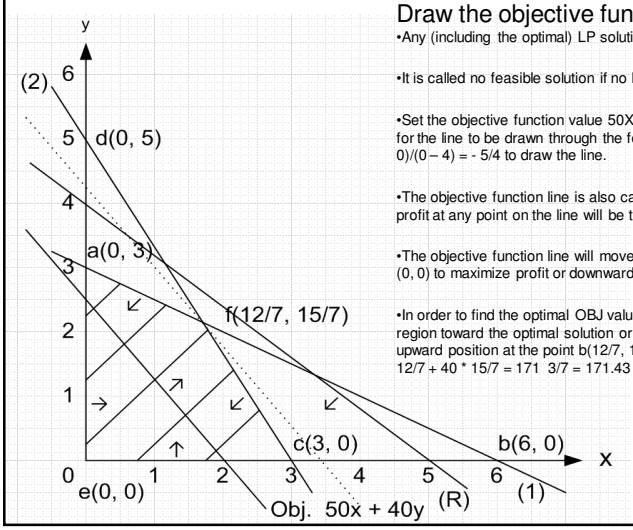
LP Graphic Solution & Excel@ Solver for LP

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <=15 (2) Raw materials in units
X, Y >= 0 (3)



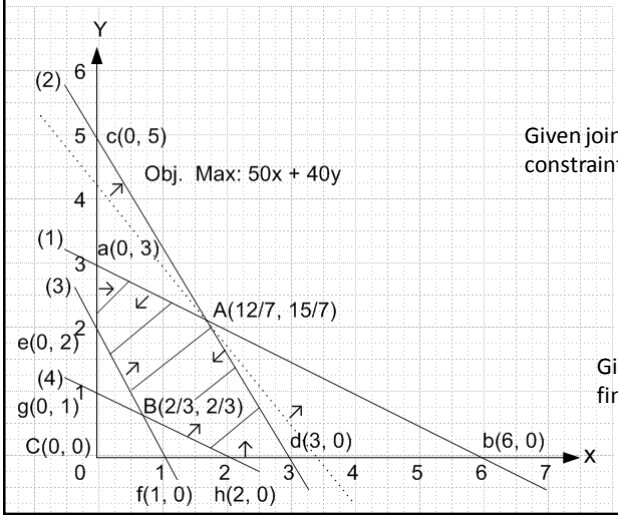
Draw the objective function line on the graph:

- Any (including the optimal) LP solution has to be part of the feasible region.
- It is called no feasible solution if no LP solution is found in the feasible region.
- Set the objective function value 50X + 40Y = 100 to get the points (0, 2.5) and (2, 0) for the line to be drawn through the feasible region. Or use the slope of OBJ = (5 – 0)/(0 – 4) = - 5/4 to draw the line.
- The objective function line is also called Equal Profit line or level curve because the profit at any point on the line will be the same of 100.
- The objective function line will move in the direction of upward away from the origin (0, 0) to maximize profit or downward toward to the origin (0, 0) to minimize costs.
- In order to find the optimal OBJ value, the OBJ line always moves in the feasible region toward the optimal solution or vertex (X, Y). The OBJ line will reach its most upward position at the point b(12/7, 15/7) with the optimal OBJ function value as 50 * 12/7 + 40 * 15/7 = 171 3/7 = 171.43

Max OBJ with more Constraints

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
(3) Customer v demand >=0
(4) Customer w demand >= 0
X, Y >= 0 (5) Non negative

Given joints e(0, 2) and f(1, 0), what is constraint for Customer v demand?

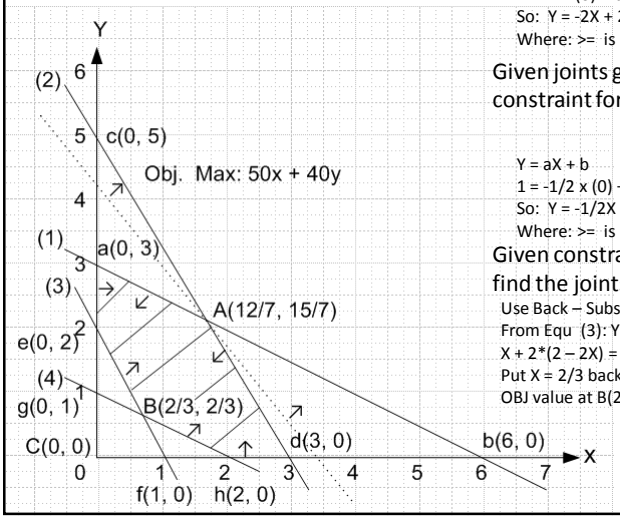


Given joints g(0, 1) and h(2, 0), what is constraint for Customer w demand?

Given constraints (3) and (4), find the joints e, f, g, h and B?

Max OBJ with more Constraints

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
(3) Customer v demand >=0
(4) Customer w demand >= 0
X, Y >= 0 (5) Non negative



Given joints e(0, 2) and f(1, 0), what is constraint for Customer v demand?

$$Slope = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{2-0}{0-1} = -2$$

Y = aX + b
2 = -2 x (0) + b or b = 2
So: Y = -2X + 2 or 2X + Y >= 2 (3)
Where: >= is used because Line (3) moves up

Given joints g(0, 1) and h(2, 0), what is constraint for Customer w demand?

$$Slope = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{1-0}{0-2} = -\frac{1}{2}$$

Y = aX + b
1 = -1/2 x (0) + b or b = 1
So: Y = -1/2X + 1 or X + 2Y >= 2 (4)
Where: >= is used because Line (4) moves up

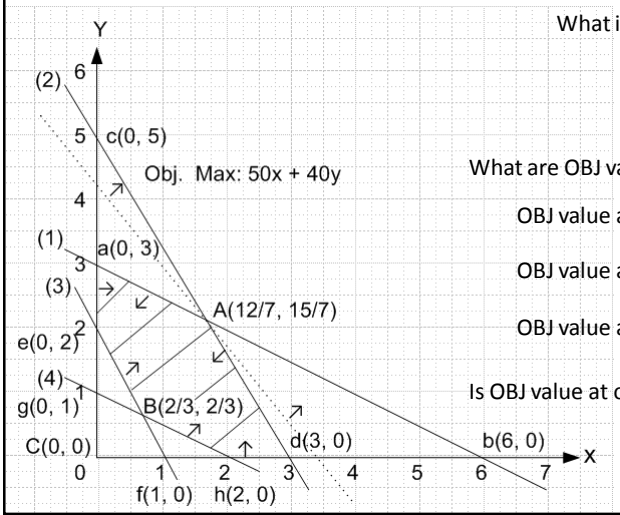
Given constraints (3) and (4), find the joints e, f, g, h and B?

Use Back – Substitution to solve B(X, Y):
From Equ (3): Y = 2 – 2X, Put in Equ (4)
X + 2*(2 – 2X) = 2, or X + 4 – 4X = 2, → X = 2/3
Put X = 2/3 back into Y = 2 – 2X = 2 – 2*(2/3) = 2/3
OBJ value at B(2/3, 2/3) = 50*(2/3) + 40*(2/3) = 60

Feasible Region May Change with Added Constraints

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

What is the new feasible region?



What is the value of Joint B(X, Y)?

What are OBJ values at joints e, ,h and B?

OBJ value at joint h(2, 0) =

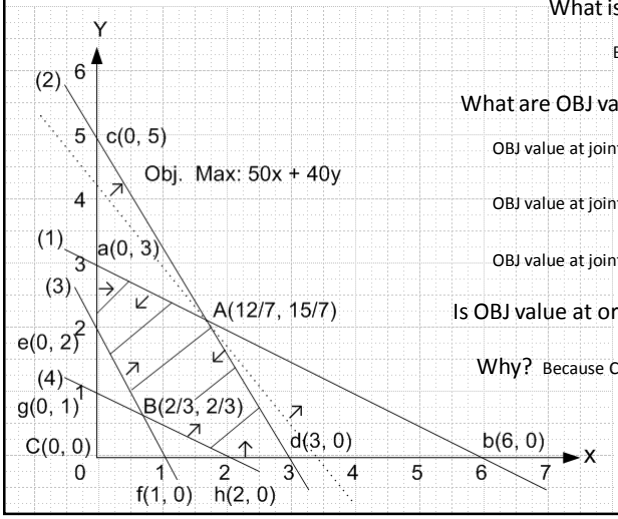
OBJ value at joint e(0, 2) =

OBJ value at joint B(2/3, 2/3) =

Is OBJ value at origin C(0, 0) feasible? Why?

Feasible Region May Change with Added Constraints

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative



What is the new feasible region?

a→e→B→h→d→A→a

What is the value of Joint B(X, Y)?

B(2/3, 2/3)

What are OBJ values at joints e, ,h and B?

OBJ value at joint h(2, 0) = 2*50 = 100

OBJ value at joint e(0, 2) = 2*40 = 80

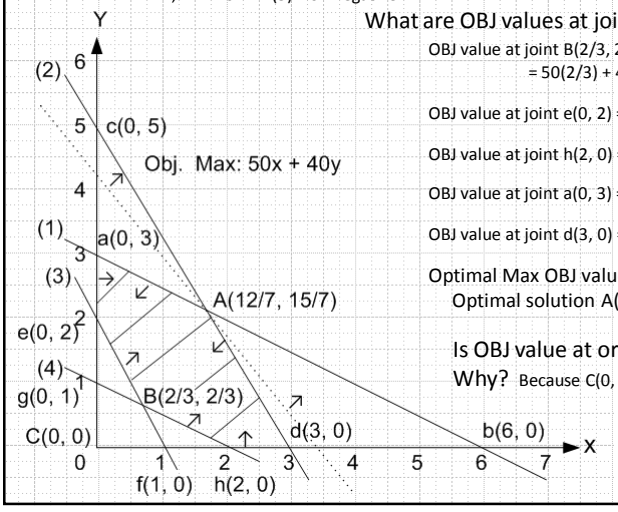
OBJ value at joint B(2/3, 2/3) = 50*2/3 + 40*2/3 = 60

Is OBJ value at origin C(0, 0) feasible? No

Why? Because C(0, 0) is outside of feasible region

Solutions and OBJ Values versus Optimal Solution and Optimal OBJ Value

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative



What is the new feasible region?
a, e, B, h, d, A and a form feasible region

What is the value of Joint B(X, Y)? B(2/3, 2/3)

What are OBJ values at joints e, ,h and B?

OBJ value at joint B(2/3, 2/3) =
= 50(2/3) + 40(2/3) = 60

OBJ value at joint e(0, 2) = 40(2) = 80

OBJ value at joint h(2, 0) = 50(2) = 100

OBJ value at joint a(0, 3) = 40(3) = 120

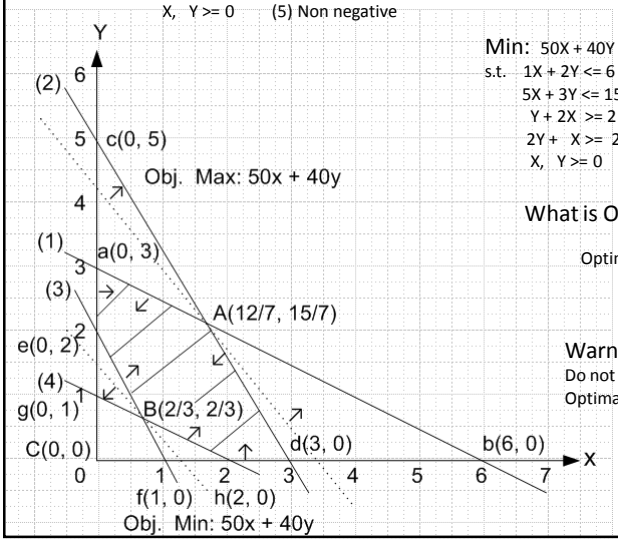
OBJ value at joint d(3, 0) = 50(3) = 150

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

Is OBJ value at origin C(0, 0) feasible? No,
Why? Because C(0, 0) is outside of feasible region

Max OBJ versus Min OBJ Optimal Solutions

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative



What is Optimal Solution to Max OBJ?

Optimal Max OBJ value at Optimal solution
joint A(12/7, 15/7) = 171 3/7

Min: 50X + 40Y Production Cost
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

What is Optimal Solution to Min OBJ?

Optimal Min OBJ value at Optimal solution
joint B(2/3, 2/3) = 60

Warning No. 1:
Do not use Optimal solution to Min OBJ as Optimal solution to Max OBJ

Excel@ LP Layout for Solver or PremSolver

Values of Decision variables

OBJ coefficients

	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C$3)		
3	Units to make	0	0	Total Profit		
4	Unit Profits	$50	$40	0		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	0	<=	6
8	Raw Material Units	5	3	0	<=	15
9	Customer v Demand	1	2	0	>=	2
10	Customer w Demand	2	1	0	>=	2
11		=SUMPRODUCT(B7:C7,$B$3:$C$3)				

OBJ value & Excel@ formula

Right Hand Side (RHS) Values

Left Hand Side (LHS) Values & Excel@ formulas

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

More Warnings:

2) Values of Decision variables be numeric

3) OBJ coefficients be numeric, $ format with Excel@ Cell Format/Currency, not typed as "$"

4) OBJ value in Excel@ formula with $$ if needed

5) RHS values be numeric, No any character, i.e., variable names

6) LHS values in Excel@ formulas with $$ if needed

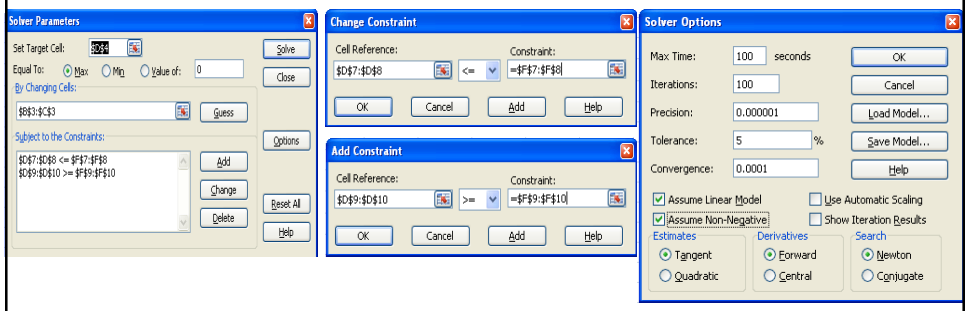
7) <=, = or >= signs match given constraints

Use Excel@ Solver to LP Problem

	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C$3)		
3	Units to make	0	0	Total Profit		
4	Unit Profits	$50	$40	0		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	0	<=	6
8	Raw Material Units	5	3	0	<=	15
9	Customer v Demand	1	2	0	>=	2
10	Customer w Demand	2	1	0	>=	2
11		=SUMPRODUCT(B7:C7,$B$3:$C$3)				

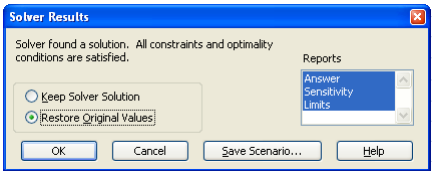
Procedure to use Excel@ Solver

1) Put Cursor at OBJ formula
2) Click Data/Solver
3) Set Target Cell = Cell with OBJ formula
4) Click Equal To: as Max for Max OBJ
5) Set By Changing Cells = Cells of OBJ coefficents
6) Set Subject to Constraints as LHS formulas vs. RHS values
7) Click Solver Options
   i) Click Assume Linear Model
   ii) Or Click Standard LP Simplex in PremSolver
   iii) Click Assume Non - negative

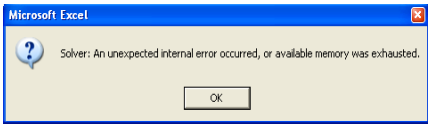


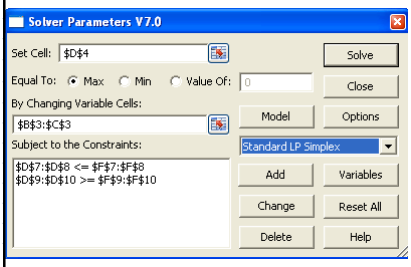
Use Excel@ Solver to Max OBJ in LP Problem

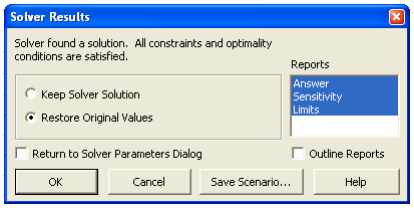
	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C$3)		
3	Units to make	1 5/7	2 1/7	Total Profit		
4	Unit Profits	$50	$40	171 3/7		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	6	<=	6
8	Raw Material Units	5	3	15	<=	15
9	Customer v Demand	1	2	6	>=	2
10	Customer w Demand	2	1	5 4/7	>=	2
11		=SUMPRODUCT(B7:C7,$B$3:$C$3)				



The Time to Install PremSolver







Excel@ Solver Reports: Answer, Sensitivity, and Limits to Max OBJ 50X + 40Y

Microsoft Excel 12.0 Answer Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM
Result: Solver found a solution. All constraints and optimality conditions are satisfied.
Engine: Standard LP Simplex
Solution Time: 00 Seconds
Iterations: 5
Subproblems: 0
Incumbent Solutions: 0

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	171.4285714

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	0	1 5/7
$C$3	Units to make Y	0	2 1/7

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	6	$D$7<=$F$7	Binding	0
$D$8	Raw Material Units Used	15	$D$8<=$F$8	Binding	0
$D$9	Customer v Demand Used	6	$D$9>=$F$9	Not Binding	4
$D$10	Customer w Demand Used	5 4/7	$D$10>=$F$10	Not Binding	3.571428571

Microsoft Excel 12.0Limits Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM

		Target
Cell	Name	Value
$D$4	Unit Profits Total Profit	171 3/7

		Adjustable	Lower	Target	Upper	Target
Cell	Name	Value	Limit	Result	Limit	Result
$B$3	Units to make X	1 5/7	0	85 5/7	1 5/7	171 3/7
$C$3	Units to make Y	2 1/7	1/7	91 3/7	2 1/7	171 3/7

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM

Target Cell (Max)

Cell	Name	Final Value
$D$4	Unit Profits Total Profit	171.4285714

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	1 5/7	0	50	16.66666667	30
$C$3	Units to make Y	2 1/7	0	40	60	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	6	7 1/7	6	4	3
$D$8	Raw Material Units Used	15	8 4/7	15	15	6
$D$9	Customer v Demand Used	6	0	2	4	1E+30
$D$10	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30

Optimal Solution

Optimal Max OBJ value

	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C$3)		
3	Units to make	1 5/7	2 1/7	Total Profit		
4	Unit Profits	$50	$40	171 3/7		
5				LHS	RHS	
6	Constraints	X	Y	Used	Available	
7	Production Minutes	1	2	6	6	
8	Raw Material Units	5	3	15	15	
9		=SUMPRODUCT(B7:C7,$B$3:$C$3)				

Microsoft Excel 12.0 Answer Report
Worksheet: [LP.xls]Ex
Report Created: 3/1/2008 1:33:29 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	171 3/7

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Number to make X	0	1 5/7
$C$3	Number to make Y	0	2 1/7

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Equ (1) Used	6	$D$7<=$E$7	Binding	0
$D$8	Equ (2) Used	15	$D$8<=$E$8	Binding	0
$B$3	Number to make X	1 5/7	$B$3>=0	Not Binding	1 5/7
$C$3	Number to make Y	2 1/7	$C$3>=0	Not Binding	2 1/7

Microsoft Excel 12.0 Limits Report
Worksheet: [LP.xls]Limits Report 1
Report Created: 3/1/2008 1:33:29 PM

		Target
Cell	Name	Value
$D$4	Unit Profits Total Profit	171 3/7

		Adjustable	Lower	Target	Upper	Target
Cell	Name	Value	Limit	Result	Limit	Result
$B$3	Number to make X	1 5/7	0	85 5/7	1 5/7	171 3/7
$C$3	Number to make Y	2 1/7	0	85 5/7	2 1/7	171 3/7

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xls]Ex
Report Created: 3/1/2008 1:33:29 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Number to make X	1 5/7	0	50	16.667	30
$C$3	Number to make Y	2 1/7	0	40	60	10

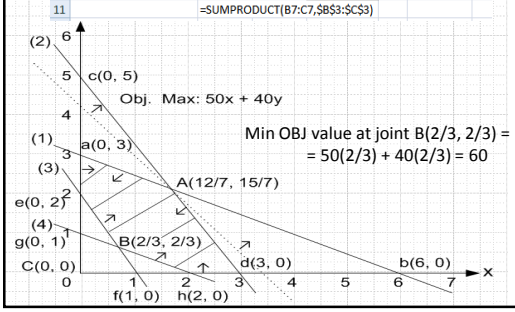
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Equ (1) Used	6	7 1/7	6	4	3
$D$8	Equ (2) Used	15	8 4/7	15	15	6

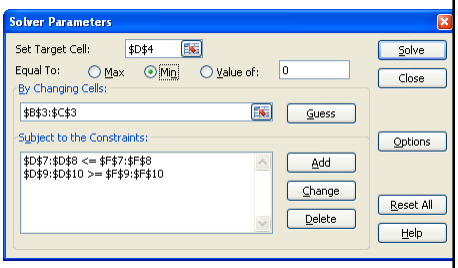
Use Excel@ Solver to Min OBJ in LP Problem

Min: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
Y + 2X >= 2 (3) Customer demand v
2Y + X >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

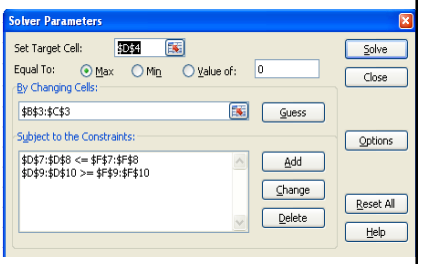
	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C$3)		
3	Units to make	2/3	2/3	Total Profit		
4	Unit Profits	$50	$40	60		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	2	<=	6
8	Raw Material Units	5	3	5 1/3	<=	15
9	Customer v Demand	1	2	2	>=	2
10	Customer w Demand	2	1	2	>=	2
11		=SUMPRODUCT(B7:C7,$B$3:$C$3)				



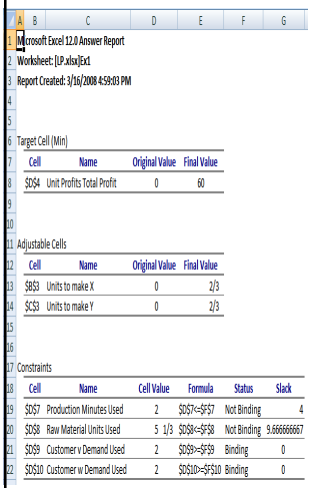
Min OBJ value at joint B(2/3, 2/3) =
= 50(2/3) + 40(2/3) = 60



Warning:
Do not Use Max OBJ for Min OBJ



Excel@ Solver Reports: Answer, Sensitivity, and Limits to Min OBJ 50X + 40Y



Microsoft Excel 12.0 Answer Report
Worksheet: [LP.xlsx]Ex1
Report Created: 3/16/2008 4:59:03 PM

Target Cell (Min)

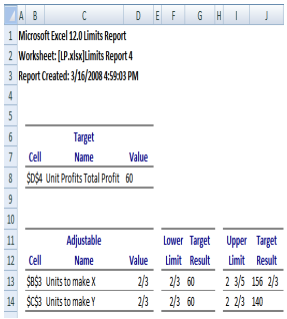
Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	60

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	0	2/3
$C$3	Units to make Y	0	2/3

Constraints

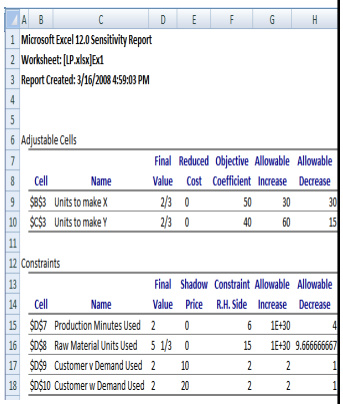
Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	2	$D$7<=$F$7	Not Binding	4
$D$8	Raw Material Units Used	5 1/3	$D$8<=$F$8	Not Binding	9.666666667
$D$9	Customer v Demand Used	2	$D$9>=$F$9	Binding	0
$D$10	Customer w Demand Used	2	$D$10>=$F$10	Binding	0



Microsoft Excel 12.0 Limits Report
Worksheet: [LP.xlsx]Limits Report 4
Report Created: 3/16/2008 4:59:03 PM

	Target		
Cell	Name	Value	
$D$4	Unit Profits Total Profit	60	

	Adjustable		Lower	Target	Upper	Target
Cell	Name	Value	Limit	Result	Limit	Result
$B$3	Units to make X	2/3	2/3	60	2 3/5	156 2/3
$C$3	Units to make Y	2/3	2/3	60	2 2/3	140



Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]Ex1
Report Created: 3/16/2008 4:59:03 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	2/3	0	50	30	30
$C$3	Units to make Y	2/3	0	40	60	15

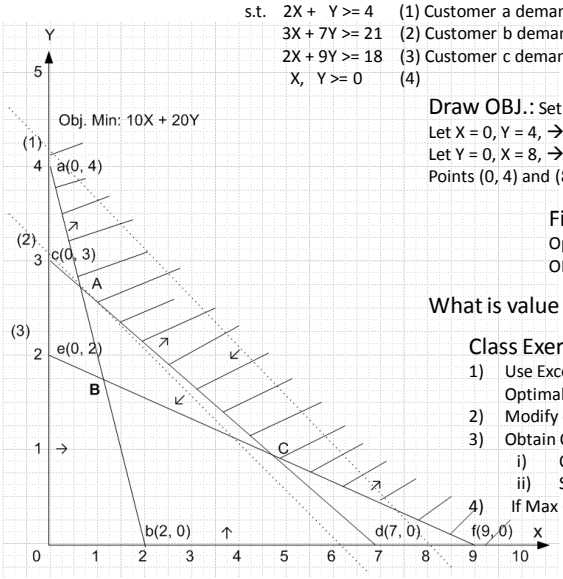
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	2	0	6	1E+30	4
$D$8	Raw Material Units Used	5 1/3	0	15	1E+30	9.666666667
$D$9	Customer v Demand Used	2	10	2	2	1
$D$10	Customer w Demand Used	2	20	2	2	1

Minimize Production Cost and Optimal Solution

Min: 10X + 20Y

s.t. 2X + Y >= 4 (1) Customer a demand
3X + 7Y >= 21 (2) Customer b demand
2X + 9Y >= 18 (3) Customer c demand
X, Y >= 0 (4)



Draw OBJ.: Set 10X + 20Y = 80
Let X = 0, Y = 4, → (0, 4)
Let Y = 0, X = 8, → (8, 0)
Points (0, 4) and (8, 0) forms the OBJ line

Find Optimal Solution A(X, Y)
Optimal Solution A(7/11, 30/11) with value of
OBJ(7/11, 30/11) = 10x(7/11) + 20x(30/11) = 60.91

What is value of OBJ at C(X, Y)?

Class Exercise:

1) Use Excel@ Solver to obtain Optimal solution and Optimal Min OBJ value
2) Modify constraints to make ABCA as feasible region
3) Obtain Optimal solution and Optimal Min OBJ value with
   i) On Graph
   ii) Solver
4) If Max OBJ 10X + 20Y, find Optimal solution on Graph