribution)**		
13	0.7	0.9	$46	Mean	15000	
14	0.9	1.0	$47	Std Deviation	4500	
15						
16						
17	**Simulation Trials**					
18						
19		Direct Labor	Parts	First-Year		
20	Trial	Cost per Unit	Cost per Unit	Demand	Profit	
21	1	47	$85.36	17,366	$1,025,570	
22	2	44	$91.68	12,900	$461,828	
23	3	45	$93.35	20,686	$1,288,906	
24	4	43	$98.56	10,888	$169,807	
25	5	45	$88.36	14,259	$648,911	
516	496	44	$98.67	8,730	($71,739)	
517	497	45	$94.38	19,257	$1,110,952	
518	498	44	$90.85	14,920	$703,118	
519	499	43	$90.37	13,471	$557,652	
520	500	46	$92.50	18,614	$1,056,847	
521						
522			**Summary Statistics**			
523			Mean Profit		$698,457	
524			Standard Deviation		$520,485	
525			Minimum Profit		($785,234)	
526			Maximum Profit		$2,367,058	
527			Number of Losses		51	
528			Probability of Loss		0.1020	

figure. If desired, the rows for these trials can be shown and the simulation results displayed for all 500 trials. The details of the Excel worksheet that provided the PortaCom simulation are described in Appendix 12.1.

Excel worksheets for all simulations presented in this chapter are available on the CD that accompanies this text.

The simulation summary statistics in Figure 12.6 provide information about the risk associated with PortaCom's new printer. The worst result obtained in a simulation of 500 trials is a loss of $785,234, and the best result is a profit of $2,367,058. The mean profit is $698,457. Fifty-one of the trials resulted in a loss; thus, the estimated probability of a loss is 51/500 = 0.1020.

Simulation studies enable an objective estimate of the probability of a loss, which is an important aspect of risk analysis.

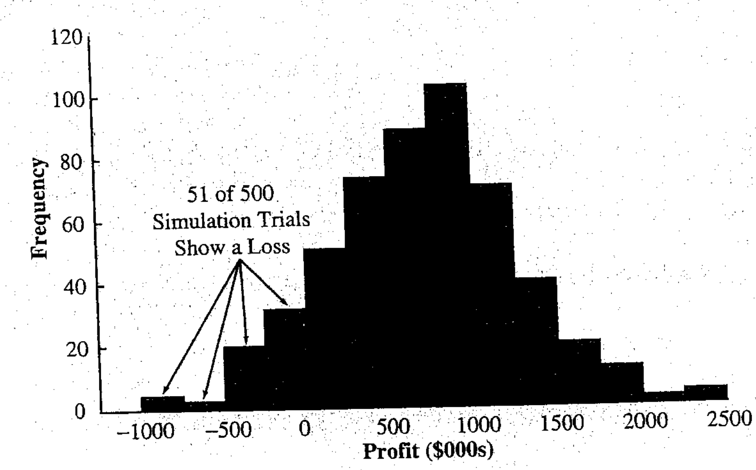
A histogram of simulated profit values is shown in Figure 12.7. We note that the distribution of profit values is fairly symmetric with a large number of values in the range of $250,000 to $1,250,000. The probability of a large loss or a large gain is small. Only 3 trials resulted in a loss of more than $500,000, and only 3 trials resulted in a profit greater than $2,000,000. However, the probability of a loss is significant. Forty-eight of the 500 trials resulted in a loss in the $0 to $500,000 range—almost 10 percent. The modal category, the one with the largest number of values, is the range of profits between $750,000 and $1,000,000.

In comparing the simulation approach to risk analysis to the what-if approach, we see that much more information is obtained by using simulation. With the what-if analysis, we learned that the base-case scenario projected a profit of $710,000. The worst-case scenario projected a loss of $847,000, and the best-case scenario projected a profit of $2,591,000. From the 500 trials of the simulation run, we see that the worst- and best-case scenarios, although possible, are unlikely. None of the 500 trials provided a loss as low as the worst-case or a profit as high as the best-case. Indeed, the advantage of simulation for risk analysis is the information it provides on the likely values of the output. We now know the probability of a loss, how the profit values are distributed over their range, and what profit values are most likely.

For practice working through a simulation problem, try Problems 9 and 14.

The simulation results help PortaCom's management better understand the profit/loss potential of the PortaCom portable printer. The 0.1020 probability of a loss may be acceptable to management given a probability of almost 0.80 (see Figure 12.7) that profit will exceed $250,000. On the other hand, PortaCom might want to conduct further market research before deciding whether to introduce the product. In any case, the simulation results should be helpful in reaching an appropriate decision. The Management Science in Action, Meeting Demand Levels at Pfizer, describes how a simulation model helped find ways to meet increasing demand for a product.

FIGURE 12.7 HISTOGRAM OF SIMULATED PROFIT FOR 500 TRIALS OF THE PORTACOM SIMULATION



MANAGEMENT SCIENCE IN ACTION

MEETING DEMAND LEVELS AT PFIZER*

Pharmacia & Upjohn's merger with Pfizer created one of the world's largest pharmaceutical firms. Demand for one of Pharmacia & Upjohn's long-standing products remained stable for several years at a level easily satisfied by the company's manufacturing facility. However, changes in market conditions caused an increase in demand to a level beyond the current capacity. A simulation model of the production process was developed to explore ways to increase production to meet the new level of demand in a cost-effective manner.

Simulation results were used to help answer the following questions:

- What is the maximum throughput of the existing facility?
- How can the existing production process be modified to increase throughput?
- How much equipment must be added to the existing facility to meet the increased demand?
- What is the desired size and configuration of the new production process?

The simulation model was able to demonstrate that the existing facilities, with some operating policy improvements, were large enough to satisfy the increased demand for the next several years. Expansion to a new production facility was not necessary. The simulation model also helped determine the number of operators required as the production level increased in the future. This result helped ensure that the proper number of operators would be trained by the time they were needed. The simulation model also provided a way reprocessed material could be used to replace fresh raw materials, resulting in a savings of approximately $3 million per year.

*Based on information provided by David B. Magerlein, James M. Magerlein, and Michael J. Goodrich.

12.2 INVENTORY SIMULATION

In this section we describe how simulation can be used to establish an inventory policy for a product that has an uncertain demand. The product is a home ventilation fan distributed by the Butler Electrical Supply Company. Each fan costs Butler \$75 and sells for \$125. Thus Butler realizes a gross profit of \$125 − \$75 = \$50 for each fan sold. Monthly demand for the fan is described by a normal probability distribution with a mean of 100 units and a standard deviation of 20 units.

Butler receives monthly deliveries from its supplier and replenishes its inventory to a level of Q at the beginning of each month. This beginning inventory level is referred to as the replenishment level. If monthly demand is less than the replenishment level, an inventory holding cost of \$15 is charged for each unit that is not sold. However, if monthly demand is greater than the replenishment level, a stock-out occurs and a shortage cost is incurred. Because Butler assigns a goodwill cost of \$30 for each customer turned away, a shortage cost of \$30 is charged for each unit of demand that cannot be satisfied. Management would like to use a simulation model to determine the average monthly net profit resulting from using a particular replenishment level. Management would also like information on the percentage of total demand that will be satisfied. This percentage is referred to as the *service level.*

The controllable input to the Butler simulation model is the replenishment level, Q. The probabilistic input is the monthly demand, D. The two output measures are the average monthly net profit and the service level. Computation of the service level requires that we keep track of the number of fans sold each month and the total demand for fans for each month. The service level will be computed at the end of the simulation run as the ratio of total units sold to total demand. A diagram of the relationship between the inputs and the outputs is shown in Figure 12.8.

When demand is less than or equal to the replenishment level ($D \leq Q$), D units are sold, and an inventory holding cost of \$15 is incurred for each of the $Q - D$ units that remain in inventory. Net profit for this case is computed as follows:

Case 1: $D \leq Q$

$$\begin{aligned}\text{Gross profit} &= \$50D \\ \text{Holding cost} &= \$15(Q - D) \\ \text{Net profit} &= \text{Gross profit} - \text{Holding cost} = \$50D - \$15(Q - D)\end{aligned} \tag{12.5}$$

When demand is greater than the replenishment level ($D >$ Q), Q fans are sold, and a shortage cost of \$30 is imposed for each of the $D - Q$ units of demand not satisfied. Net profit for this case is computed as follows:

Case 2: $D > Q$

$$\begin{aligned}\text{Gross profit} &= \$50Q \\ \text{Shortage cost} &= \$30(D - Q) \\ \text{Net profit} &= \text{Gross profit} - \text{Shortage cost} = \$50Q - \$30(D - Q)\end{aligned} \tag{12.6}$$

Figure 12.9 shows a flowchart that defines the sequence of logical and mathematical operations required to simulate the Butler inventory system. Each trial in the simulation represents one month of operation. The simulation is run for 300 months using a given

FIGURE 12.8 BUTLER INVENTORY SIMULATION MODEL

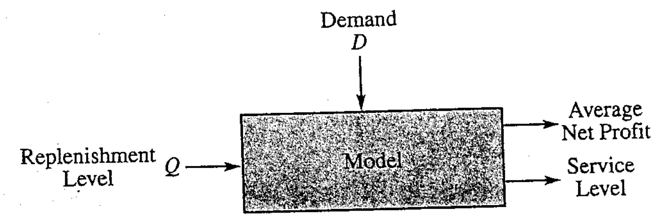
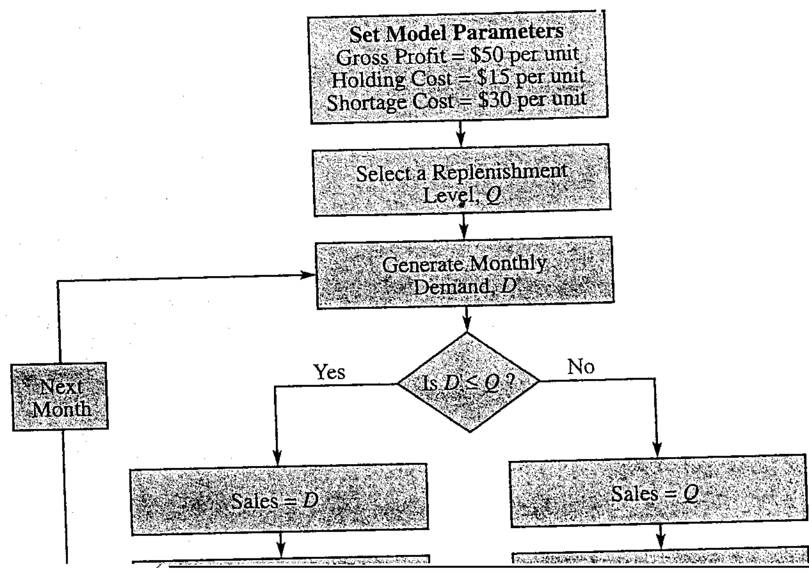


FIGURE 12.9 FLOWCHART FOR THE BUTLER INVENTORY SIMULATION



replenishment level, Q. Then the average profit and service level output measures are computed. Let us describe the steps involved in the simulation by illustrating the results for the first two months of a simulation run using a replenishment level of $Q = 100$.

The first block of the flowchart in Figure 12.9 sets the values of the model parameters: gross profit = \$50 per unit, holding cost = \$15 per unit, and shortage cost = \$30 per unit. The next block shows that a replenishment level of Q is selected; in our illustration, $Q = 100$. Then a value for monthly demand is generated. Because monthly demand is normally distributed with a mean of 100 units and a standard deviation of 20 units, we can use the Excel function =NORMINV(RAND(),100,20), as described in Section 12.1, to generate a value for monthly demand. Suppose that a value of $D = 79$ is generated on the first trial. This value of demand is then compared with the replenishment level, Q. With the replenishment level set at $Q = 100$, demand is less than the replenishment level, and the left branch of the flowchart is followed. Sales are set equal to demand (79), and gross profit, holding cost, and net profit are computed as follows:

$$\begin{aligned} \text{Gross profit} &= 50D = 50(79) = 3950 \\ \text{Holding cost} &= 15(Q - D) = 15(100 - 79) = 315 \\ \text{Net profit} &= \text{Gross profit} - \text{Holding cost} = 3950 - 315 = 3635 \end{aligned}$$

The values of demand, sales, gross profit, holding cost, and net profit are recorded for the first month. The first row of Table 12.8 summarizes the information for this first trial.

For the second month, suppose that a value of 111 is generated for monthly demand. Because demand is greater than the replenishment level, the right branch of the flowchart is followed. Sales are set equal to the replenishment level (100), and gross profit, shortage cost, and net profit are computed as follows:

$$\begin{aligned} \text{Gross profit} &= 50Q = 50(100) = 5000 \\ \text{Shortage cost} &= 30(D - Q) = 30(111 - 100) = 330 \\ \text{Net profit} &= \text{Gross profit} - \text{Shortage cost} = 5000 - 330 = 4670 \end{aligned}$$

The values of demand, sales, gross profit, holding cost, shortage cost, and net profit are recorded for the second month. The second row of Table 12.8 summarizes the information generated in the second trial.

Results for the first five months of the simulation are shown in Table 12.8. The totals show an accumulated total net profit of \$22,310, which is an average monthly net profit of \$22,310/5 = \$4462. Total unit sales are 472, and total demand is 501. Thus, the service

TABLE 12.8 BUTLER INVENTORY SIMULATION RESULTS FOR FIVE TRIALS WITH $Q = 100$

Month	Demand	Sales	Gross Profit (\$)	Holding Cost (\$)	Shortage Cost (\$)	Net Profit (\$)
1	79	79	3,950	315	0	3,635
2	111	100	5,000	0	330	4,670
3	93	93	4,650	105	0	4,545
4	100	100	5,000	0	0	5,000
5	118	100	5,000	0	540	4,460
Totals	501	472	23,600	420	870	22,310
Average	100	94	\$4,720	\$ 84	\$174	\$4,462

level is 472/501 = 0.942, indicating Butler has been able to satisfy 94.2% of demand during the five-month period.

Butler Inventory Simulation

Using Excel, we simulated the Butler inventory operation for 300 months. The worksheet used to carry out the simulation is shown in Figure 12.10. Note that the simulation results for months 6 through 295 have been hidden so that the results can be shown in a reasonably sized figure. If desired, the rows for these months can be shown and the simulation results displayed for all 300 months.

The summary statistics in Figure 12.10 show what can be anticipated over 300 months if Butler operates its inventory system using a replenishment level of 100. The average net profit is $4293 per month. Because 27,917 units of the total demand of 30,181 units were satisfied,

FIGURE 12.10 EXCEL WORKSHEET FOR THE BUTLER INVENTORY SIMULATION

EXCELfile
Butler

	A	B	C	D	E	F	G	H
1	**Butler Inventory**							
2								
3	Gross Profit per Unit		$50					
4	Holding Cost per Unit		$15					
5	Shortage Cost per Unit		$30					
6								
7	**Replenishment Level**		100					
8								
9	**Demand (Normal Distribution)**							
10	Mean	100						
11	Std Deviation	20						
12								
13								
14	**Simulation**							
15								
16	Month	Demand	Sales	Gross Profit	Holding Cost	Shortage Cost	Net Profit	
17	1	79	79	$3,950	$315	$0	$3,635	
18	2	111	100	$5,000	$0	$330	$4,670	
19	3	93	93	$4,650	$105	$0	$4,545	
20	4	100	100	$5,000	$0	$0	$5,000	
21	5	118	100	$5,000	$0	$540	$4,460	
312	296	89	89	$4,450	$165	$0	$4,285	
313	297	91	91	$4,550	$135	$0	$4,415	
314	298	122	100	$5,000	$0	$660	$4,340	
315	299	93	93	$4,650	$105	$0	$4,545	
316	300	126	100	$5,000	$0	$780	$4,220	
317								
318	**Totals**	30,181	27,917		**Summary Statistics**			
319					Mean Profit		[redacted]	
320					Standard Deviation		[redacted]	
321					Minimum Profit		[redacted]	
322					Maximum Profit		[redacted]	
323					Service Level		[redacted]	

TABLE 12.9 BUTLER INVENTORY SIMULATION RESULTS FOR 300 TRIALS

Replenishment Level	Average Net Profit ($)	Service Level (%)
100	4293	92.5
110	4524	96.5
120	4575	98.6
130	4519	99.6
140	4399	99.9

Simulation allows the user to consider different operating policies and changes to model parameters and then to observe the impact of the changes on output measures such as profit or service level.

the service level is 27,917/30,181 = 92.5%. We are now ready to use the simulation model to consider other replenishment levels that may improve the net profit and the service level.

At this point, we conducted a series of simulation experiments by repeating the Butler inventory simulation with replenishment levels of 110, 120, 130, and 140 units. The average monthly net profits and the service levels are shown in Table 12.9. The highest monthly net profit of $4575 occurs with a replenishment level of $Q = 120$. The associated service level is 98.6 percent. On the basis of these results, Butler selected a replenishment level of $Q = 120$.

Experimental simulation studies, such as this one for Butler's inventory policy, can help identify good operating policies and decisions. Butler's management used simulation to choose a replenishment level of 120 for its home ventilation fan. With the simulation model in place, management can also explore the sensitivity of this decision to some of the model parameters. For instance, we assigned a shortage cost of $30 for any customer demand not met. With this shortage cost, the replenishment level was $Q = 120$ and the service level was 98.6%. If management felt a more appropriate shortage cost was $10 per unit, running the simulation again using $10 as the shortage cost would be a simple matter.

Problem 18 gives you a chance to develop a different simulation model.

We mentioned earlier that simulation is not an optimization technique. Even though we used simulation to choose a replenishment level, it does not guarantee that this choice is optimal. All possible replenishment levels were not tested. Perhaps a manager would like to consider additional simulation runs with replenishment levels of $Q = 115$ and $Q = 125$ to search for an even better inventory policy. Also, we have no guarantee that with another set of 300 randomly generated demand values that the replenishment level with the highest profit would not change. However, with a large number of simulation trials, we should find a good and, at least, near optimal solution. The Management Science in Action, Petroleum Distribution in the Gulf of Mexico, describes a simulation application for 15 petroleum companies in the state of Florida.

MANAGEMENT SCIENCE IN ACTION

PETROLEUM DISTRIBUTION IN THE GULF OF MEXICO*

Domestic suppliers who operate oil refineries along the Gulf Coast are helping to satisfy Florida's increasing demand for refined petroleum products. Barge fleets, operated either by independent shipping companies or by the petroleum companies themselves, are used to transport more than 20 different petroleum products to 15 Florida petroleum companies. The petroleum products are loaded at refineries in Texas, Louisiana, and Mississippi and are discharged at tank terminals concentrated in Tampa, Port Everglades, and Jacksonville.

Barges operate under three types of contracts between the fleet operator and the client petroleum company:

- The client assumes total control of a barge and uses it for trips between its own refinery and one or more discharging ports.
- The client is guaranteed a certain volume will be moved during the contract period. Schedules vary considerably depending

upon the customer's needs and the fleet operator's capabilities.
- The client hires a barge for a single trip.

A simulation model was developed to analyze the complex process of operating barge fleets in the Gulf of Mexico. An appropriate probability distribution was used to simulate requests for shipments by the petroleum companies. Additional probability distributions were used to simulate the travel times depending upon the size and type of barge. Using this information, the simulation model was used to track barge loading times, barge discharge times, barge utilization, and total cost.

Analysts used simulation runs with a variety of what-if scenarios to answer questions about the petroleum distribution system and to make recommendations for improving the efficiency of the operation. Simulation helped determine the following:

- The optimal trade-off between fleet utilization and on-time delivery
- The recommended fleet size
- The recommended barge capacities
- The best service contract structure to balance the trade-off between customer service and delivery cost

Implementation of the simulation-based recommendations demonstrated a significant improvement in the operation and a significant lowering of petroleum distribution costs.

*Based on E. D. Chajakis, "Sophisticated Crude Transportation," *OR/MS Today* (December 1997): 30–34.

12.3 WAITING LINE SIMULATION

The simulation models discussed thus far have been based on independent trials in which the results for one trial do not affect what happens in subsequent trials. In this sense, the system being modeled does not change or evolve over time. Simulation models such as these are referred to as **static simulation models.** In this section, we develop a simulation model of a waiting line system where the state of the system, including the number of customers in the waiting line and whether the service facility is busy or idle, changes or evolves over time. To incorporate time into the simulation model, we use a simulation clock to record the time that each customer arrives for service as well as the time that each customer completes service. Simulation models that must take into account how the system changes or evolves over time are referred to as **dynamic simulation models.** In situations where the arrivals and departures of customers are **events** that occur at *discrete* points in time, the simulation model is also referred to as a **discrete-event simulation model.**

In Chapter 11, we presented formulas that could be used to compute the steady-state operating characteristics of a waiting line, including the average waiting time, the average number of units in the waiting line, the probability of waiting, and so on. In most cases, the waiting line formulas were based on specific assumptions about the probability distribution for arrivals, the probability distribution for service times, the queue discipline, and so on. Simulation, as an alternative for studying waiting lines, is more flexible. In applications where the assumptions required by the waiting line formulas are not reasonable, simulation may be the only feasible approach to studying the waiting line system. In this section we discuss the simulation of the waiting line for the Hammondsport Savings Bank automated teller machine (ATM).

Hammondsport Savings Bank ATM Waiting Line

Hammondsport Savings Bank will open several new branch banks during the coming year. Each new branch is designed to have one automated teller machine (ATM). A concern is that during busy periods several customers may have to wait to use the ATM. This concern prompted the bank to undertake a study of the ATM waiting line system. The bank's vice president wants to determine whether one ATM will be sufficient. The bank established service guidelines for its ATM system stating that the average customer waiting time for an

ATM should be one minute or less. Let us show how a simulation model can be used to study the ATM waiting line at a particular branch.

Customer Arrival Times

One probabilistic input to the ATM simulation model is the arrival times of customers who use the ATM. In waiting line simulations, arrival times are determined by randomly generating the time between two successive arrivals, referred to as the *interarrival time*. For the branch bank being studied, the customer interarrival times are assumed to be uniformly distributed between 0 and 5 minutes, as shown in Figure 12.11. With r denoting a random number between 0 and 1, an interarrival time for two successive customers can be simulated by using the formula for generating values from a uniform probability distribution.

$$\text{Interarrival time} = a + r(b - a) \tag{12.7}$$

where

r = random number between 0 and 1
a = minimum interarrival time
b = maximum interarrival time

A uniform probability distribution of interarrival times is used here to illustrate the simulation computations. Actually, any interarrival time probability distribution can be assumed, and the logic of the waiting line simulation model will not change.

For the Hammondsport ATM system, the minimum interarrival time is $a = 0$ minutes, and the maximum interarrival time is $b = 5$ minutes; therefore, the formula for generating an interarrival time is

$$\text{Interarrival time} = 0 + r(5 - 0) = 5r \tag{12.8}$$

Assume that the simulation run begins at time = 0. A random number of $r = 0.2804$ generates an interarrival time of $5(0.2804) = 1.4$ minutes for customer 1. Thus, customer 1 arrives 1.4 minutes after the simulation run begins. A second random number of $r = 0.2598$ generates an interarrival time of $5(0.2598) = 1.3$ minutes, indicating that customer 2 arrives 1.3 minutes after customer 1. Thus, customer 2 arrives $1.4 + 1.3 = 2.7$ minutes after

FIGURE 12.11 UNIFORM PROBABILITY DISTRIBUTION OF INTERARRIVAL TIMES FOR THE ATM WAITING LINE SYSTEM

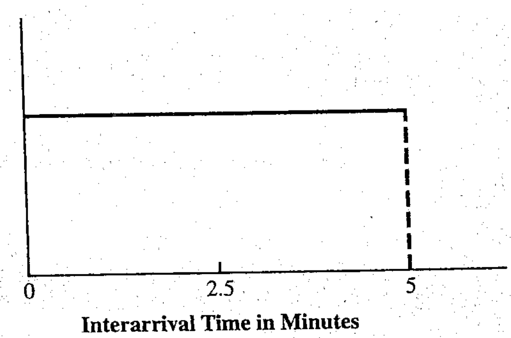
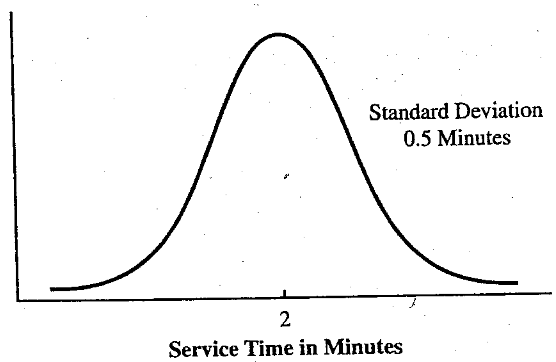


FIGURE 12.12 NORMAL PROBABILITY DISTRIBUTION OF SERVICE TIMES FOR THE ATM WAITING LINE SYSTEM



the simulation begins. Continuing, a third random number of $r = 0.9802$ indicates that customer 3 arrives 4.9 minutes after customer 2, which is 7.6 minutes after the simulation begins.

Customer Service Times

Another probabilistic input in the ATM simulation model is the service time, which is the time a customer spends using the ATM machine. Past data from similar ATMs indicate that a normal probability distribution with a mean of 2 minutes and a standard deviation of 0.5 minutes, as shown in Figure 12.12, can be used to describe service times. As discussed in Sections 12.1 and 12.2, values from a normal probability distribution with mean 2 and standard deviation 0.5 can be generated using the Excel function =NORMINV(RAND(),2,0.5). For example, the random number of 0.7257 generates a customer service time of 2.3 minutes.

Simulation Model

The probabilistic inputs to the Hammondsport Savings Bank ATM simulation model are the interarrival time and the service time. The controllable input is the number of ATMs used. The output will consist of various operating characteristics such as the probability of waiting, the average waiting time, the maximum waiting time, and so on. We show a diagram of the ATM simulation model in Figure 12.13.

FIGURE 12.13 HAMMONDSPORT SAVINGS BANK ATM SIMULATION MODEL

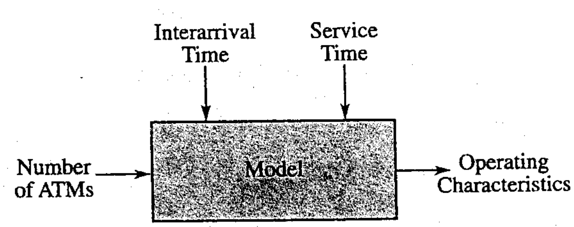
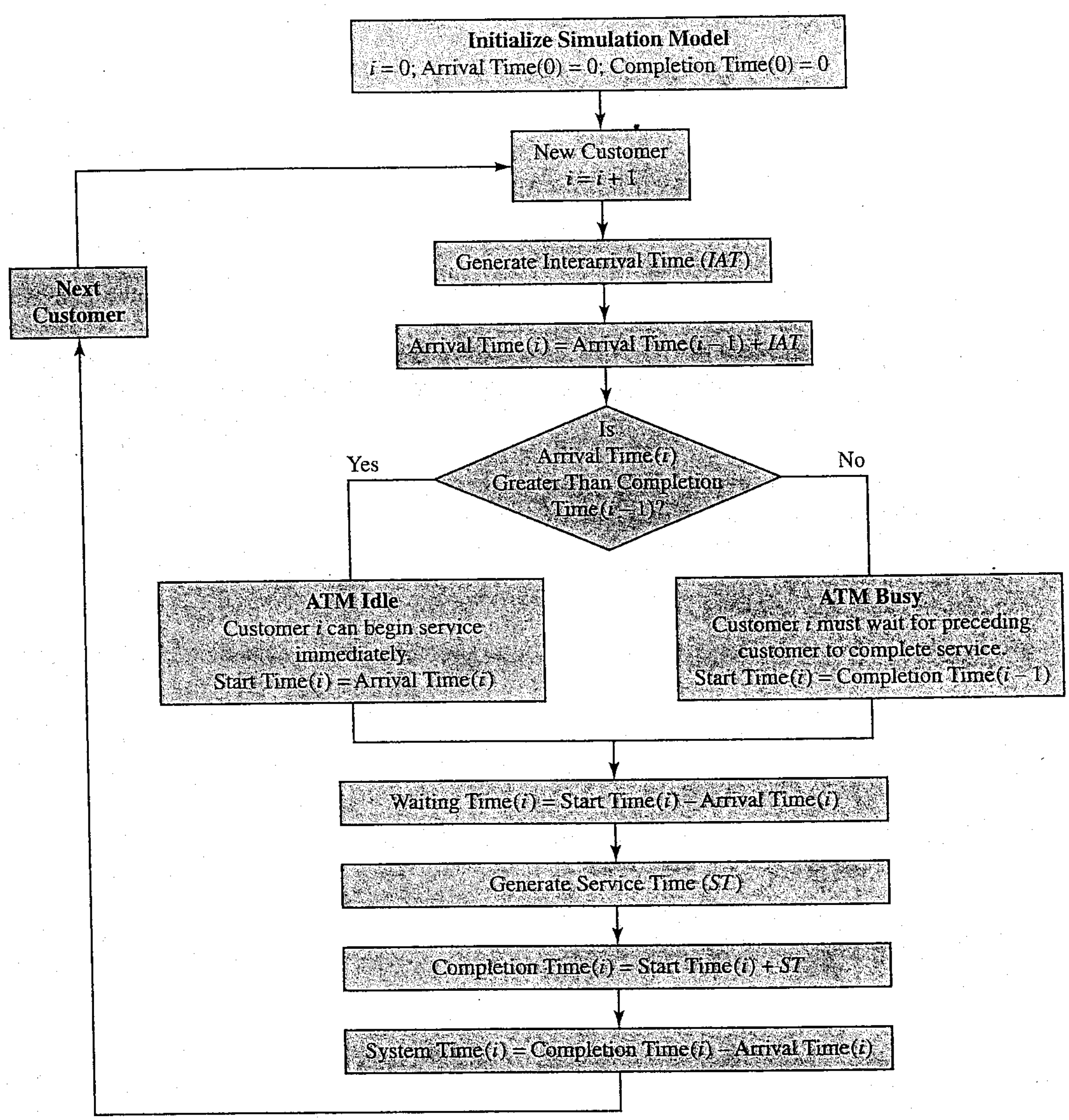


Figure 12.14 shows a flowchart that defines the sequence of logical and mathematical operations required to simulate the Hammondsport ATM system. The flowchart uses the following notation:

IAT = Interarrival time generated
Arrival time (i) = Time at which customer i arrives
Start time (i) = Time at which customer i starts service
Wait time (i) = Waiting time for customer i
ST = Service time generated
Completion time (i) = Time at which customer i completes service
System time (i) = System time for customer i (completion time − arrival time)

FIGURE 12.14 FLOWCHART OF THE HAMMONDSPORT SAVINGS BANK ATM WAITING LINE SIMULATION



Referring to Figure 12.14, we see that the simulation is initialized in the first block of the flowchart. Then a new customer is created. An interarrival time is generated to determine the time since the preceding customer arrived.[2] The arrival time for the new customer is then computed by adding the interarrival time to the arrival time of the preceding customer.

The decision rule for deciding whether the ATM is idle or busy is the most difficult aspect of the logic in a waiting line simulation model.

The arrival time for the new customer must be compared to the completion time of the preceding customer to determine whether the ATM is idle or busy. If the arrival time of the new customer is greater than the completion time of the preceding customer, the preceding customer will have finished service prior to the arrival of the new customer. In this case, the ATM will be idle, and the new customer can begin service immediately. The service start time for the new customer is equal to the arrival time of the new customer. However, if the arrival time for the new customer is not greater than the completion time of the preceding customer, the new customer arrived before the preceding customer finished service. In this case, the ATM is busy; the new customer must wait to use the ATM until the preceding customer completes service. The service start time for the new customer is equal to the completion time of the preceding customer.

Note that the time the new customer has to wait to use the ATM is the difference between the customer's service start time and the customer's arrival time. At this point, the customer is ready to use the ATM, and the simulation run continues with the generation of the customer's service time. The time at which the customer begins service plus the service time generated determine the customer's completion time. Finally, the total time the customer spends in the system is the difference between the customer's service completion time and the customer's arrival time. At this point, the computations are complete for the current customer, and the simulation continues with the next customer. The simulation is continued until a specified number of customers have been served by the ATM.

Simulation results for the first 10 customers are shown in Table 12.10. We discuss the computations for the first three customers to illustrate the logic of the simulation model and to show how the information in Table 12.10 was developed.

TABLE 12.10 SIMULATION RESULTS FOR 10 ATM CUSTOMERS

Customer	Interarrival Time	Arrival Time	Service Start Time	Waiting Time	Service Time	Completion Time	Time in System
1	1.4	1.4	1.4	0.0	2.3	3.7	2.3
2	1.3	2.7	3.7	1.0	1.5	5.2	2.5
3	4.9	7.6	7.6	0.0	2.2	9.8	2.2
4	3.5	11.1	11.1	0.0	2.5	13.6	2.5
5	0.7	11.8	13.6	1.8	1.8	15.4	3.6
6	2.8	14.6	15.4	0.8	2.4	17.8	3.2
7	2.1	16.7	17.8	1.1	2.1	19.9	3.2
8	0.6	17.3	19.9	2.6	1.8	21.7	4.4
9	2.5	19.8	21.7	1.9	2.0	23.7	3.9
10	1.9	21.7	23.7	2.0	2.3	26.0	4.3
Totals	21.7			11.2	20.9		32.1
Averages	2.17			1.12	2.09		3.21

[2] For the first customer, the interarrival time determines the time since the simulation started. Thus, the first interarrival time determines the time the first customer arrives.

Customer 1

- An interarrival time of $IAT = 1.4$ minutes is generated.
- Because the simulation run begins at time 0, the arrival time for customer 1 is $0 + 1.4 = 1.4$ minutes.
- Customer 1 may begin service immediately with a start time of 1.4 minutes.
- The waiting time for customer 1 is the start time minus the arrival time: $1.4 - 1.4 = 0$ minutes.
- A service time of $ST = 2.3$ minutes is generated for customer 1.
- The completion time for customer 1 is the start time plus the service time: $1.4 + 2.3 = 3.7$ minutes.
- The time in the system for customer 1 is the completion time minus the arrival time: $3.7 - 1.4 = 2.3$ minutes.

Customer 2

- An interarrival time of $IAT = 1.3$ minutes is generated.
- Because the arrival time of customer 1 is 1.4, the arrival time for customer 2 is $1.4 + 1.3 = 2.7$ minutes.
- Because the completion time of customer 1 is 3.7 minutes, the arrival time of customer 2 is not greater than the completion time of customer 1; thus, the ATM is busy when customer 2 arrives.
- Customer 2 must wait for customer 1 to complete service before beginning service. Customer 1 completes service at 3.7 minutes, which becomes the start time for customer 2.
- The waiting time for customer 2 is the start time minus the arrival time: $3.7 - 2.7 = 1$ minute.
- A service time of $ST = 1.5$ minutes is generated for customer 2.
- The completion time for customer 2 is the start time plus the service time: $3.7 + 1.5 = 5.2$ minutes.
- The time in the system for customer 2 is the completion time minus the arrival time: $5.2 - 2.7 = 2.5$ minutes.

Customer 3

- An interarrival time of $IAT = 4.9$ minutes is generated.
- Because the arrival time of customer 2 was 2.7 minutes, the arrival time for customer 3 is $2.7 + 4.9 = 7.6$ minutes.
- The completion time of customer 2 is 5.2 minutes, so the arrival time for customer 3 is greater than the completion time of customer 2. Thus, the ATM is idle when customer 3 arrives.
- Customer 3 begins service immediately with a start time of 7.6 minutes.
- The waiting time for customer 3 is the start time minus the arrival time: $7.6 - 7.6 = 0$ minutes.
- A service time of $ST = 2.2$ minutes is generated for customer 3.
- The completion time for customer 3 is the start time plus the service time: $7.6 + 2.2 = 9.8$ minutes.
- The time in the system for customer 3 is the completion time minus the arrival time: $9.8 - 7.6 = 2.2$ minutes.

Using the totals in Table 12.10, we can compute an average waiting time for the 10 customers of $11.2/10 = 1.12$ minutes, and an average time in the system of $32.1/10 = 3.21$ minutes. Table 12.10 shows that seven of the 10 customers had to wait. The total time for the 10-customer simulation is given by the completion time of the 10th customer: 26.0 minutes. However, at this point, we realize that a simulation for 10 customers is much too short a period to draw any firm conclusions about the operation of the waiting line.

Hammondsport Savings Bank ATM Simulation

Using an Excel worksheet, we simulated the operation of the Hammondsport ATM waiting line system for 1000 customers. The worksheet used to carry out the simulation is shown in Figure 12.15. Note that the simulation results for customers 6 through 995 have been hidden so that the results can be shown in a reasonably sized figure. If desired, the rows for these customers can be shown and the simulation results displayed for all 1000 customers.

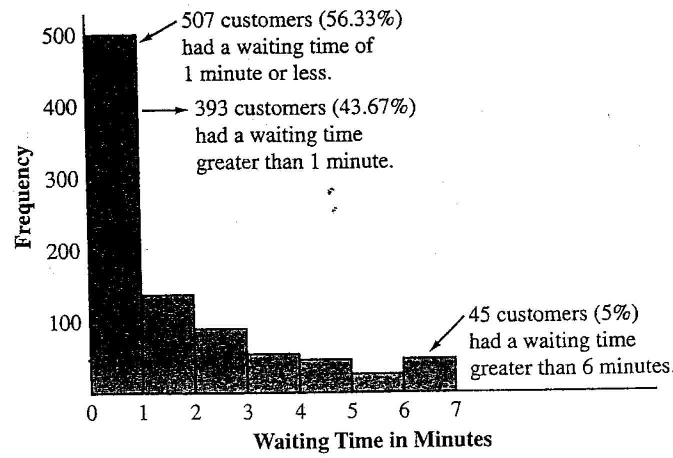
Ultimately, summary statistics will be collected in order to describe the results of 1000 customers. Before collecting the summary statistics, let us point out that most simulation studies of dynamic systems focus on the operation of the system during its long-run or steady-state operation. To ensure that the effects of start-up conditions are not included in the

FIGURE 12.15 EXCEL WORKSHEET FOR THE HAMMONDSPORT SAVINGS BANK WITH ONE ATM

EXCELfile
Hammondsport1

	A	B	C	D	E	F	G	H	I
1	**Hammondsport Savings Bank with One ATM**								
2									
3	**Interarrival Times (Uniform Distribution)**								
4	Smallest Value	0							
5	Largest Value	5							
6									
7	**Service Times (Normal Distribution)**								
8	Mean	2							
9	Std Deviation	0.5							
10									
11									
12	**Simulation**								
13									
14		Interarrival	Arrival	Service	Waiting	Service	Completion	Time	
15	Customer	Time	Time	Start Time	Time	Time	Time	in System	
16	1	1.4	1.4	1.4	0.0	2.3	3.7	2.3	
17	2	1.3	2.7	3.7	1.0	1.5	5.2	2.5	
18	3	4.9	7.6	7.6	0.0	2.2	9.8	2.2	
19	4	3.5	11.1	11.1	0.0	2.5	13.6	2.5	
20	5	0.7	11.8	13.6	1.8	1.8	15.4	3.6	
1011	996	0.5	2496.8	2498.1	1.3	0.6	2498.7	1.9	
1012	997	0.2	2497.0	2498.7	1.7	2.0	2500.7	3.7	
1013	998	2.7	2499.7	2500.7	1.0	1.8	2502.5	2.8	
1014	999	3.7	2503.4	2503.4	0.0	2.4	2505.8	2.4	
1015	1000	4.0	2507.4	2507.4	0.0	1.9	2509.3	1.9	
1016									
1017		**Summary Statistics**							
1018		Number Waiting			[redacted]				
1019		Probability of Waiting			[redacted]				
1020		Average Waiting Time			[redacted]				
1021		Maximum Waiting Time			[redacted]				
1022		Utilization of ATM			[redacted]				
1023		Number Waiting > 1 Min			[redacted]				
1024		Probability of Waiting > 1 Min			[redacted]				

FIGURE 12.16 HISTOGRAM SHOWING THE WAITING TIME FOR 900 ATM CUSTOMERS



steady-state calculations, a dynamic simulation model is usually run for a specified period without collecting any data about the operation of the system. The length of the start-up period can vary depending on the application. For the Hammondsport Savings Bank ATM simulation, we treated the results for the first 100 customers as the start-up period. Thus, the summary statistics shown in Figure 12.15 are for the 900 customers arriving during the steady-state period.

The summary statistics show that 549 of the 900 Hammondsport customers had to wait. This result provides a 549/900 = 0.61 probability that a customer will have to wait for service. In other words, approximately 61% of the customers will have to wait because the ATM is in use. The average waiting time is 1.59 minutes per customer with at least one customer waiting the maximum time of 13.5 minutes. The utilization rate of 0.7860 indicates that the ATM is in use 78.6% of the time. Finally, 393 of the 900 customers had to wait more than 1 minute (43.67% of all customers). A histogram of waiting times for the 900 customers is shown in Figure 12.16. This figure shows that 45 customers (5%) had a waiting time greater than 6 minutes.

The simulation supports the conclusion that the branch will have a busy ATM system. With an average customer wait time of 1.59 minutes, the branch does not satisfy the bank's customer service guideline. This branch is a good candidate for installation of a second ATM.

Simulation with Two ATMs

We extended the simulation model to the case of two ATMs. For the second ATM we also assume that the service time is normally distributed with a mean of 2 minutes and a standard deviation of 0.5 minutes. Table 12.11 shows the simulation results for the first 10 customers. In comparing the two-ATM system results in Table 12.11 with the single ATM simulation results shown in Table 12.10, we see that two additional columns are needed. These two columns show when each ATM becomes available for customer service. We assume that, when a new customer arrives, the customer will be served by the ATM that frees up first. When the simulation begins, the first customer is assigned to ATM 1.

Table 12.11 shows that customer 7 is the first customer who has to wait to use an ATM. We describe how customers 6, 7, and 8 are processed to show how the logic of the simulation run for two ATMs differs from that with a single ATM.

TABLE 12.11 SIMULATION RESULTS FOR 10 CUSTOMERS FOR A TWO-ATM SYSTEM

Customer	Interarrival Time	Arrival Time	Service Start Time	Waiting Time	Service Time	Completion Time	Time in System	Time Available ATM 1	Time Available ATM 2
1	1.7	1.7	1.7	0.0	2.1	3.8	2.1	3.8	0.0
2	0.7	2.4	2.4	0.0	2.0	4.4	2.0	3.8	4.4
3	2.0	4.4	4.4	0.0	1.4	5.8	1.4	5.8	4.4
4	0.1	4.5	4.5	0.0	0.9	5.4	0.9	5.8	5.4
5	4.6	9.1	9.1	0.0	2.2	11.3	2.2	5.8	11.3
6	1.3	10.4	10.4	0.0	1.6	12.0	1.6	12.0	11.3
7	0.6	11.0	11.3	0.3	1.7	13.0	2.0	12.0	13.0
8	0.3	11.3	12.0	0.7	2.2	14.2	2.9	14.2	13.0
9	3.4	14.7	14.7	0.0	2.9	17.6	2.9	14.2	17.6
10	0.1	14.8	14.8	0.0	2.8	17.6	2.8	17.6	17.6
Totals	14.8			1.0	19.8		20.8		
Averages	1.48			0.1	1.98		2.08		

Customer 6

- An interarrival time of 1.3 minutes is generated, and customer 6 arrives 9.1 + 1.3 = 10.4 minutes into the simulation.
- From the customer 5 row, we see that ATM 1 frees up at 5.8 minutes, and ATM 2 will free up at 11.3 minutes into the simulation. Because ATM 1 is free, customer 6 does not wait and begins service on ATM 1 at the arrival time of 10.4 minutes.
- A service time of 1.6 minutes is generated for customer 6. So customer 6 has a completion time of 10.4 + 1.6 = 12.0 minutes.
- The time ATM 1 will next become available is set at 12.0 minutes; the time available for ATM 2 remains 11.3 minutes.

Customer 7

- An interarrival time of 0.6 minute is generated, and customer 7 arrives 10.4 + 0.6 = 11.0 minutes into the simulation.
- From the previous row, we see that ATM 1 will not be available until 12.0 minutes, and ATM 2 will not be available until 11.3 minutes. So customer 7 must wait to use an ATM. Because ATM 2 will free up first, customer 7 begins service on that machine at a start time of 11.3 minutes. With an arrival time of 11.0 and a service start time of 11.3, customer 7 experiences a waiting time of 11.3 − 11.0 = 0.3 minute.
- A service time of 1.7 minutes is generated, leading to a completion time of 11.3 + 1.7 = 13.0 minutes.
- The time available for ATM 2 is updated to 13.0 minutes, and the time available for ATM 1 remains at 12.0 minutes.

Customer 8

- An interarrival time of 0.3 minute is generated, and customer 8 arrives 11.0 + 0.3 = 11.3 minutes into the simulation.
- From the previous row, we see that ATM 1 will be the first available. Thus, customer 8 starts service on ATM 1 at 12.0 minutes resulting in a waiting time of 12.0 − 11.3 = 0.7 minute.
- A service time of 2.2 minutes is generated, resulting in a completion time of 12.0 + 2.2 = 14.2 minutes and a system time of 0.7 + 2.2 = 2.9 minutes.
- The time available for ATM 1 is updated to 14.2 minutes, and the time available for ATM 2 remains at 13.0 minutes.

From the totals in Table 12.11, we see that the average waiting time for these 10 customers is only 1.0/10 = 0.1 minute. Of course, a much longer simulation will be necessary before any conclusions can be drawn.

Simulation Results with Two ATMs

Worksheets for the Hammondsport one-ATM and two-ATM systems are available on the CD that accompanies this text.

The Excel worksheet that we used to conduct a simulation for 1000 customers using two ATMs is shown in Figure 12.17. Results for the first 100 customers were discarded to account for the start-up period. With two ATMs, the number of customers who had to wait was reduced from 549 to 78. This reduction provides a 78/900 = 0.0867 probability that a customer will have to wait for service when two ATMs are used. The two-ATM system also reduced the average waiting time to 0.07 minute (4.2 seconds) per customer. The maximum waiting time was reduced from 13.5 to 2.9 minutes, and each ATM was in use 40.84% of the time. Finally, only 23 of the 900 customers had to wait more than 1 minute for an ATM to become available. Thus, only 2.56% of customers had to wait more than 1 minute. The simulation results provide evidence that Hammondsport Savings Bank needs to expand to the two-ATM system.

FIGURE 12.17 EXCEL WORKSHEET FOR THE HAMMONDSPORT SAVINGS BANK WITH TWO ATMs

EXCELfile
Hammondsport2

	A	B	C	D	E	F	G	H	I	J	K
1	**Hammondsport Savings Bank with Two ATMs**										
2											
3	**Interarrival Times (Uniform Distribution)**										
4	Smallest Value	0									
5	Largest Value	5									
6											
7	**Service Times (Normal Distribution)**										
8	Mean	2									
9	Std Deviation	0.5									
10											
11											
12	**Simulation**										
13											
14		Interarrival	Arrival	Service	Waiting	Service	Completion	Time	Time Available		
15	Customer	Time	Time	Start Time	Time	Time	Time	in System	ATM 1	ATM 2	
16	1	1.7	1.7	1.7	0.0	2.1	3.8	2.1	3.8	0.0	
17	2	0.7	2.4	2.4	0.0	2.0	4.4	2.0	3.8	4.4	
18	3	2.0	4.4	4.4	0.0	1.4	5.8	1.4	5.8	4.4	
19	4	0.1	4.5	4.5	0.0	0.9	5.4	0.9	5.8	5.4	
20	5	4.6	9.1	9.1	0.0	2.2	11.3	2.2	5.8	11.3	
1011	996	3.3	2483.2	2483.2	0.0	2.2	2485.4	2.2	2485.4	2482.1	
1012	997	4.5	2487.7	2487.7	0.0	1.9	2489.6	1.9	2485.4	2489.6	
1013	998	3.8	2491.5	2491.5	0.0	3.2	2494.7	3.2	2494.7	2489.6	
1014	999	0.0	2491.5	2491.5	0.0	2.4	2493.9	2.4	2494.7	2493.9	
1015	1000	2.6	2494.1	2494.1	0.0	2.8	2496.9	2.8	2494.7	2496.9	
1016											
1017		**Summary Statistics**									
1018		Number Waiting			78						
1019		Probability of Waiting			0.0867						
1020		Average Waiting Time			0.07						
1021		Maximum Waiting Time			2.9						
1022		Utilization of ATMs			0.4084						
1023		Number Waiting > 1 Min			23						
1024		Probability of Waiting > 1 Min			0.0256						

The simulation models that we developed can now be used to study the ATM operation at other branch banks. In each case, assumptions must be made about the appropriate interarrival time and service time probability distributions. However, once appropriate assumptions have been made, the same simulation models can be used to determine the operating characteristics of the ATM waiting line system. The Management Science in Action, Preboard Screening at Vancouver International Airport, describes another use of simulation for a queueing system.

MANAGEMENT SCIENCE IN ACTION

PREBOARD SCREENING AT VANCOUVER INTERNATIONAL AIRPORT*

Following the September 11, 2001, terrorist attacks in the United States, long lines at airport security checkpoints became commonplace. In order to reduce passenger waiting time, the Vancouver International Airport Authority teamed up with students and faculty at the University of British Columbia's Centre for Operations Excellence (COE) to build a simulation model of the airport's preboard screening security checkpoints. The goal was to use the simulation model to help achieve acceptable service standards.

Prior to building the simulation model, students from the COE observed the flow of passengers through the screening process and collected data on the service time at each process step. In addition to service time data, passenger demand data provided input to the simulation model. Two triangular probability distributions were used to simulate passenger arrivals at the preboarding facilities. For flights to Canadian destinations a 90-40-20 triangle was used. This distribution assumes that, for each flight, the first passenger will arrive at the screening checkpoint 90 minutes before departure, the last passenger will arrive 20 minutes before departure, and the most likely arrival time is 40 minutes before departure. For international flights a 150-80-20 triangle was used.

Output statistics from the simulation model provided information concerning resource utilization, waiting line lengths, and the time passengers spend in the system. The simulation model provided information concerning the number of personnel needed to process 90% of the passengers with a waiting time of 10 minutes or less. Ultimately the airport authority was able to design and staff the preboarding checkpoints in such a fashion that waiting times for 90% of the passengers were a maximum of 10 minutes.

*Based on Derek Atkins et al., "Right on Queue," *OR/MS Today* (April 2003): 26–29.

NOTES AND COMMENTS

1. The ATM waiting line model was based on uniformly distributed interarrival times and normally distributed service times. One advantage of simulation is its flexibility in accommodating a variety of different probability distributions. For instance, if we believe an exponential distribution is more appropriate for interarrival times, the ATM simulation could be repeated by simply changing the way the interarrival times are generated.
2. At the beginning of this section, we defined *discrete-event simulation* as involving a dynamic system that evolves over time. The simulation computations focus on the sequence of events as they occur at discrete points in time. In the ATM waiting line example, customer arrivals and the customer service completions were the discrete events. Referring to the arrival times and completion times in Table 12.10, we see that the first five discrete events for the ATM waiting line simulation were as follows:

Event	Time
Customer 1 arrives	1.4
Customer 2 arrives	2.7
Customer 1 finished	3.7
Customer 2 finished	5.2
Customer 3 arrives	7.6

3. We did not keep track of the number of customers in the ATM waiting line as we carried out the ATM simulation computations on a customer-by-customer basis. However, we can determine the average number of customers in

(continued)

the waiting line from other information in the simulation output. The following relationship is valid for any waiting line system:

$$\text{Average number in waiting line} = \frac{\text{Total waiting time}}{\text{Total time of simulation}}$$

For the system with one ATM, the 100th customer completed service at 247.8 minutes into the simulation. Thus, the total time of the simulation for the next 900 customers was $2509.3 - 247.8 = 2261.5$ minutes. The average waiting time was 1.59 minutes. During the simulation, the 900 customers had a total waiting time of $900(1.59) = 1431$ minutes. Therefore, the average number of customers in the waiting line is

$$\begin{aligned}\text{Average number in waiting line} &= 1431/2261.5\\ &= 0.63 \text{ customer}\end{aligned}$$

12.4 OTHER SIMULATION ISSUES

Because simulation is one of the most widely used quantitative analysis techniques, various software tools have been developed to help analysts implement a simulation model on a computer. In this section we comment on the software available and discuss some issues involved in verifying and validating a simulation model. We close the section with a discussion of some of the advantages and disadvantages of using simulation to study a real system.

Computer Implementation

The use of spreadsheets for simulation has grown rapidly in recent years, and third-party software vendors have developed spreadsheet add-ins that make building simulation models on a spreadsheet much easier. These add-in packages provide an easy facility for generating random values from a variety of probability distributions and provide a rich array of statistics describing the simulation output. Two popular spreadsheet add-ins are Crystal Ball from Decisioneering and @RISK from Palisade Corporation. Although spreadsheets can be a valuable tool for some simulation studies, they are generally limited to smaller, less complex systems.

With the growth of simulation applications, both users of simulation and software developers began to realize that computer simulations have many common features: model development, generating values from probability distributions, maintaining a record of what happens during the simulation, and recording and summarizing the simulation output. A variety of special-purpose simulation packages are available, including GPSS®, SIMSCRIPT®, SLAM®, and Arena®. These packages have built-in simulation clocks, simplified methods for generating probabilistic inputs, and procedures for collecting and summarizing the simulation output. Special-purpose simulation packages enable quantitative analysts to simplify the process of developing and implementing the simulation model. Indeed, Arena 6.0 was used to develop the simulation model described in the Management Science in Action, Preboard Screening at Vancouver International Airport.

The computational and record-keeping aspects of simulation models are assisted by special simulation software packages. The packages ease the tasks of developing a computer simulation model.

Simulation models can also be developed using general-purpose computer programming languages such as BASIC, FORTRAN, PASCAL, C, and C++. The disadvantage of using these languages is that special simulation procedures are not built in. One command in a special-purpose simulation package often performs the computations and record-keeping tasks that would require several BASIC, FORTRAN, PASCAL, C, or C++ statements to duplicate. The advantage of using a general-purpose programming language is that they offer greater flexibility in terms of being able to model more complex systems.

To decide which software to use, an analyst will have to consider the relative merits of a spreadsheet, a special-purpose simulation package, and a general-purpose computer programming language. The goal is to select the method that is easy to use while still providing an adequate representation of the system being studied.

Verification and Validation

An important aspect of any simulation study involves confirming that the simulation model accurately describes the real system. Inaccurate simulation models cannot be expected to provide worthwhile information. Thus, before using simulation results to draw conclusions about a real system, one must take steps to verify and validate the simulation model.

Verification is the process of determining that the computer procedure that performs the simulation calculations is logically correct. Verification is largely a debugging task to make sure that no errors are in the computer procedure that implements the simulation. In some cases, an analyst may compare computer results for a limited number of events with independent hand calculations. In other cases, tests may be performed to verify that the probabilistic inputs are being generated correctly and that the output from the simulation model seems reasonable. The verification step is not complete until the user develops a high degree of confidence that the computer procedure is error free.

Validation is the process of ensuring that the simulation model provides an accurate representation of a real system. Validation requires an agreement among analysts and managers that the logic and the assumptions used in the design of the simulation model accurately reflect how the real system operates. The first phase of the validation process is done prior to, or in conjunction with, the development of the computer procedure for the simulation process. Validation continues after the computer program has been developed, with the analyst reviewing the simulation output to see whether the simulation results closely approximate the performance of the real system. If possible, the output of the simulation model is compared to the output of an existing real system to make sure that the simulation output closely approximates the performance of the real system. If this form of validation is not possible, an analyst can experiment with the simulation model and have one or more individuals experienced with the operation of the real system review the simulation output to determine whether it is a reasonable approximation of what would be obtained with the real system under similar conditions.

Verification and validation are not tasks to be taken lightly. They are key steps in any simulation study and are necessary to ensure that decisions and conclusions based on the simulation results are appropriate for the real system.

Advantages and Disadvantages of Using Simulation

The primary advantages of simulation are that it is easy to understand and that the methodology can be used to model and learn about the behavior of complex systems that would be difficult, if not impossible, to deal with analytically. Simulation models are flexible; they can be used to describe systems without requiring the assumptions that are often required by mathematical models. In general, the larger the number of probabilistic inputs a system has, the more likely that a simulation model will provide the best approach for studying the system. Another advantage of simulation is that a simulation model provides a convenient experimental laboratory for the real system. Changing assumptions or operating policies in the simulation model and rerunning it can provide results that help predict how such changes will affect the operation of the real system. Experimenting directly with a real system is often not feasible.

Using simulation, we can ask what-if questions and project how the real system will behave. Although simulation does not guarantee optimality, it will usually provide near-optimal solutions. In addition, simulation models often warn against poor decision strategies by projecting disastrous outcomes such as system failures, large financial losses, and so on.

Simulation is not without some disadvantages. For complex systems, the process of developing, verifying, and validating a simulation model can be time-consuming and expensive. In addition, each simulation run provides only a sample of how the real system will operate. As such, the summary of the simulation data provides only estimates or approximations about the real system. Consequently, simulation does not guarantee an optimal solution. Nonetheless, the danger of obtaining poor solutions is slight if the analyst exercises good judgment in developing the simulation model and if the simulation process is run long enough under a wide variety of conditions so that the analyst has sufficient data to predict how the real system will operate.

SUMMARY

Simulation is a method for learning about a real system by experimenting with a model that represents the system. Some of the reasons simulation is frequently used are

1. It can be used for a wide variety of practical problems.
2. The simulation approach is relatively easy to explain and understand. As a result, management confidence is increased, and acceptance of the results is more easily obtained.
3. Spreadsheet packages now provide another alternative for model implementation, and third-party vendors have developed add-ins that expand the capabilities of the spreadsheet packages.
4. Computer software developers have produced simulation packages that make it easier to develop and implement simulation models for more complex problems.

We first showed how simulation can be used for risk analysis by analyzing a situation involving the development of a new product: the PortaCom printer. We then showed how simulation can be used to select an inventory replenishment level that would provide both a good profit and a good customer service level. Finally, we developed a simulation model for the Hammondsport Savings Bank ATM waiting line system. This model is an example of a dynamic simulation model in which the state of the system changes or evolves over time.

Our approach was to develop a simulation model that contained both controllable inputs and probabilistic inputs. Procedures were developed for randomly generating values for the probabilistic inputs, and a flowchart was developed to show the sequence of logical and mathematical operations that describe the steps of the simulation process. Simulation results obtained by running the simulation for a suitable number of trials or length of time provided the basis for conclusions drawn about the operation of the real system.

The Management Science in Action, Netherlands Company Improves Warehouse Order-Picking Efficiency, describes how a simulation model determined the warehouse storage location for 18,000 products and the sequence in which products were retrieved by order-picking personnel.

MANAGEMENT SCIENCE IN ACTION

NETHERLANDS COMPANY IMPROVES WAREHOUSE ORDER-PICKING EFFICIENCY*

As a wholesaler of tools, hardware, and garden equipment, Ankor, based in The Netherlands, warehouses more than 18,000 different products for customers who are primarily retail store chains, do-it-yourself businesses, and garden centers. Warehouse managers store the fastest-moving products on the ends of the aisles on the ground floor, the medium-moving products in the middle section of the aisles on the ground floor, and the slow-moving products on the mezzanine.

When a new order is received, a warehouse order-picker travels to each product location and selects the requested number of units. An average order includes 25 different products, which requires the order-picker to travel to 25 different locations in the warehouse. In order to minimize damage to the products, heavier products are picked first and breakable products are picked last. Order-picking is typically one of the most time-consuming and expensive aspects of operating the warehouse. The company is under continuous pressure to improve the efficiency of this operation.

To increase efficiency, researchers developed a simulation model of the warehouse order-picking system. Using a sequence of 1098 orders received for 27,790 products over a seven-week period, the researchers used the model to simulate the required order-picking times. The researchers, with the help of the model, varied the assignment of products to storage locations and the sequence in which products were retrieved from the storage locations. The model simulated order-picking times for a variety of product storage location alternatives and four different routing policies that determined the sequence in which products were picked.

Analysis of the simulation results provided a new storage assignment policy for the warehouse as well as new routing rules for the sequence in which to retrieve products from storage. Implementation of the new storage and routing procedures reduced the average route length of the order-picking operation by 31%. Due to the increased efficiency of the operation, the number of order pickers was reduced by more than 25%, saving the company an estimated €140,00 per year.

*Based on R. Dekker, M. B. M. de Koster, K. J. Roodbergen, and H. van Kalleveen, "Improving Order-Picking Response Time at Ankor's Warehouse," *Interfaces* (July/August 2004): 303–313.

GLOSSARY

Simulation A method for learning about a real system by experimenting with a model that represents the system.

Simulation experiment The generation of a sample of values for the probabilistic inputs of a simulation model and computing the resulting values of the model outputs.

Controllable input Input to a simulation model that is selected by the decision maker.

Probabilistic input Input to a simulation model that is subject to uncertainty. A probabilistic input is described by a probability distribution.

Risk analysis The process of predicting the outcome of a decision in the face of uncertainty.

Parameters Numerical values that appear in the mathematical relationships of a model. Parameters are considered known and remain constant over all trials of a simulation.

What-if analysis A trial-and-error approach to learning about the range of possible outputs for a model. Trial values are chosen for the model inputs (these are the what-ifs) and the value of the output(s) is computed.

Base-case scenario Determining the output given the most likely values for the probabilistic inputs of a model.

Worst-case scenario Determining the output given the worst values that can be expected for the probabilistic inputs of a model.

Best-case scenario Determining the output given the best values that can be expected for the probabilistic inputs of a model.

Static simulation model A simulation model used in situations where the state of the system at one point in time does not affect the state of the system at future points in time. Each trial of the simulation is independent.

Dynamic simulation model A simulation model used in situations where the state of the system affects how the system changes or evolves over time.

Event An instantaneous occurrence that changes the state of the system in a simulation model.

Discrete-event simulation model A simulation model that describes how a system evolves over time by using events that occur at discrete points in time.

Verification The process of determining that a computer program implements a simulation model as it is intended.

Validation The process of determining that a simulation model provides an accurate representation of a real system.

Managerial Report

Prepare a report that discusses the general development of the spreadsheet simulation model, and make any recommendations that you have regarding the best store design and staffing plan for County Beverage. One additional consideration is that the design allowing for a two-channel system will cost an additional $10,000 to build.

1. List the information the spreadsheet simulation model should generate so that a decision can be made on the store design and the desired number of clerks.
2. Run the simulation for 1000 customers for each alternative considered. You may want to consider making more than one run with each alternative. [*Note:* Values from an exponential probability distribution with mean μ can be generated in Excel using the following function: $= -\mu$*LN(RAND()).]
3. Be sure to note the number of customers County Beverage is likely to lose due to long customer waiting times with each design alternative.

Appendix 12.1 SIMULATION WITH EXCEL

Excel enables small and moderate-sized simulation models to be implemented relatively easily and quickly. In this appendix we show the Excel worksheets for the three simulation models presented in the chapter.

The PortaCom Simulation Model

Tutorial 7: Simulation Using Basic Excel

We simulated the PortaCom problem 500 times. The worksheet used to carry out the simulation is shown again in Figure 12.19. Note that the simulation results for trials 6 through 495 have been hidden so that the results can be shown in a reasonably sized figure. If desired, the rows for these trials can be shown and the simulation results displayed for all 500 trials. Let us describe the details of the Excel worksheet that provided the PortaCom simulation.

First, the PortaCom data are presented in the first 14 rows of the worksheet. The selling price per unit, administrative cost, and advertising cost parameters are entered directly into cells C3, C4, and C5. The discrete probability distribution for the direct labor cost per unit is shown in a tabular format. Note that the random number intervals are entered first followed by the corresponding cost per unit. For example, 0.0 in cell A10 and 0.1 in cell B10 show that a cost of $43 per unit will be assigned if the random number is in the interval 0.0 but less than 0.1. Thus, approximately 10% of the simulated direct labor costs will be $43 per unit. The uniform probability distribution with a smallest value of $80 in cell E8 and a largest value of $100 in cell E9 describes the parts cost per unit. Finally, a normal probability distribution with a mean of 15,000 units in cell E13 and a standard deviation of 4500 units in cell E14 describes the first-year demand distribution for the product. At this point we are ready to insert the Excel formulas that will carry out each simulation trial.

Simulation information for the first trial appears in row 21 of the worksheet. The cell formulas for row 21 are as follows:

Cell A21 Enter 1 for the first simulation trial

Cell B21 Simulate the direct labor cost per unit*
=VLOOKUP(RAND(),$A$10:$C$14,3)

Cell C21 Simulate the parts cost per unit (uniform distribution)
=$E$8+($E$9−$E$8)*RAND()

*The VLOOKUP function generates a random number using the RAND() function. Then, using the table defined by the region from cells $A$10 to $C$14, the function identifies the row containing the RAND() random number and assigns the corresponding direct labor cost per unit shown in column C.

FIGURE 12.19 WORKSHEET FOR THE PORTACOM PROBLEM

EXCELfile
PortaCom

	A	B	C	D	E	F
1	**PortaCom Risk Analysis**					
2						
3	Selling Price per Unit		$249			
4	Administrative Cost		$400,000			
5	Advertising Cost		$600,000			
6						
7	**Direct Labor Cost**			**Parts Cost (Uniform Distribution)**		
8	Lower	Upper		Smallest Value	$80	
9	Random No.	Random No.	Cost per Unit	Largest Value	$100	
10	0.0	0.1	$43			
11	0.1	0.3	$44			
12	0.3	0.7	$45	**Demand (Normal Distribution)**		
13	0.7	0.9	$46	Mean	15000	
14	0.9	1.0	$47	Std Deviation	4500	
15						
16						
17	**Simulation Trials**					
18						
19		Direct Labor	Parts	First-Year		
20	Trial	Cost per Unit	Cost per Unit	Demand	Profit	
21	1	47	$85.36	17,366	$1,025,570	
22	2	44	$91.68	12,900	$461,828	
23	3	45	$93.35	20,686	$1,288,906	
24	4	43	$98.56	10,888	$169,807	
25	5	45	$88.36	14,259	$648,911	
516	496	44	$98.67	8,730	($71,739)	
517	497	45	$94.38	19,257	$1,110,952	
518	498	44	$90.85	14,920	$703,118	
519	499	43	$90.37	13,471	$557,652	
520	500	46	$92.50	18,614	$1,056,847	
521						
522			**Summary Statistics**			
523			Mean Profit		$698,457	
524			Standard Deviation		$520,485	
525			Minimum Profit		($785,234)	
526			Maximum Profit		$2,367,058	
527			Number of Losses		51	
528			Probability of Loss		0.1020	

Cell D21 Simulate the first-year demand (normal distribution)
=NORMINV(RAND(),$E$13,$E$14)

Cell E21 The profit obtained for the first trial
=($C$3−B21−C21)*D21−$C$4−$C$5

Cells A21:E21 can be copied to A520:E520 in order to provide the 500 simulation trials.

Ultimately, summary statistics will be collected in order to describe the results of the 500 simulated trials. Using the standard Excel functions, the following summary statistics are computed for the 500 simulated profits appearing in cells E21 to E520.

Cell E523 The mean profit per trial = AVERAGE(E21:E520)

Cell E524 The standard deviation of profit = STDEV(E21:E520)

Cell E525 The minimum profit = MIN(E21:E520)

Cell E526 The maximum profit = MAX(E21:E520)

Cell E527 The count of the number of trials where a loss occurred (i.e., profit < $0) = COUNTIF(E21:E520,"<0")

Cell E528 The percentage or probability of a loss based on the 500 trials = E527/500

The F9 key can be used to perform another complete simulation of PortaCom. In this case, the entire worksheet will be recalculated and a set of new simulation results will be provided. Any data summaries, measures, or functions that have been built into the worksheet earlier will be updated automatically.

The Butler Inventory Simulation Model

We simulated the Butler inventory operation for 300 months. The worksheet used to carry out the simulation is shown again in Figure 12.20. Note that the simulation results for months 6 through 295 have been hidden so that the results can be shown in a reasonably sized figure. If desired, the rows for these months can be shown and the simulation results displayed for all 300 months. Let us describe the details of the Excel worksheet that provided the Butler inventory simulation.

First, the Butler inventory data are presented in the first 11 rows of the worksheet. The gross profit per unit, holding cost per unit, and shortage cost per unit data are entered directly into cells C3, C4, and C5. The replenishment level is entered into cell C7, and the mean and standard deviation of the normal probability distribution for demand are entered into cells B10 and B11. At this point we are ready to insert Excel formulas that will carry out each simulation month or trial.

Simulation information for the first month or trial appears in row 17 of the worksheet. The cell formulas for row 17 are as follows:

Cell A17 Enter 1 for the first simulation month

Cell B17 Simulate demand (normal distribution) =NORMINV(RAND(),$B$10,$B$11)

Next compute the sales, which is equal to demand (cell B17) if demand is less than or equal to the replenishment level, or is equal to the replenishment level (cell C7) if demand is greater than the replenishment level.

Cell C17 Compute sales =IF(B17<=$C$7,B17,$C$7)

Cell D17 Calculate gross profit =$C$3*C17

Cell E17 Calculate the holding cost if demand is less than or equal to the replenishment level =IF(B17<= $C$7,$C$4*($C$7−B17),0)

Cell F17 Calculate the shortage cost if demand is greater than the replenishment level =IF(B17>$C$7,$C$5*(B17−$C$7),0)

Cell G17 Calculate net profit =D17−E17−F17

Cells A17:G17 can be copied to cells A316:G316 in order to provide the 300 simulation months.

FIGURE 12.20 WORKSHEET FOR THE BUTLER INVENTORY PROBLEM

EXCELfile
Butler

	A	B	C	D	E	F	G	H
1	**Butler Inventory**							
2								
3	Gross Profit per Unit		$50					
4	Holding Cost per Unit		$15					
5	Shortage Cost per Unit		$30					
6								
7	**Replenishment Level**		100					
8								
9	**Demand (Normal Distribution)**							
10	Mean	100						
11	Std Deviation	20						
12								
13								
14	**Simulation**							
15								
16	Month	Demand	Sales	Gross Profit	Holding Cost	Shortage Cost	Net Profit	
17	1	79	79	$3,950	$315	$0	$3,635	
18	2	111	100	$5,000	$0	$330	$4,670	
19	3	93	93	$4,650	$105	$0	$4,545	
20	4	100	100	$5,000	$0	$0	$5,000	
21	5	118	100	$5,000	$0	$540	$4,460	
312	296	89	89	$4,450	$165	$0	$4,285	
313	297	91	91	$4,550	$135	$0	$4,415	
314	298	122	100	$5,000	$0	$660	$4,340	
315	299	93	93	$4,650	$105	$0	$4,545	
316	300	126	100	$5,000	$0	$780	$4,220	
317								
318	**Totals**	30,181	27,917		**Summary Statistics**			
319					Mean Profit		$4,293	
320					Standard Deviation		$658	
321					Minimum Profit		($206)	
322					Maximum Profit		$5,000	
323					Service Level		92.5%	

Finally, summary statistics will be collected in order to describe the results of the 300 simulated trials. Using the standard Excel functions, the following totals and summary statistics are computed for the 300 months.

Cell B318 Total demand =SUM(B17:B316)
Cell C319 Total sales =SUM(C17:C316)
Cell G319 The mean profit per month =AVERAGE(G17:G316)
Cell G320 The standard deviation of net profit =STDEV(G17:G316)
Cell G321 The minimum net profit =MIN(G17:G316)
Cell G322 The maximum net profit =MAX(G17:G316)
Cell G323 The service level =C318/B318

The Hammondsport ATM Simulation Model

We simulated the operation of the Hammondsport ATM waiting line system for 1000 customers. The worksheet used to carry out the simulation is shown again in Figure 12.21. Note that the simulation results for customers 6 through 995 have been hidden so that the results can be shown in a reasonably sized figure. If desired, the rows for these customers can be shown and the simulation results displayed for all 1000 customers. Let us describe the details of the Excel worksheet that provided the Hammondsport ATM simulation.

The data are presented in the first 9 rows of the worksheet. The interarrival times are described by a uniform distribution with a smallest time of 0 minutes (cell B4) and a largest time of 5 minutes (cell B5). A normal probability distribution with a mean of 2 minutes (cell B8) and a standard deviation of 0.5 minute (cell B9) describes the service time distribution.

FIGURE 12.21 WORKSHEET FOR THE HAMMONDSPORT SAVINGS BANK WITH ONE ATM

EXCELfile
Hammondsport1

	A	B	C	D	E	F	G	H	I
1	**Hammondsport Savings Bank with One ATM**								
2									
3	**Interarrival Times (Uniform Distribution)**								
4	Smallest Value	0							
5	Largest Value	5							
6									
7	**Service Times (Normal Distribution)**								
8	Mean	2							
9	Std Deviation	0.5							
10									
11									
12	**Simulation**								
13									
14		Interarrival	Arrival	Service	Waiting	Service	Completion	Time	
15	Customer	Time	Time	Start Time	Time	Time	Time	in System	
16	1	1.4	1.4	1.4	0.0	2.3	3.7	2.3	
17	2	1.3	2.7	3.7	1.0	1.5	5.2	2.5	
18	3	4.9	7.6	7.6	0.0	2.2	9.8	2.2	
19	4	3.5	11.1	11.1	0.0	2.5	13.6	2.5	
20	5	0.7	11.8	13.6	1.8	1.8	15.4	3.6	
1011	996	0.5	2496.8	2498.1	1.3	0.6	2498.7	1.9	
1012	997	0.2	2497.0	2498.7	1.7	2.0	2500.7	3.7	
1013	998	2.7	2499.7	2500.7	1.0	1.8	2502.5	2.8	
1014	999	3.7	2503.4	2503.4	0.0	2.4	2505.8	2.4	
1015	1000	4.0	2507.4	2507.4	0.0	1.9	2509.3	1.9	
1016									
1017		**Summary Statistics**							
1018		Number Waiting			549				
1019		Probability of Waiting			0.6100				
1020		Average Waiting Time			1.59				
1021		Maximum Waiting Time			13.5				
1022		Utilization of ATM			0.7860				
1023		Number Waiting > 1 Min			393				
1024		Probability of Waiting > 1 Min			0.4367				

Simulation information for the first customer appears in row 16 of the worksheet. The cell formulas for row 16 are as follows:

Cell A16	Enter 1 for the first customer
Cell B16	Simulate the interarrival time for customer 1 (uniform distribution) =$B$4+RAND()*($B$5−$B$4)
Cell C16	Compute the arrival time for customer 1 =B16
Cell D16	Compute the start time for customer 1 =C16
Cell E16	Compute the waiting time for customer 1 =D1−C16
Cell F16	Simulate the service time for customer 1 (normal distribution) =NORMINV(RAND(),$B$8,$B$9)
Cell G16	Compute the completion time for customer 1 =D16+F16
Cell H16	Compute the time in the system for customer 1 =G16−C16

Simulation information for the second customer appears in row 17 of the worksheet. The cell formulas for row 17 are as follows:

Cell A17	Enter 2 for the second customer
Cell B17	Simulate the interarrival time for customer 2 (uniform distribution) =$B$4+RAND()*($B$5−$B$4)
Cell C17	Compute the arrival time for customer 2 =C16+B17
Cell D17	Compute the start time for customer 2 =IF(C17>G16,C17,G16)
Cell E17	Compute the waiting time for customer 2 =D17−C17
Cell F17	Simulate the service time for customer 2 (normal distribution) =NORMINV(RAND(),$B$8,$B$9)
Cell G17	Compute the completion time for customer 2 =D17+F17
Cell H17	Compute the time in the system for customer 2 =G17−C17

Cells A17:H17 can be copied to cells A1015:H1015 in order to provide the 1000-customer simulation.

Ultimately, summary statistics will be collected in order to describe the results of 1000 customers. Before collecting the summary statistics, let us point out that most simulation studies of dynamic systems focus on the operation of the system during its long-run or steady-state operation. To ensure that the effects of start-up conditions are not included in the steady-state calculations, a dynamic simulation model is usually run for a specified period without collecting any data about the operation of the system. The length of the start-up period can vary depending on the application. For the Hammondsport Savings Bank ATM simulation, we treated the results for the first 100 customers as the start-up period. The simulation information for customer 100 appears in row 115 of the spreadsheet. Cell G115 shows that the completion time for the 100th customer is 247.8. Thus the length of the start-up period is 247.8 minutes.

Summary statistics are collected for the next 900 customers corresponding to rows 116 to 1015 of the worksheet. The following Excel formulas provided the summary statistics.

Cell E1018	Number of customers who had to wait (i.e., waiting time > 0) =COUNTIF(E116:E1015,">0")
Cell E1019	Probability of waiting =E1018/900
Cell E1020	The average waiting time =AVERAGE(E116:E1015)

Cell E1021 The maximum waiting time =MAX(E116:E1015)

Cell E1022 The utilization of the ATM* =SUM(F116:F1015)/(G1015 − G115)

Cell E1023 The number of customers who had to wait more than 1 minute =COUNTIF(E116:E1015, " >1")

Cell E1024 Probability of waiting more than 1 minute =E1023/900

Appendix 12.2 SIMULATION USING CRYSTAL BALL

In Section 12.1 we used simulation to perform risk analysis for the PortaCom problem, and in Appendix 12.1 we showed how to construct the Excel worksheet that provided the simulation results. Developing the worksheet simulation for the PortaCom problem using the basic Excel package was relatively easy. The use of add-ins enables larger and more complex simulation problems to be easily analyzed using spreadsheets. In this appendix, we show how Crystal Ball, an add-in package, can be used to perform the PortaCom simulation. We will run the simulation for 1000 trials here. Instructions for installing and starting Crystal Ball are included with the Crystal Ball software.

Formulating a Crystal Ball Model

We begin by entering the problem data into the top portion of the worksheet. For the PortaCom problem, we must enter the following data: selling price, administrative cost, advertising cost, probability distribution for the direct labor cost per unit, smallest and largest values for the parts cost per unit (uniform distribution), and the mean and standard deviation for first-year demand (normal distribution). These data with appropriate descriptive labels are shown in cells A1:E13 of Figure 12.22.

For the PortaCom problem, the Crystal Ball model contains the following two components: (1) cells for the probabilistic inputs (direct labor cost, parts cost, first-year demand), and (2) a cell containing a formula for computing the value of the simulation model output (profit). In Crystal Ball the cells that contain the values of the probabilistic inputs are called *assumption cells*, and the cells that contain the formulas for the model outputs are referred to as *forecast cells*. The PortaCom problem requires only one output (profit), and thus the Crystal Ball model only contains one forecast cell. In more complex simulation problems more than one forecast cell may be necessary.

The assumption cells may only contain simple numeric values. In this model-building stage, we entered PortaCom's best estimates of the direct labor cost ($45), the parts cost ($90), and the first-year demand (15,000) into cells C21:C23, respectively. The forecast cells in a Crystal Ball model contain formulas that refer to one or more of the assumption cells. Because only one forecast cell in the PortaCom problem corresponds to profit, we entered the following formula into cell C27:

=(C3−C21−C22)*C23−C4−C5

The resulting value of $710,000 is the profit corresponding to the base-case scenario discussed in Section 12.1.

*The proportion of time the ATM is in use is equal to the sum of the 900 customer service times in column F divided by the total elapsed time required for the 900 customers to complete service. This total elapsed time is the difference between the completion time of customer 1000 and the completion time of customer 100.