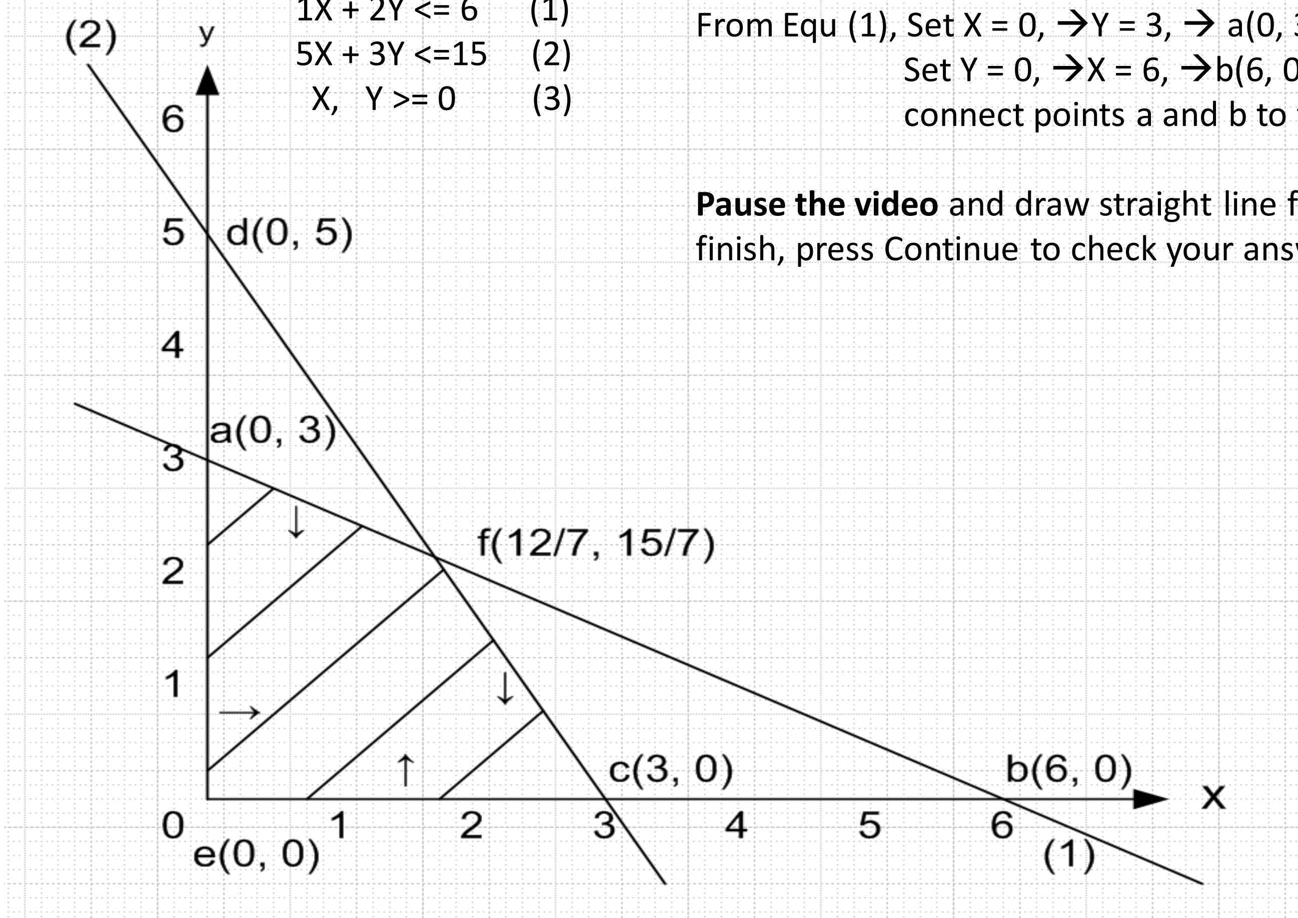
Algebra Reviews & LP Graphic Solutions

Given Constraints to Draw Straight Lines and Identify Feasible Region

Constraints:

1X + 2Y <= 6 (1)
5X + 3Y <=15 (2)
X, Y >= 0 (3)



Draw Straight Lines for Each Constraint:

From Equ (1), Set X = 0, →Y = 3, → a(0, 3);
Set Y = 0, →X = 6, →b(6, 0),
connect points a and b to form Line (1)

Pause the video and draw straight line for Constraint (2), after you finish, press Continue to check your answer:

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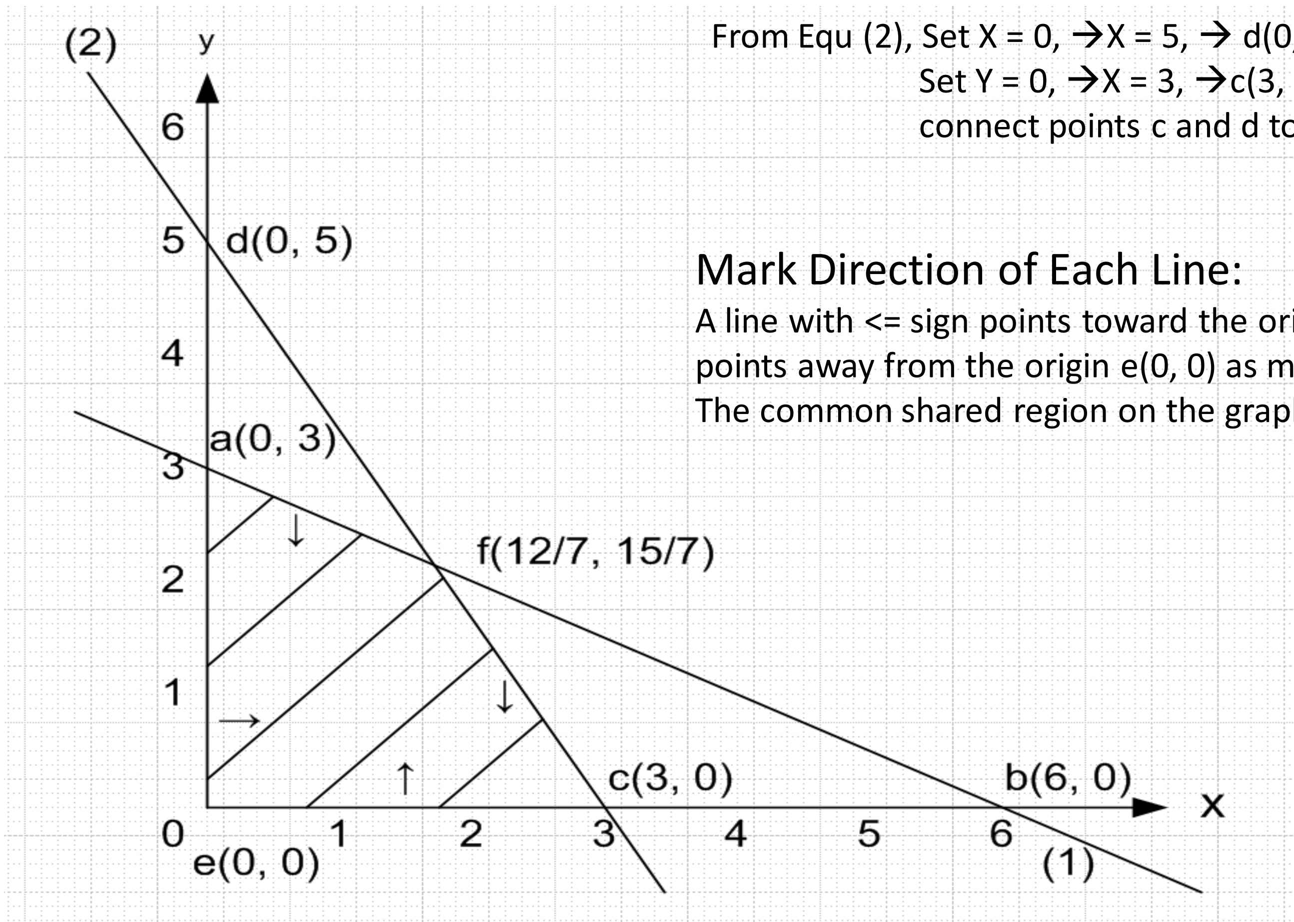
connect points a and b to form Line (1)

From Equ (2), Set X = 0, →X = 5, → d(0, 5);
Set Y = 0, →X = 3, →c(3, 0),
connect points c and d to form Line (2)



Mark Direction of Each Line:

A line with <= sign points toward the origin e(0, 0), and a line with >= sign points away from the origin e(0, 0) as marked on the graph.
The common shared region on the graph is called the feasible region

Given Two Points (X_1, Y_1) and (X_2, Y_2) on a Line to Find Equation Y = aX + b

Constraints:

1X + 2Y <= 6 (1)

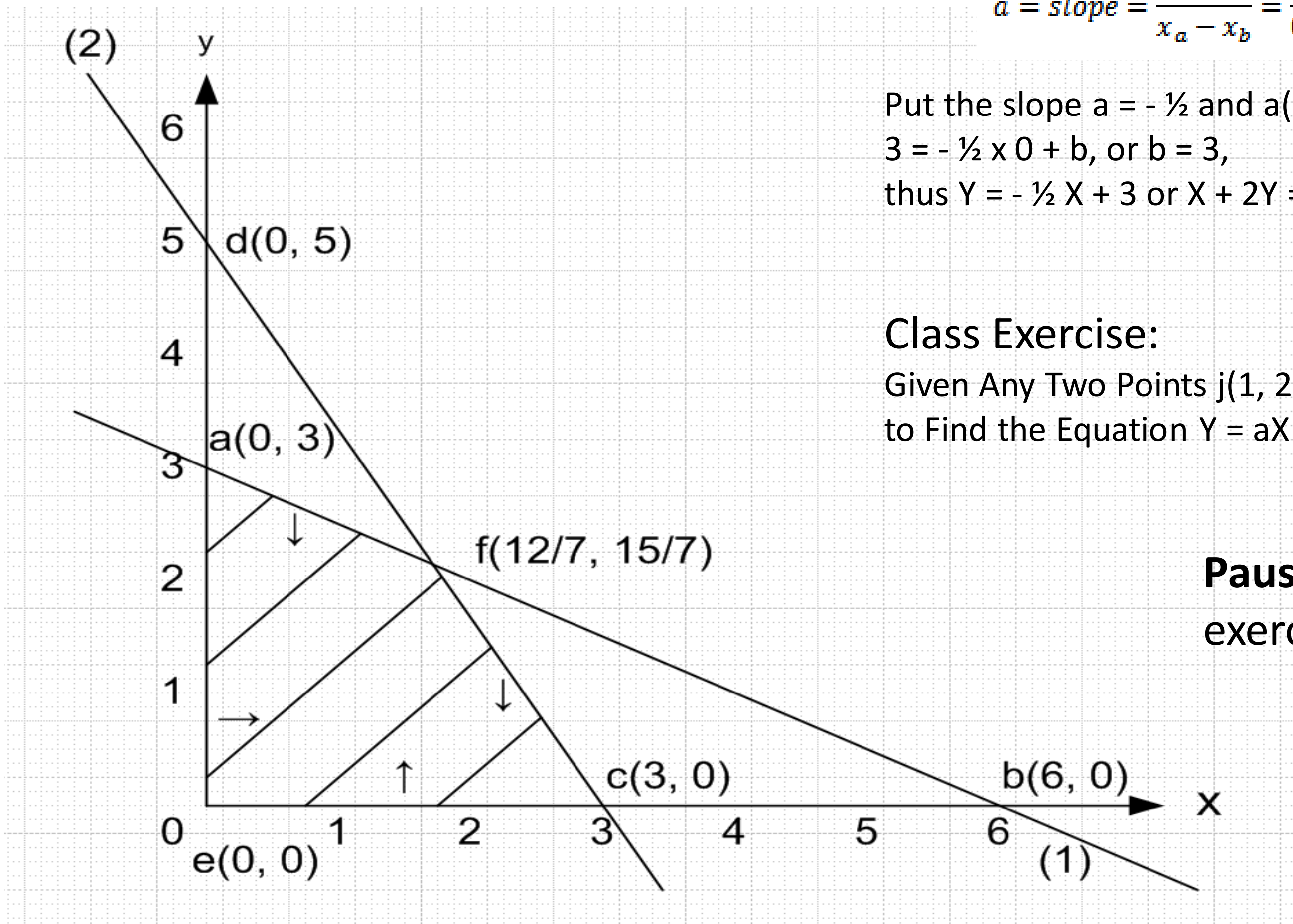
5X + 3Y <=15 (2)

X, Y >= 0 (3)

Given Any Two Points a(0, 3) and b(6, 0) on a Line to Find the Equation Y = aX + b that Represents the Line:

$$a = slope = \frac{y_a - y_b}{x_a - x_b} = \frac{3-0}{0-6} = -1/2$$

Put the slope a = - ½ and a(0, 3) into Y = aX + b, →
3 = - ½ x 0 + b, or b = 3,
thus Y = - ½ X + 3 or X + 2Y = 6



Class Exercise:

Given Any Two Points j(1, 2.5) and k(2, 2) on a Line
to Find the Equation Y = aX + b that Represents the Line:

Pause the video, finish the exercise, then Continue:

Given Two Points (X_1, Y_1) and (X_2, Y_2) on a Line to Find Equation Y = aX + b

Constraints:

1X + 2Y <= 6 (1)
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X, Y >= 0 (3)

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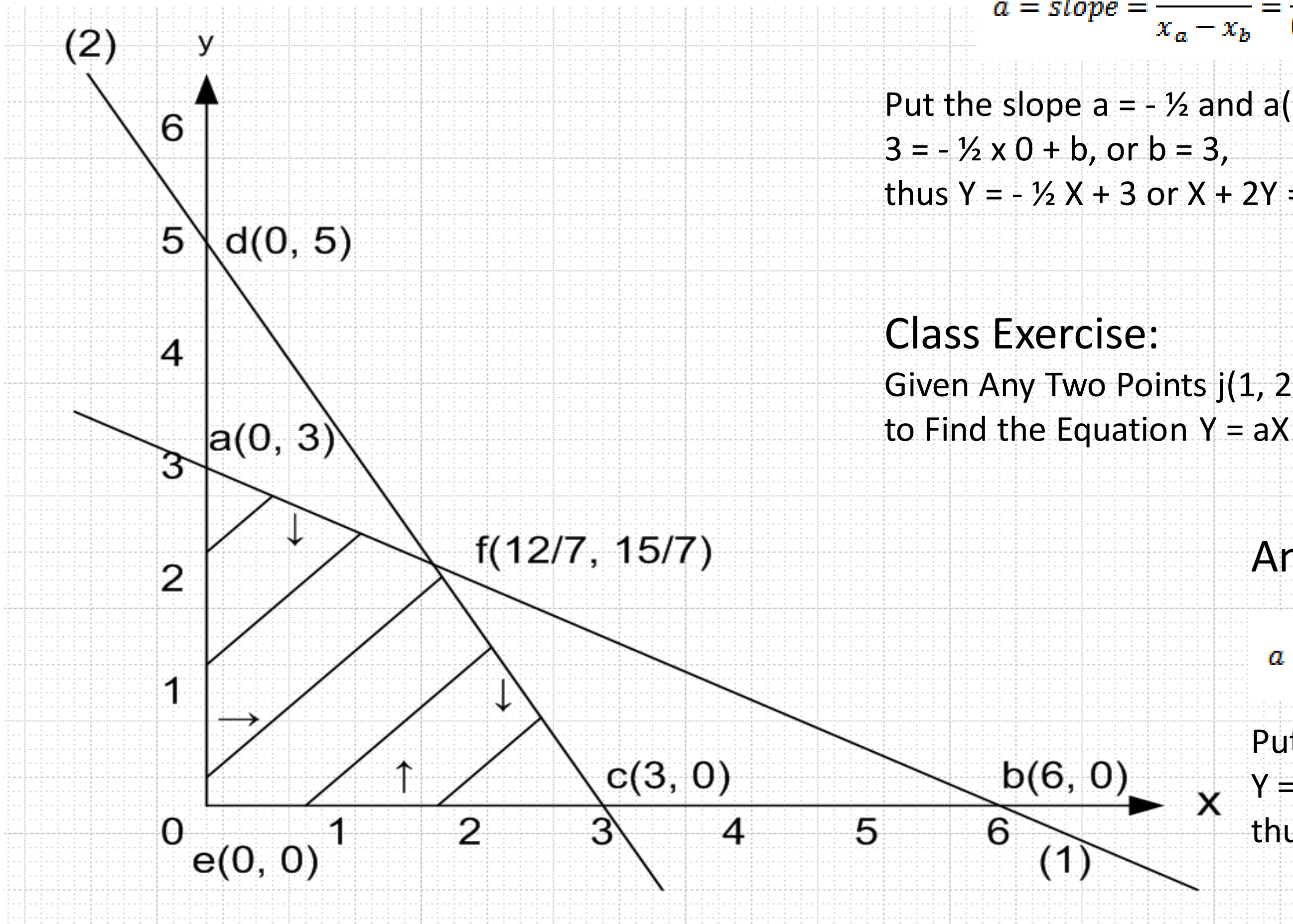


Class Exercise:

Given Any Two Points j(1, 2.5) and k(2, 2) on a Line
to Find the Equation Y = aX + b that Represents the Line:

Answer for Class Exercise:

$$a = slope = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2.5-2}{1-2} = -1/2$$

Put the slope a = - ½ and a(0, 3) into
Y = aX + b, → 3 = - ½ x 0 + b, or b = 3,
thus Y = - ½ X + 3 or X + 2Y = 6

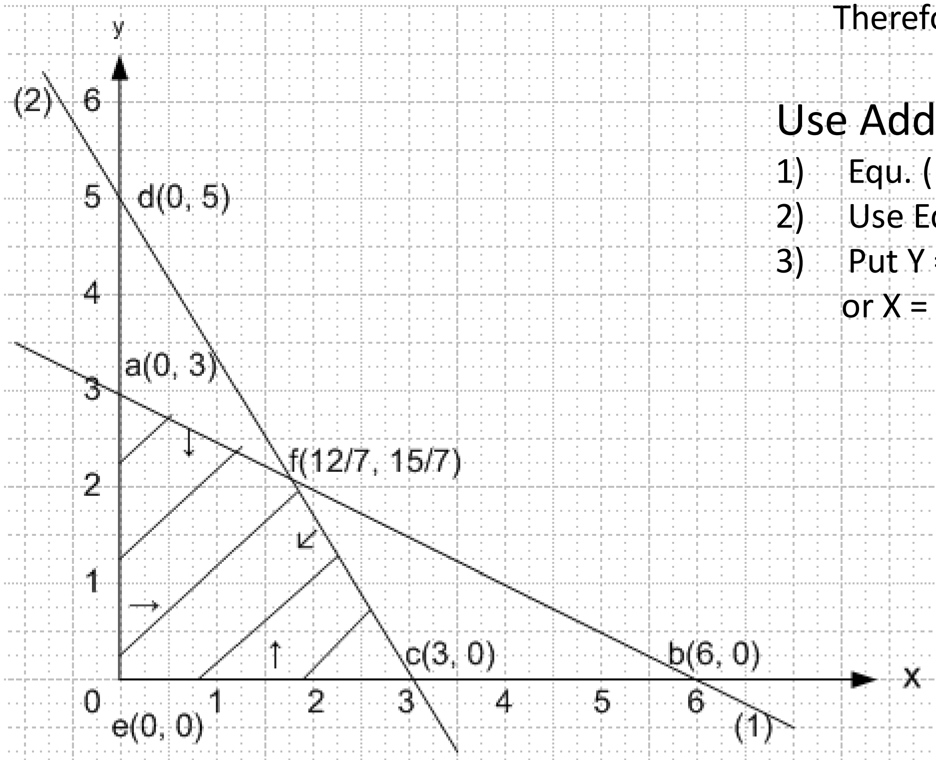
Find Values of Joint(X, Y) of Any Two Lines

Constraints:

1X + 2Y <= 6 (1)

5X + 3Y <=15 (2)

X, Y >= 0 (3)



Use Back-Substitution to find values of f(X, Y)

1) From Equ. (1), X = 6 – 2Y (4)
2) Use Equ. (4) to substitute X in Equ. (2),
   5(6 – 2Y) + 3Y = 30 – 10Y + 3Y = 15, or Y = 15/7 (5)
3) Put Y = 15/7 back in Equ. (4), X = 6 – 2 (15/7) = 12/7
   Therefore, f(X = 12/7, Y = 15/7)

Use Addition – Subtraction to find values of f(X, Y)

1) Equ. (1) times 5, → 5X + 10Y = 30 (4)
2) Use Equ. (4) – Equ. (2), → 0X + 7Y = 15, Y = 15/7
3) Put Y = 15/7 back in Equ. (1), → X + 2 x(15/7) = 6
   or X = 6 – 30/7 = 12/7

Find Values of Joint(X, Y) of Any Two Lines

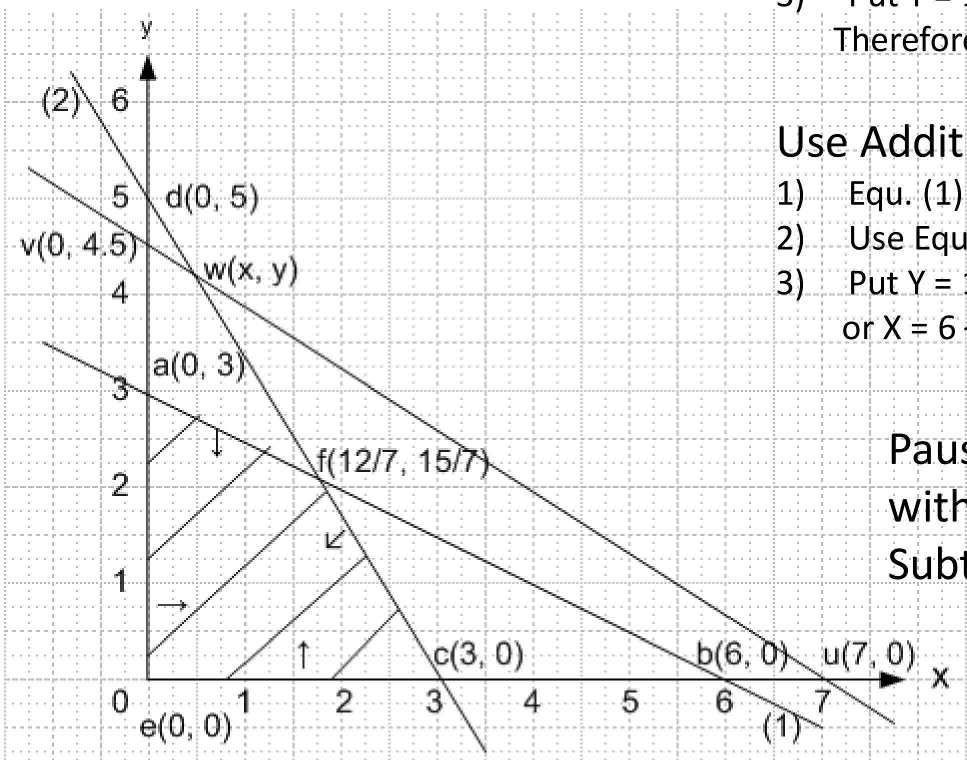
Constraints:

1X + 2Y <= 6 (1)

5X + 3Y <=15 (2)

X, Y >= 0 (3)

9X + 14Y<=63 (4)



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3) Put Y = 15/7 back in Equ. (1), → X + 2 x(15/7) = 6
   or X = 6 – 30/7 = 12/7

Pause the Video, find the values of w(x, y) with both Back-Substitution and Add-Subtract methods before continue:

Find Values of Joint(X, Y) of Any Two Lines

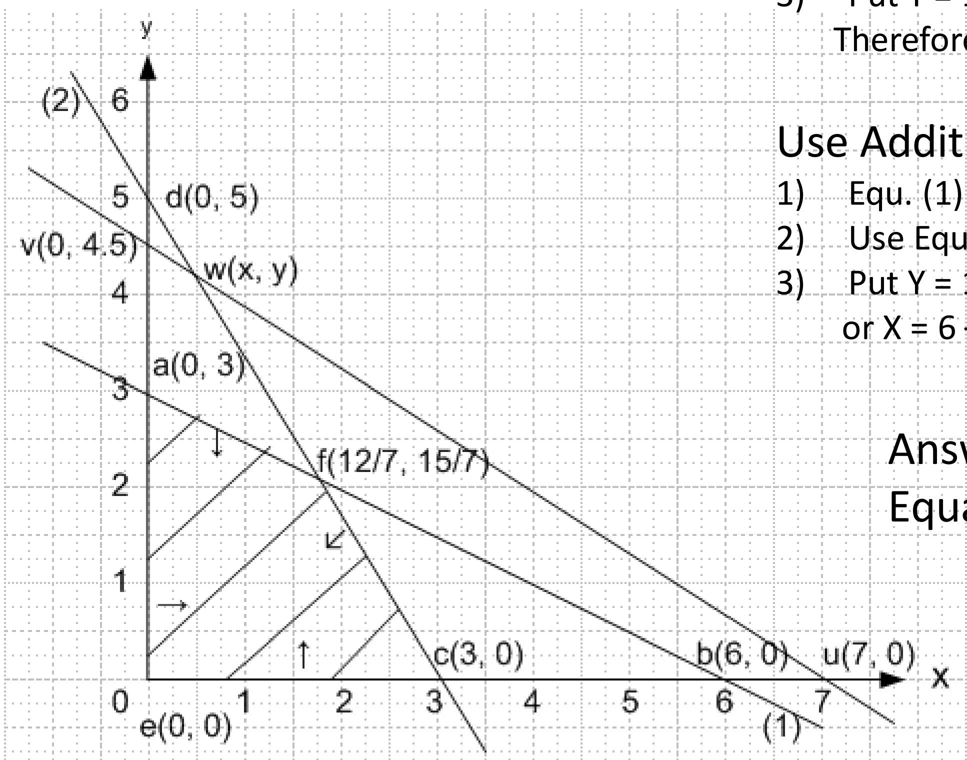
Constraints:

$1X + 2Y <= 6$ (1)

$5X + 3Y <=15$ (2)

$X, \quad Y >= 0$ (3)

$9X + 14Y<=63$ (4)



Use Back-Substitution to find values of f(X, Y)

1) From Equ. (1), $X = 6 – 2Y$ (4)
2) Use Equ. (4) to substitute X in Equ. (2),
   $5(6 – 2Y) + 3Y = 30 – 10Y + 3Y = 15$, or $Y = 15/7$ (5)
3) Put $Y = 15/7$ back in Equ. (4), $X = 6 – 2 (15/7) = 12/7$
   Therefore, $f(X = 12/7, Y = 15/7)$

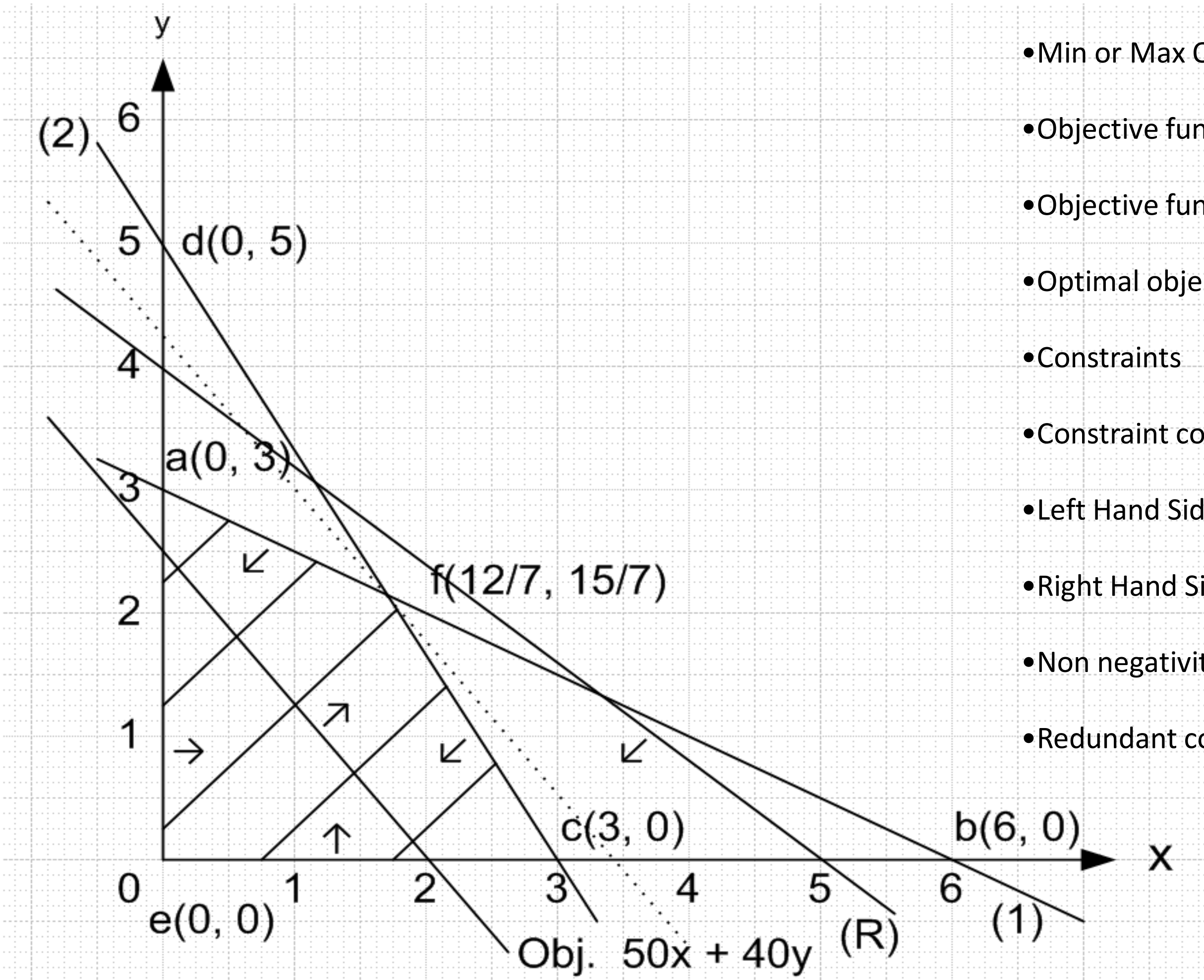
Use Addition – Subtraction to find values of f(X, Y)

1) Equ. (1) times 5, → $5X + 10Y = 30$ (4)
2) Use Equ. (4) – Equ. (2), → $0X + 7Y = 15$, $Y = 15/7$
3) Put $Y = 15/7$ back in Equ. (1), → $X + 2 x(15/7) = 6$
   or $X = 6 – 30/7 = 12/7$

Answer: w(x=0.4884, y = 4.186) from Equations (2) and (4)

LP Graphic Solution

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <=15 (2) Raw materials in units
X, Y >= 0 (3)



Terms or Definitions:

- Decision variables X and Y
- Optimal solution (X, Y)
- Min or Max Objective function
- Objective function coefficients 50 or 40
- Objective function value
- Optimal objective function value
- Constraints
- Constraint coefficients (1, 2, 5, 3)
- Left Hand Side (LHS) values of constraints
- Right Hand Side (RHS) values of constraints
- Non negativity constraints (>= 0)
- Redundant constraint (R)

LP Graphic Solution

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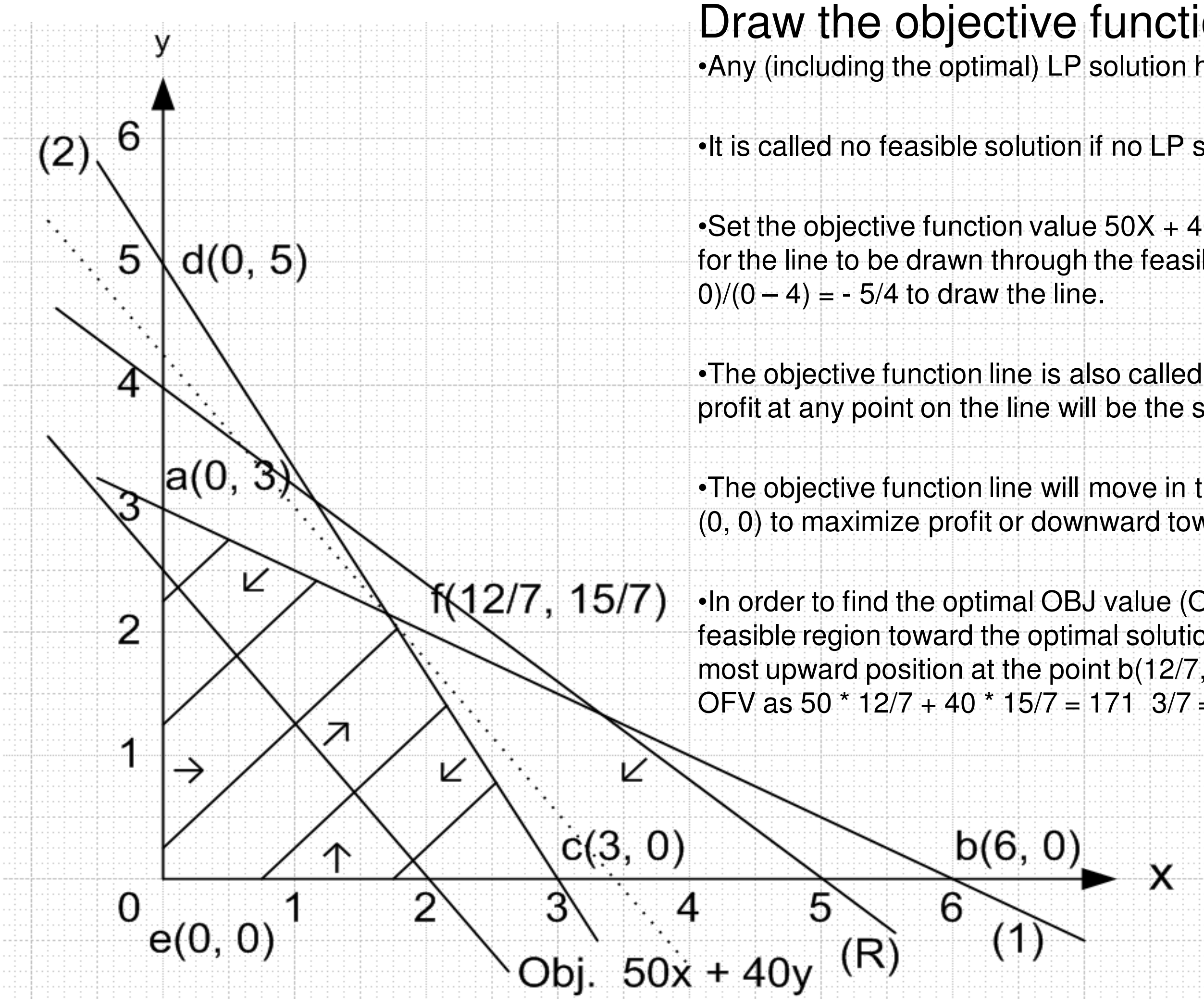


Draw the objective function line on the graph:

- Any (including the optimal) LP solution has to be part of the feasible region.
- It is called no feasible solution if no LP solution is found in the feasible region.
- Set the objective function value 50X + 40Y = 100 to get the points (0, 2.5) and (2, 0) for the line to be drawn through the feasible region. Or use the slope of OBJ = (5 – 0)/(0 – 4) = - 5/4 to draw the line.
- The objective function line is also called Equal Profit line or level curve because the profit at any point on the line will be the same of 100.
- The objective function line will move in the direction of upward away from the origin (0, 0) to maximize profit or downward toward to the origin (0, 0) to minimize costs.
- In order to find the optimal OBJ value (OFV), the OBJ line always moves in the feasible region toward the optimal solution or vertex (X, Y). The OBJ line will reach its most upward position at the point b(12/7, 15/7) with the optimal OBJ function value OFV as 50 * 12/7 + 40 * 15/7 = 171 3/7 = 171.43

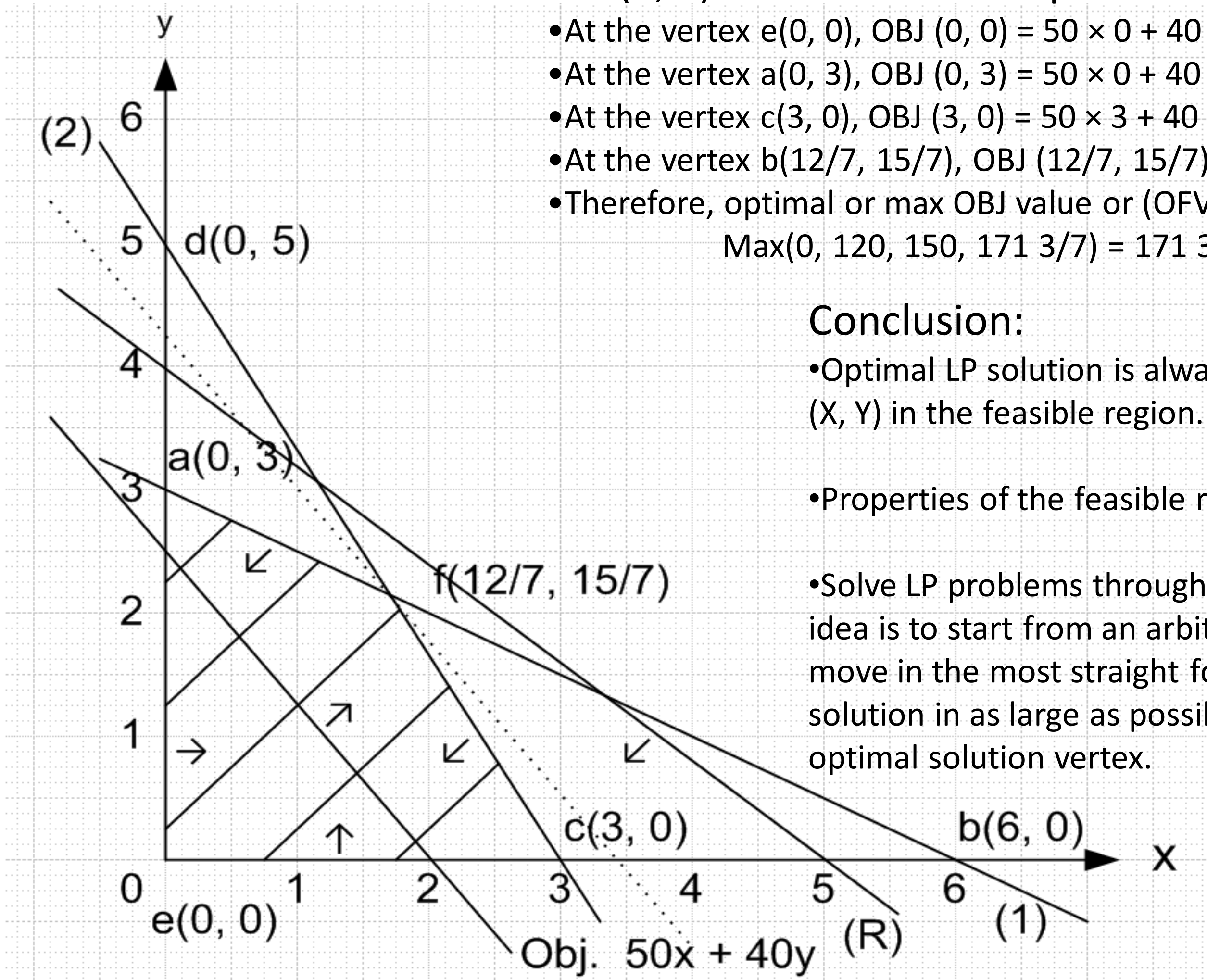
LP Solution Through Enumerating Extreme Points(X, Y)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6   (1) Production time in minutes
 5X + 3Y <=15 (2) Raw materials in units
 X, Y >= 0   (3)

OBJ(X, Y) value at extreme point or vertex (X, Y)

- At the vertex e(0, 0), OBJ (0, 0) = 50 × 0 + 40 × 0 = 0
- At the vertex a(0, 3), OBJ (0, 3) = 50 × 0 + 40 × 3 = 120
- At the vertex c(3, 0), OBJ (3, 0) = 50 × 3 + 40 × 0 = 150
- At the vertex b(12/7, 15/7), OBJ (12/7, 15/7) = 50 × 12/7 + 40 × 15/7 = 171 3/7
- Therefore, optimal or max OBJ value or (OFV) is given by

  Max(0, 120, 150, 171 3/7) = 171 3/7 at vertex f(12/7, 15/7)



Conclusion:

- Optimal LP solution is always found at one of extreme vertexes (X, Y) in the feasible region.
- Properties of the feasible region: It is always a convex hull.
- Solve LP problems through Interior point method. The basic idea is to start from an arbitrary point within the feasible region, move in the most straight forward direction toward the optimal solution in as large as possible steps, and eventually reach the optimal solution vertex.

Minimize Production Cost and Optimal Solution

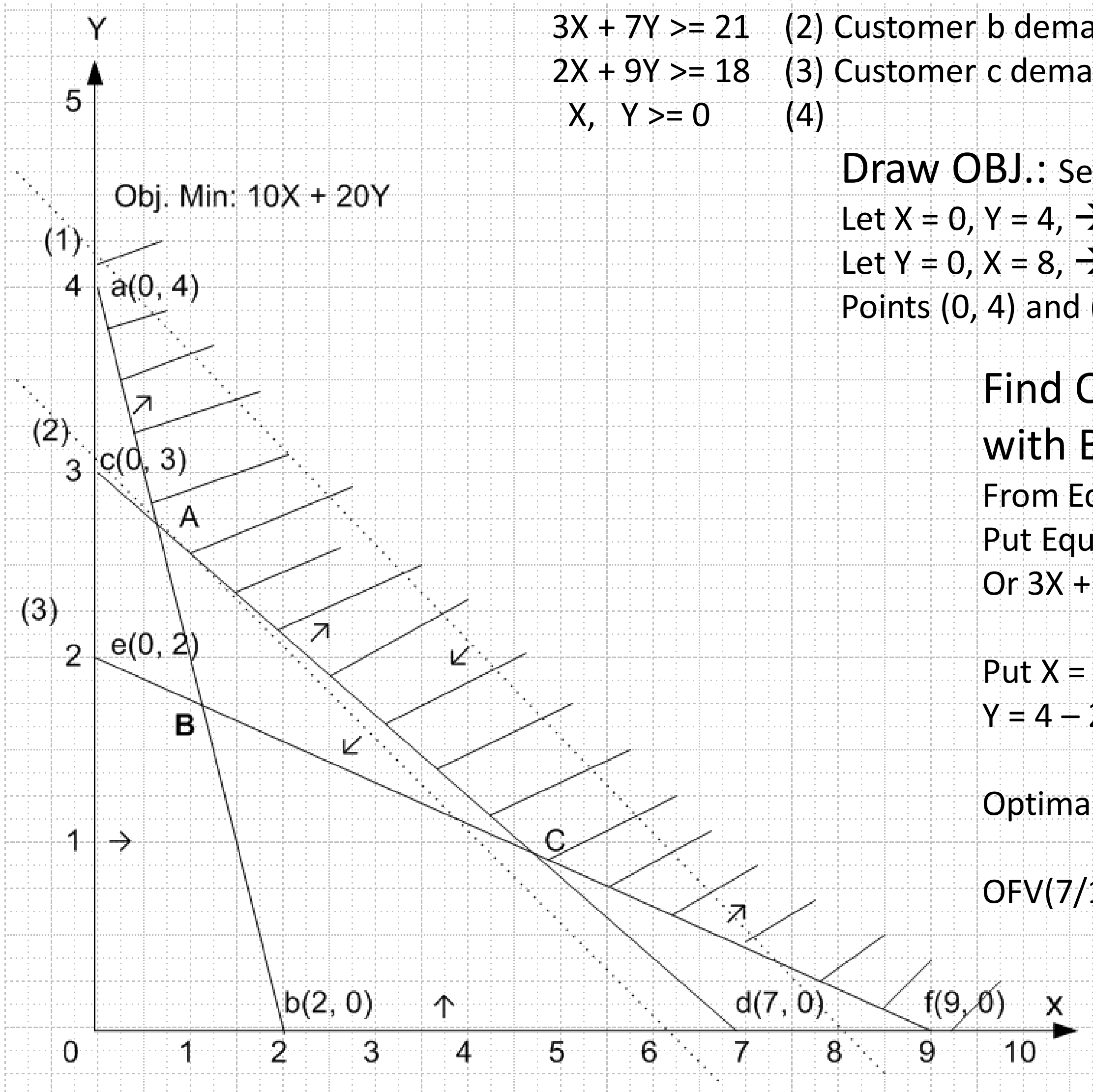
Min: 10X + 20Y

s.t. 2X + Y >= 4 (1) Customer a demand

3X + 7Y >= 21 (2) Customer b demand

2X + 9Y >= 18 (3) Customer c demand

X, Y >= 0 (4)



Draw OBJ.: Set 10X + 20Y = 80

Let X = 0, Y = 4, → (0, 4)

Let Y = 0, X = 8, → (8, 0)

Points (0, 4) and (8, 0) forms the OBJ line

Find Optimal Solution A(X, Y) with Back - Substitution

From Equ (1), Y = 4 – 2X, (5)

Put Equ (5) into Equ (2), 3X + 7 x (4 – 2X) = 21

Or 3X + 28 – 14X = 21, X = 7/11

Put X = 7/11 into Equ (5)

Y = 4 – 2 x (7/11) = 4 – 14/11 = 30/11

Optimal Solution A(7/11, 30/11) with the value of

OFV(7/11, 30/11) = 10x(7/11) + 20x(30/11) = 60.91

What is value of OBJ at C(X, Y)?

Minimize Production Cost and Optimal Solution

Min: $10X + 20Y$

s.t. $2X + Y >= 4$ (1) Customer a demand

$3X + 7Y >= 21$ (2) Customer b demand

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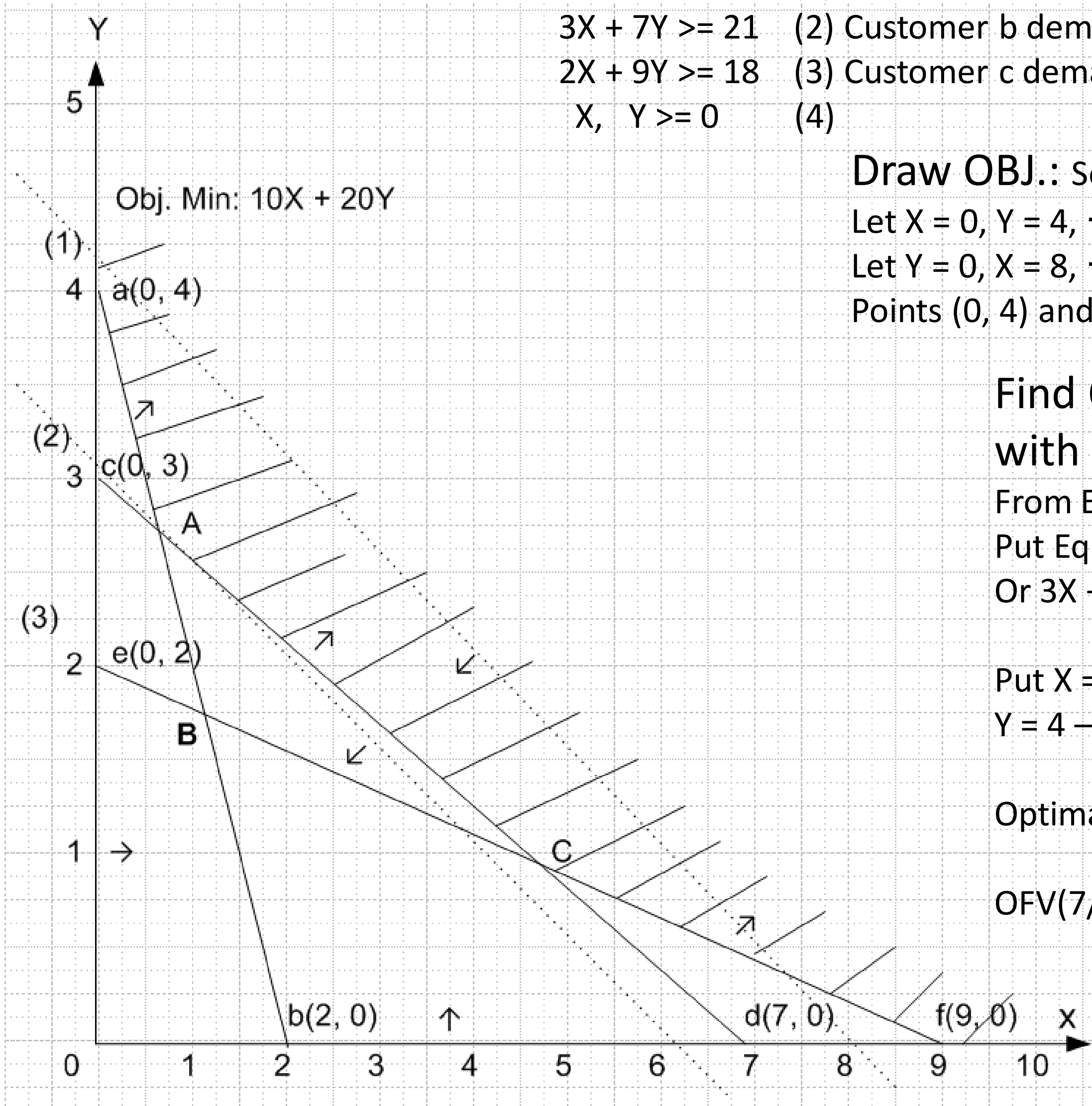
$X, Y >= 0$ (4)



Draw OBJ.: Set $10X + 20Y = 80$

Let $X = 0$, $Y = 4$, → (0, 4)

Let $Y = 0$, $X = 8$, → (8, 0)

Points (0, 4) and (8, 0) forms the OBJ line

Find Optimal Solution A(X, Y) with Back - Substitution

From Equ (1), $Y = 4 – 2X$, (5)

Put Equ (5) into Equ (2), $3X + 7 x (4 – 2X) = 21$

Or $3X + 28 – 14X = 21$, $X = 7/11$

Put $X = 7/11$ into Equ (5)

$Y = 4 – 2 x (7/11) = 4 – 14/11 = 30/11$

Optimal Solution A(7/11, 30/11) with the value of

OFV(7/11, 30/11) = $10x(7/11) + 20x(30/11) = 60.91$

What is value of OBJ at C(X, Y)?

Answer: C(X=63/13, Y=12/13)

OFV (63/13, 12/13) = 30