

$$\text{Min } -10X + 20Y$$

S.t.

Production Cost

$$2X + 1Y \geq 4 \quad (1)$$

$$3X + 7Y \geq 21 \quad (2)$$

$$2X + 9Y \geq 18 \quad (3)$$

$$X, Y \geq 0 \quad (4)$$

$\geq, =, \leq$

Graphic Solution:

1) Draw a straight line for each constraint (equ.) (1) to (4)

Equ (1).  $X=0, Y=4/1=4, \rightarrow a(0, 4); Y=0, X=4/2=2, \rightarrow b(2, 0)$

Equ (2).  $X=0, Y=21/7=3, \rightarrow c(0, 3); Y=0, X=21/3=7, \rightarrow d(7, 0)$

Equ (3).  $X=0, Y=18/9=2, \rightarrow e(0, 2); Y=0, X=18/2=9, \rightarrow f(9, 0)$

2) Mark direction of each line (equ.)

for Equ. (2):

$$3X + 7Y \geq 21 \quad (2-1)$$

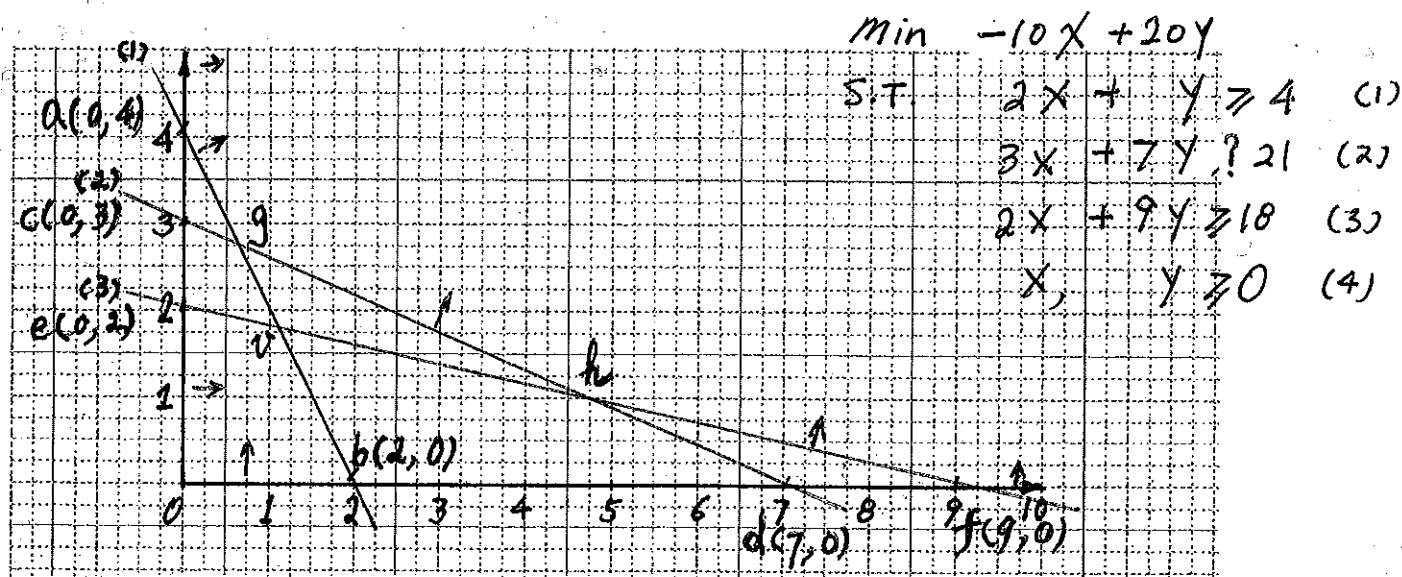
$$3X + 7Y = 21 \quad (2-2)$$

$$3X + 7Y \leq 21 \quad (2-3)$$

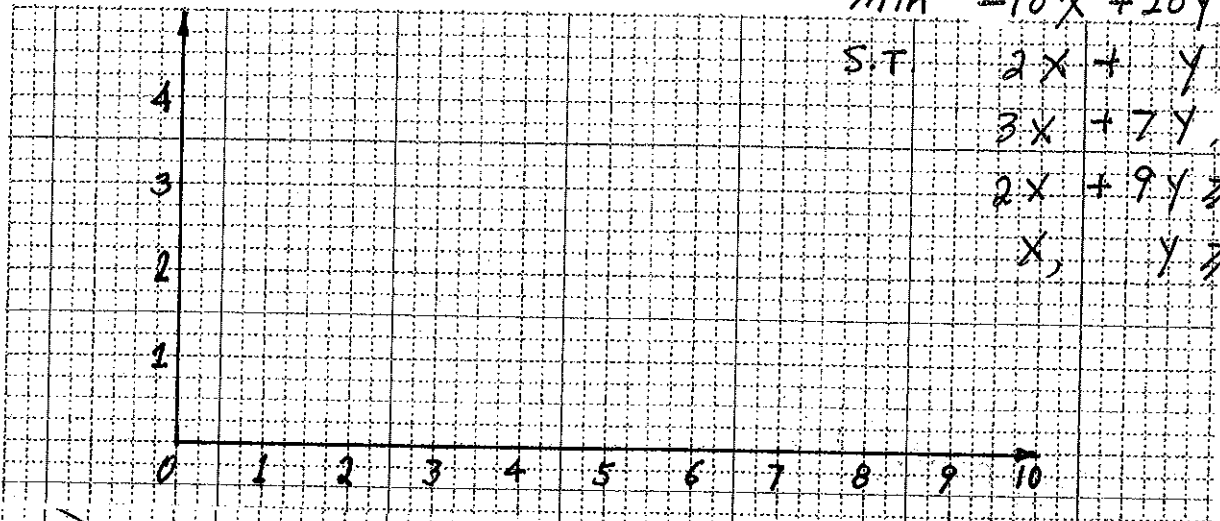
feasible region  
area

line or point

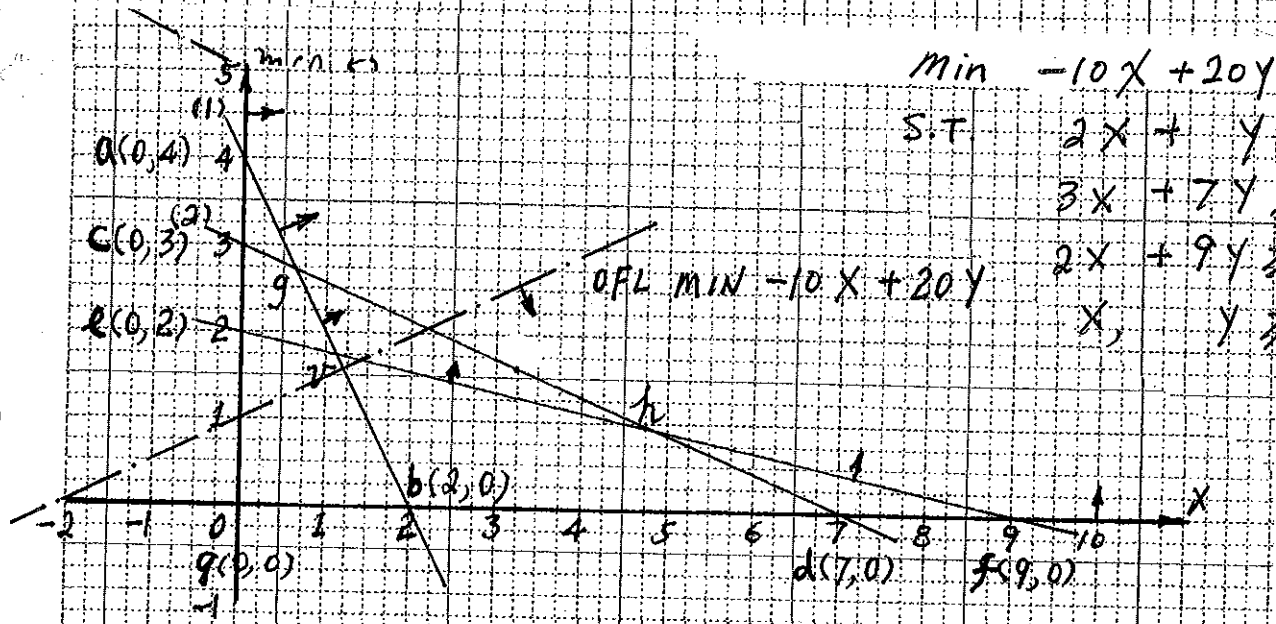
area



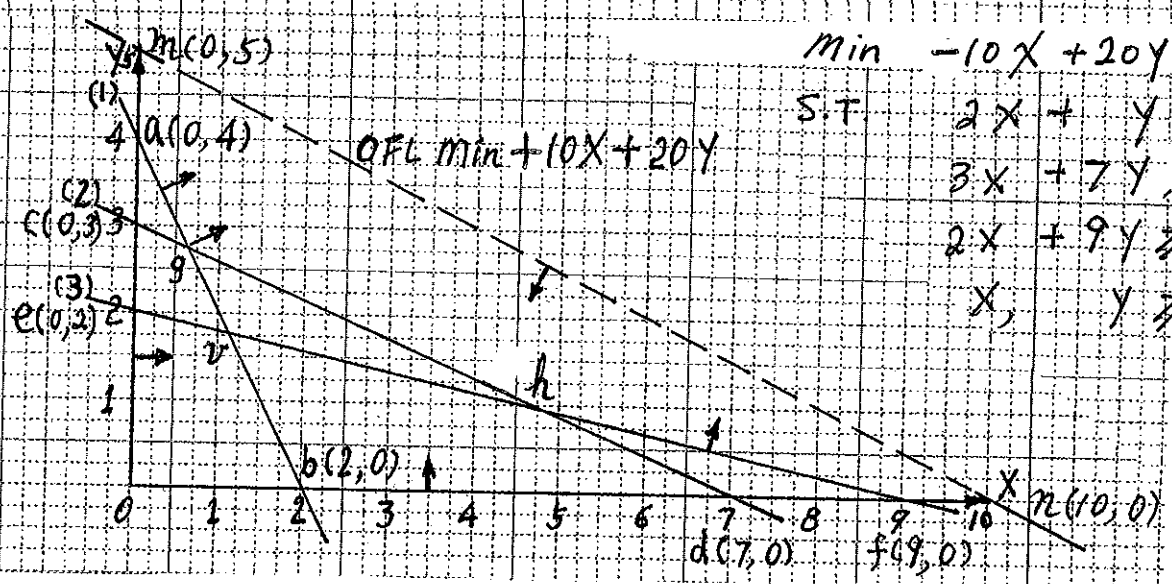
$$\begin{aligned} \text{Min } & -10x + 20y \\ \text{s.t. } & 2x + y \geq 4 \quad (1) \\ & 3x + 7y \geq 21 \quad (2) \\ & 2x + 9y \geq 18 \quad (3) \\ & x, y \geq 0 \quad (4) \end{aligned}$$



$$\begin{aligned} \text{Min } & -10x + 20y \\ \text{s.t. } & 2x + y \geq 4 \quad (1) \\ & 3x + 7y \geq 21 \quad (2) \\ & 2x + 9y \geq 18 \quad (3) \\ & x, y \geq 0 \quad (4) \end{aligned}$$



$$\begin{aligned} \text{Min } & -10x + 20y \\ \text{s.t. } & 2x + y \geq 4 \quad (1) \\ & 3x + 7y \geq 21 \quad (2) \\ & 2x + 9y \geq 18 \quad (3) \\ & x, y \geq 0 \quad (4) \end{aligned}$$



3) Identify the feasible Region:

Feasible region: satisfy all of constraints

i.e. Equ. (2)  $3X + 7Y \leq 21$

Equ. (2)  $\leq$  less than or equal to, at most, no more (greater) than  
area g-v-h-g, bounded region

Equ. (2)  $\geq$  greater than or equal to, at least, no less (smaller) than  
wide open area a-g-h-f upward, unbounded region

Equ. (2) = equal to, the same as.

line segment g-h, bounded region (line)

IF Equ. (1)  $2X + 1Y = 4$ , bounded region (single point g)

OR IF Equ. (3)  $2X + 9Y = 18$ , bounded region (single point h)

WHAT IF  $2X + 1Y = 4$  (1-1)

$3X + 7Y = 21$  (2-1)

$2X + 9Y = 18$  (3-1)

$X, Y \geq 0$  (4)

Infeasible region: outside of feasible region

i.e. Any region outside of g-v-h-g IF Equ. (2)  $3X + 7Y \leq 21$

No feasible region:

at least one constraint (equ.) is NOT satisfied

i.e. Equ. (2)  $\neq$

$2X + Y \leq 4$  (1-1)

$3X + 7Y \geq 21$  (2-1)

$2X + 9Y \leq 18$  (3-1)

$X, Y \geq 0$  (4)

} plus (4), area a-c-g-a

} area h-d-f-h

Solution: a intercept, joint or vertex.

Infeasible solution:

a solution outside of feasible region

i.e. IF Equ. (2)  $3X + 7Y \leq 21$

THEN, solutions:

a, b, c, d, e, f, g

Feasible solution:

a solution in (on boundary) feasible region

i.e. IF Equ. (2)  $3X + 7Y \leq 21$

THEN, solutions:

g,

v,

h

Optimal solution:

a solution (feasible) that max or min OBJ (OFL) value

4) obtain a solution  $(x, y)$

i.e.  $g(x, y)$

with back substitution:

step 1.

from Equ. (1)  $y = 4 - 2x$  (1-1)

step 2.

put Equ. (1-1) into Equ. (2)

$$3x + 7(4 - 2x) = 21 \quad (2-1)$$

$$3x + 28 - 14x = 21$$

$$28 - 21 = (14 - 3)x$$

$$x = (28 - 21) / (14 - 3) = 7/11 \quad (2-1')$$

step 3.

put Equ. (2-1')  $x = 7/11$  into Equ. (1-1)

$$y = 4 - 2(7/11) = (44 - 14)/11 = 30/11 \quad (1-1')$$

Therefore:  $x = 7/11$  and  $y = 30/11$  at point  $g$  or  $g(7/11, 30/11)$

with Addition or subtraction:

step 1a

Equ. (1)  $\times 7$ :  $14x + 7y = 28$  (1-1)

step 2a

Equ. (1-1) - Equ. (2)

$$\frac{3x + 7y = 21}{11x + 0y = 7} \quad (2)$$

$$11x + 0y = 7 \quad (1-1')$$

$$x = 7/11 \quad (1-1'')$$

step 3a = step 3

5) Derive Equation  $y = b_0 + b_1 X$  or  $y = aX + b$   
from two points on the same line

$a(0, 4)$  and  $b(2, 0)$  on line (1)

$$b_1 = \frac{Y_a - Y_b}{X_a - X_b} = \frac{4 - 0}{0 - 2} = -2$$

use  $a(0, 4)$ ,  $y = b_0 + b_1 X$  and  $b_1 = -2$   
 $4 = b_0 - 2(0)$   
 $b_0 = 4$

Therefore:  $y = 4 - 2X$  or  $2X + y = 4$

Identify the sign ( $\leq$ ,  $=$  or  $\geq$ ) for the equation

i. e. line (1) away from origin  $(0, 0)$

randomly select point  $(1, 4)$  in the same side of  $\nearrow$   
 LHS  $2X + y = 2(1) + 4 = 6 > \text{RHS} = 4$

6) Objective function line and its direction (OFL or OBJ)

objective  
Min  
s.t.

OFL coefficients  
 $-10X + 20Y$   
 Decision Variables

constraints:

|           | LHS       | RHS |                |
|-----------|-----------|-----|----------------|
| $2X + 1Y$ | $\geq 4$  | (1) | Demand #1      |
| $3X + 7Y$ | $\leq 21$ | (2) | ... #2         |
| $2X + 9Y$ | $\geq 18$ | (3) | ... #3         |
| $X, Y$    | $\geq 0$  | (4) | Non Negativity |

constraint coefficients

7) draw OFL on the graph and mark its direction:  
 — arbitrary value

step 1: Set OFL  $-10X + 20Y = 20$

step 2: Let  $X=0, Y=1$  or point  $(0, 1)$

$X=-2, Y=0$  or point  $(-2, 0)$

step 3: connect points  $(0, 1)$  and  $(-2, 0)$  to form OFL

Observation: OFL is an Equal Profit (Max) or Cost (Min) line

i.e. OFL value = 20 at  $(0, 1)$

$(-2, 0)$

Any other point

Step 4: min OFL toward making OFL as small as possible

i.e. OFL  $-10X + 20Y = 40$

vs.  $-10X + 20Y = 20$

moving downward

Max OFL toward making OFL as large as possible  
 moving upward

8) Visualize optimal (Max or Min) solution

i.e. Min  $-10X + 20Y$  and Equ. (2) with  $\leq 21$

feasible region: g-v-h-g

feasible solutions: g, v and h

optimal solution:

a feasible solution that min OFL value. h

or joint of lines (2) and (3)

Obtain optimal solution  $h(x, y)$

With Addition and Subtraction:

Step 1. Equ. (2)  $\times 2$ :  $6x + 14y = 42$  (2-1)

Equ. (3)  $\times 3$ :  $6x + 27y = 54$  (3-1)

Step 2. Equ. (3-1) - Equ. (2-1)

$$\begin{array}{r} 0x + 13y = 12 \quad (3-2) \\ y = \frac{12}{13} \quad (3-2') \end{array}$$

Step 3. put Equ. (3-2')  $y = \frac{12}{13}$  into Equ. (3)

$$2x + 9\left(\frac{12}{13}\right) = 18 \quad (3-3)$$

$$x + \left(\frac{9}{2}\right)\left(\frac{12}{13}\right) = \frac{18}{2}$$

$$x + (9)\left(\frac{6}{13}\right) = 9$$

$$x = 9 - (9)\left(\frac{6}{13}\right) = (9)\left(1 - \frac{6}{13}\right) = (9)\left(\frac{7}{13}\right) = \frac{63}{13} = 4\frac{11}{13} \quad (3-3')$$

Step 4. put Equ. (3-3')  $x = \frac{63}{13}$  into Equ. (3)

$$(2)\left(\frac{63}{13}\right) + 9y = 18 \quad (3-4)$$

$$\left(\frac{2}{9}\right)\left(\frac{63}{13}\right) + \frac{9}{9}y = \frac{18}{9}$$

$$(2)\left(\frac{7}{13}\right) + y = 2$$

$$y = 2 - (2)\left(\frac{7}{13}\right) = 2 - \frac{14}{13} = \frac{26-14}{13} = \frac{12}{13} \quad (3-4')$$

Therefore: the optimal solution  $h\left(\frac{63}{13}, \frac{12}{13}\right)$

obtain optimal OFL value at  $h\left(\frac{63}{13}, \frac{12}{13}\right)$

$$\begin{aligned} (-10)\left(\frac{63}{13}\right) + (20)\left(\frac{12}{13}\right) &= \frac{1}{13}(20 \times 12 - 10 \times 63) \\ &= \frac{-390}{13} = -30 \end{aligned}$$

- 10) special cases of LP modeling  
alternative optimal solutions

Redundant Constraints

$$x \geq 0,$$

$$y \geq 0$$

Unbounded Solutions

Infeasible Solution

- 11) Reports from premSolver

binding constraints: slack or surplus = 0, LHS = RHS

i.e. Equ. (2) and Equ. (3)

at  $(\frac{63}{13}, \frac{12}{13})$  joints Equs (2) & (3).

Answer Report:  
pp. 21, 22

Not binding constraints: slack = RHS - LHS > 0

surplus = LHS - RHS > 0

for Equ. (1): surplus =  $10\frac{8}{13} - 4 = 6\frac{8}{13}$

Sensitivity Reports pp. 23

OFL coefficients (Adjustable cells) change within Allowable limits.

- The optimal solution  $h(\frac{63}{13}, \frac{12}{13})$  will NOT change.

- Original optimal OFL value will CHANGE.

$$\text{OFL value} \quad ? X + ? Y =$$
$$\frac{63}{13} \quad \frac{12}{13}$$

- alternative optimal solutions (unlimited number of) possible at allowable limits.

- Reduced Cost = OFL coef. - Sum (Const. coef. \* shadow price)

$$\text{Reduced Cost of } X = -10 - \begin{pmatrix} 2 * 0 \\ 3 * -10 \\ 2 * 10 \end{pmatrix} = -10 + 10 = 0$$

$$\text{Reduced Cost of } Y = 20 - \begin{pmatrix} 1 * 0 \\ 7 * -10 \\ 9 * 10 \end{pmatrix} = 20 - 20 = 0$$

Read Ragsdale pages 148 to 151.

OFL coef. of X range:  $(-10 - \infty, -10 + 14.44]$

when OFL coef. of X = -5 or  $\uparrow$  by +5

$$\begin{aligned} \text{OFL value} &= (-5)\left(\frac{63}{13}\right) + (20)\left(\frac{12}{13}\right) = \frac{1}{13}(20*12 - 5*63) \\ &= -\frac{75}{13} = -5\frac{10}{13} = -30 + (5)*\frac{63}{13} \end{aligned}$$

OFL coef. of Y range:  $(20 - 43.33, 20 + \infty)$

when OFL coef. of Y = 10 or decreased by 10

$$\begin{aligned} \text{OFL value} &= -30 - (10)\left(\frac{12}{13}\right) = -30 - 9\frac{3}{13} = -39\frac{3}{13} \\ &= -(10)\left(\frac{63}{13}\right) + (10)\left(\frac{12}{13}\right) \end{aligned}$$

RHS values (constraints) change within allowable limits

- The optimal solution  $h(\frac{63}{13}, \frac{12}{13})$  will CHANGE.

- The original optimal OFL value will CHANGE.

$$\text{OFL value } -10X + 20Y =$$

- Always moving parallelly from original line.

- Shadow price:  $\pm$  OFL value due to one unit change of RHS  
 $\downarrow$  marginal change of OFL value  
in allowable range

For nonbinding constraints:

Equ.(1) shadow price  $\equiv 0$ , no contribution to OFL value  
because either slack or surplus  $> 0$   
when Equ.(1) RHS from  $(4-\infty, 4+6.615)$

For binding constraints:

Shadow price Equ.(2)  $= -10$

slack or surplus  $= 0$

when Equ.(2) RHS from  $(21-5.375, 21+6)$

When Equ.(2) RHS from 21 to 22,

$$\text{OFL value} = -30 - 10 \times 1 = -40$$

Shadow price Equ.(3)  $= 10$

slack or surplus  $= ?$

when Equ.(3) RHS from ( , )

range  $(18-4, 18+7.818)$

when Equ.(3) RHS from 18 to 23 or  $+5$

$$\text{OFL value} = -30 + 10 \times 5 = +20$$

MIN -10 X + 20 Y

|    | A                   | B      | C    | D                                | E  | F  | G | H | I | J | K | L | M |
|----|---------------------|--------|------|----------------------------------|----|----|---|---|---|---|---|---|---|
| 1  | Min Production Cost |        |      |                                  |    |    |   |   |   |   |   |   |   |
| 2  |                     | X      | Y    | =SUMPRODUCT(B4:C4,\$B\$3:\$C\$3) |    |    |   |   |   |   |   |   |   |
| 3  | Units to make       | 0      | 0    | Total Profit                     |    |    |   |   |   |   |   |   |   |
| 4  | Unit Cost           | (\$10) | \$20 |                                  |    |    |   |   |   |   |   |   |   |
| 5  |                     |        |      | LHS                              |    |    |   |   |   |   |   |   |   |
| 6  | Constraints         | X      | Y    | Used                             |    |    |   |   |   |   |   |   |   |
| 7  | Demand #1           | 2      | 1    | 0                                | => | 4  |   |   |   |   |   |   |   |
| 8  | Demand #2           | 3      | 7    | 0                                | =< | 21 |   |   |   |   |   |   |   |
| 9  | Demand #3           | 2      | 9    | 0                                | => | 18 |   |   |   |   |   |   |   |
| 10 |                     |        |      | =SUMPRODUCT(B7:C7,\$B\$3:\$C\$3) |    |    |   |   |   |   |   |   |   |
| 11 |                     |        |      |                                  |    |    |   |   |   |   |   |   |   |
| 12 |                     |        |      |                                  |    |    |   |   |   |   |   |   |   |
| 13 |                     |        |      |                                  |    |    |   |   |   |   |   |   |   |

**Solver Parameters V7.0**

Set Cell:

Equal To: ☐ Max ☒ Min ☐ Value Of:

By Changing Variable Cells:

Subject to the Constraints:

- 
- 
- 

Standard LP Simplex

Buttons: Solve, Close, Model, Options, Add, Variables, Change, Reset All, Delete, Help

**LP Simplex Solver Options**

General | Integer | Problem |

Max Time:  seconds

Iterations:

Precision:

Pivot Tol:

Reduced Tol:

Buttons: Load Model..., Save Model...

☐ Show Iteration Results

☐ Use Automatic Scaling

☒ Assume Non-Negative

☐ Bypass Solver Reports

Buttons: OK, Cancel, Help

|    | A                   | B       | C     | D                                | E  | F  | G | H | I | J | K | L | M |
|----|---------------------|---------|-------|----------------------------------|----|----|---|---|---|---|---|---|---|
| 1  | Min Production Cost |         |       |                                  |    |    |   |   |   |   |   |   |   |
| 2  |                     | X       | Y     | =SUMPRODUCT(B4:C4,\$B\$3:\$C\$3) |    |    |   |   |   |   |   |   |   |
| 3  | Units to make       | 4/10/03 | 12/13 | Total Profit                     |    |    |   |   |   |   |   |   |   |
| 4  | Unit Cost           | (\$10)  | \$20  | 30                               |    |    |   |   |   |   |   |   |   |
| 5  |                     |         |       | LHS                              |    |    |   |   |   |   |   |   |   |
| 6  | Constraints         | X       | Y     | Used                             |    |    |   |   |   |   |   |   |   |
| 7  | Demand #1           | 2       | 1     | 10/8/13                          | => | 4  |   |   |   |   |   |   |   |
| 8  | Demand #2           | 3       | 7     | 21                               | =< | 21 |   |   |   |   |   |   |   |
| 9  | Demand #3           | 2       | 9     | 18                               | => | 18 |   |   |   |   |   |   |   |
| 10 |                     |         |       | =SUMPRODUCT(B7:C7,\$B\$3:\$C\$3) |    |    |   |   |   |   |   |   |   |
| 11 |                     |         |       |                                  |    |    |   |   |   |   |   |   |   |

**Solver Results**

Solver found a solution. All constraints and optimality conditions are satisfied.

Buttons: Keep Solver Solution, Restore Original Values

☐ Return to Solver Parameters Dialog

Reports: Answer, Sensitivity, Limits

Buttons: OK, Cancel, Save Scenario..., Help

|    | A                                                                                         | B                      | C              | D              | E           | F           | G |
|----|-------------------------------------------------------------------------------------------|------------------------|----------------|----------------|-------------|-------------|---|
| 1  | Microsoft Excel 12.0 Answer Report                                                        |                        |                |                |             |             |   |
| 2  | Worksheet: [LP.xlsx]lecture2                                                              |                        |                |                |             |             |   |
| 3  | Report Created: 02/11/2010 16:53:51 PM                                                    |                        |                |                |             |             |   |
| 4  | Result: Solver found a solution. All constraints and optimality conditions are satisfied. |                        |                |                |             |             |   |
| 5  | Engine: Standard LP Simplex                                                               |                        |                |                |             |             |   |
| 6  | Solution Time: 00 Seconds                                                                 |                        |                |                |             |             |   |
| 7  | Iterations: 3                                                                             |                        |                |                |             |             |   |
| 8  | Subproblems: 0                                                                            |                        |                |                |             |             |   |
| 9  | Incumbent Solutions: 0                                                                    |                        |                |                |             |             |   |
| 10 |                                                                                           |                        |                |                |             |             |   |
| 11 |                                                                                           |                        |                |                |             |             |   |
| 12 | Target Cell (Min)                                                                         |                        |                |                |             |             |   |
| 13 | Cell                                                                                      | Name                   | Original Value | Final Value    |             |             |   |
| 14 | \$D\$4                                                                                    | Unit Cost Total Profit | 0              | -30            |             |             |   |
| 15 |                                                                                           |                        |                |                |             |             |   |
| 16 |                                                                                           |                        |                |                |             |             |   |
| 17 | Adjustable Cells                                                                          |                        |                |                |             |             |   |
| 18 | Cell                                                                                      | Name                   | Original Value | Final Value    |             |             |   |
| 19 | \$B\$3                                                                                    | Units to make X        | 0              | 4 11/13        |             |             |   |
| 20 | \$C\$3                                                                                    | Units to make Y        | 0              | 12/13          |             |             |   |
| 21 |                                                                                           |                        |                |                |             |             |   |
| 22 | Constraints                                                                               |                        |                |                |             |             |   |
| 23 | Cell                                                                                      | Name                   | Cell Value     | Formula        | Status      | Slack       |   |
| 24 | \$D\$7                                                                                    | Demand #1 Used         | 10 8/13        | \$D\$7>=\$F\$7 | Not Binding | 6.615384615 |   |
| 25 | \$D\$8                                                                                    | Demand #2 Used         | 21             | \$D\$8<=\$F\$8 | Binding     | 0           |   |
| 26 | \$D\$9                                                                                    | Demand #3 Used         | 18             | \$D\$9>=\$F\$9 | Binding     | 0           |   |

|    | A                                      | B | C                      | D | E       | F      | G       | H      | I       | J      |
|----|----------------------------------------|---|------------------------|---|---------|--------|---------|--------|---------|--------|
| 1  | Microsoft Excel 12.0Limits Report      |   |                        |   |         |        |         |        |         |        |
| 2  | Worksheet: [LP.xlsx]lecture2           |   |                        |   |         |        |         |        |         |        |
| 3  | Report Created: 02/11/2010 16:53:52 PM |   |                        |   |         |        |         |        |         |        |
| 4  |                                        |   |                        |   |         |        |         |        |         |        |
| 5  |                                        |   |                        |   |         |        |         |        |         |        |
| 6  | Target                                 |   |                        |   |         |        |         |        |         |        |
| 7  | Cell                                   |   | Value                  |   |         |        |         |        |         |        |
| 8  | \$D\$4                                 |   | Unit Cost Total Profit |   | -30     |        |         |        |         |        |
| 9  |                                        |   |                        |   |         |        |         |        |         |        |
| 10 |                                        |   |                        |   |         |        |         |        |         |        |
| 11 | Adjustable                             |   |                        |   | Lower   | Target | Upper   | Target |         |        |
| 12 | Cell                                   |   | Name                   |   | Value   |        | Limit   | Result | Limit   | Result |
| 13 | \$B\$3                                 |   | Units to make X        |   | 4 11/13 |        | 4 11/13 | -30    | 4 11/13 | -30    |
| 14 | \$C\$3                                 |   | Units to make Y        |   | 12/13   |        | 12/13   | -30    | 12/13   | -30    |

|    | A                                       | B                      | C           | D       | E           | F           | G           | H |
|----|-----------------------------------------|------------------------|-------------|---------|-------------|-------------|-------------|---|
| 1  | Microsoft Excel 12.0 Sensitivity Report |                        |             |         |             |             |             |   |
| 2  | Worksheet: [LP.xlsx]lecture2            |                        |             |         |             |             |             |   |
| 3  | Report Created: 02/11/2010 16:53:51 PM  |                        |             |         |             |             |             |   |
| 4  |                                         |                        |             |         |             |             |             |   |
| 5  | Target Cell (Min)                       |                        |             |         |             |             |             |   |
| 6  | Cell                                    | Name                   | Final Value |         |             |             |             |   |
| 7  | \$D\$4                                  | Unit Cost Total Profit | -30         |         |             |             |             |   |
| 8  |                                         |                        |             |         |             |             |             |   |
| 9  | Adjustable Cells                        |                        |             |         |             |             |             |   |
| 10 |                                         |                        | Final       | Reduced | Objective   | Allowable   | Allowable   |   |
| 11 | Cell                                    | Name                   | Value       | Cost    | Coefficient | Increase    | Decrease    |   |
| 12 | \$B\$3                                  | Units to make X        | 4 11/13     | 0       | -10         | 14.44444444 | 1E+30       |   |
| 13 | \$C\$3                                  | Units to make Y        | 12/13       | 0       | 20          | 1E+30       | 43.33333333 |   |
| 14 |                                         |                        |             |         |             |             |             |   |
| 15 | Constraints                             |                        |             |         |             |             |             |   |
| 16 |                                         |                        | Final       | Shadow  | Constraint  | Allowable   | Allowable   |   |
| 17 | Cell                                    | Name                   | Value       | Price   | R.H. Side   | Increase    | Decrease    |   |
| 18 | \$D\$7                                  | Demand #1 Used         | 10 8/13     | 0       | 4           | 6.615384615 | 1E+30       |   |
| 19 | \$D\$8                                  | Demand #2 Used         | 21          | -10     | 21          | 6           | 5.375       |   |
| 20 | \$D\$9                                  | Demand #3 Used         | 18          | 10      | 18          | 7.818181818 | 4           |   |

Min  $-10X + 20Y$

s. t.

$$2X + Y \Rightarrow 4 \quad (1)$$

$$3X + 7Y \leq 21 \quad (2)$$

$$2X + 9Y \Rightarrow 18 \quad (3)$$

$$X, Y \Rightarrow 0 \quad (4)$$