Blue Ridge Hot Tubs — Product Mix Problem

Decision Variable: Aqua-Spa (A) — # of Units of A to make
Hydro-Lux (H) — # of units of H to make

Max Profit $350A + 300H$

Subject to:

		LHS	RHS
Resources Used:	Labor:	$9A + 6H$	≤ 1566 hrs
	Tubing:	$12A + 16H$	≤ 2880 ft
	Pumps:	$1A + 1H$	≤ 200
Limited Proportion of A:		$1A/(1A + 1H)$	$\leq 30\%$

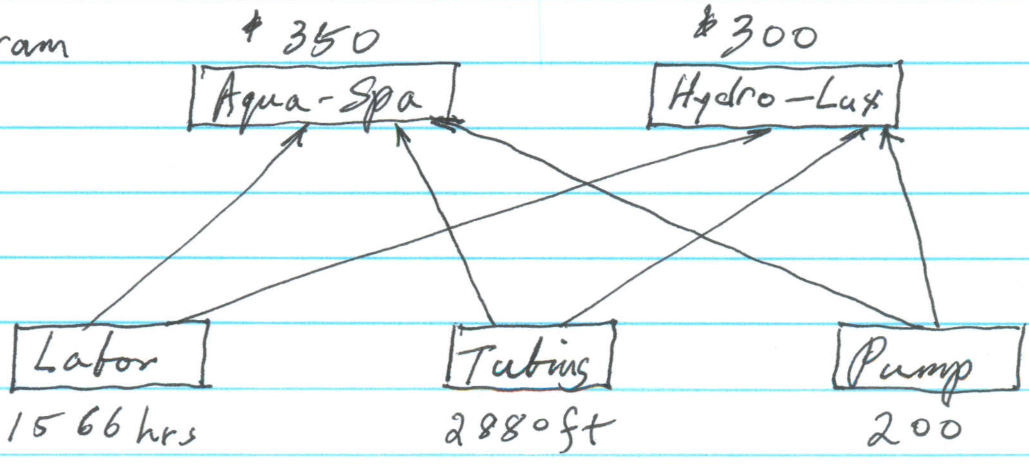
No more than 30% of total products in A.

	non negativity:	$A, \quad H$	$\geq \emptyset$

Steps:

1. Understand the problem
2. Identify the decision variable
3. State the objective function
4. State the constraints
5. Identify any lower and upper bounds for decision Variables

Diagram



$$\frac{A}{A+H} \leq .30$$

$$A \leq .30(A+H)$$

$$A \leq .30A + .30H$$

$$A - .30A - .30H \leq \phi$$

$$(1 - .30)A - .30H \leq \phi$$

$$.70A - .30H \leq \phi$$

or multiply 10 to both sides of the equation

$$7A - 3H \leq \phi$$

Make or Buy Decision

Decision variable:

M_i: # of rings to make in house for model $i = 1, 2, 3$

B_i: # of rings to Buy for model$_i$ $i = 1, 2, 3$

Min Total Cost:

$$\$50 M_1 + \$83 M_2 + \$130 M_3 + \$61 B_1 + \$97 B_2 + \$145 B_3$$

└ Cost/unit to make Model 1 └ Cost/unit to buy Model 2

Subject to: LHS RHS

Demand for Model 1: $M_1 + B_1 = 3000$ units

Model 2: $M_2 + B_2 = 2000$ units

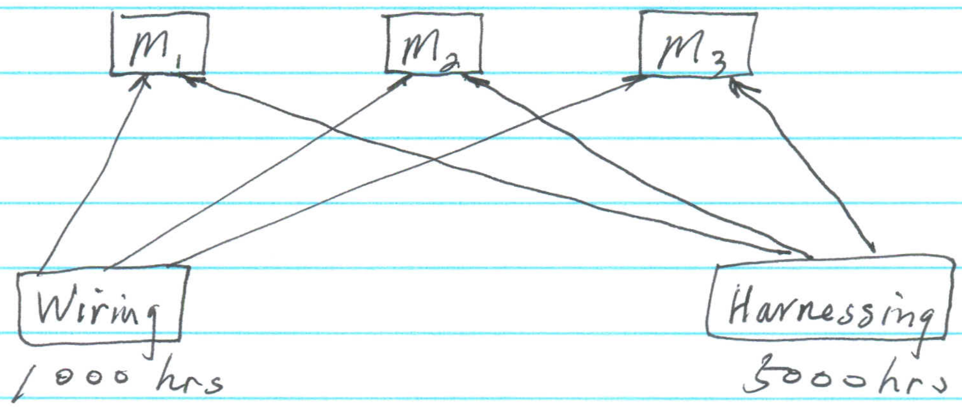
Model 3: $M_3 + B_3 = 900$ units

Hours of Wiring: $2M_1 + 1.5M_2 + 3M_3 \leq 10{,}000$ hrs

Hours of Harnessing: $1M_1 + 2M_2 + 1M_3 \leq 5000$ hrs

Non Negativity: $M_1, M_2, M_3, B_1, B_2, B_3 \geq \phi$

Cost to Make/unit:	\$50	\$83	\$130
Demand:	3000	2000	900



Investment Problem

Decision variable: Amount to invest in each bond:

AC DS EV MM, OP SS

annual Acme Chemical, Dyna Star, Eagle Vision, Micromodeling, OptiPro, Sebre Systems

Max Return of Investment:

annual $.0865AC + .095DS + .10EV + .0875MM + .0925OP + .09SS$

Subject to:

		LHS	RHS
Total to invest:	$AC + DS + EV + MM + OP + SS$		$= 750000$
	AC		$.25 \times 750000$ ≤ 187500
no more than	DS		≤ 187500
25% of total	EV		≤ 187500
be invested in any	MM		≤ 187500
one:	OP		≤ 187500
	SS		≤ 187500
maturn ≥ 10 years:	$AC + DS + MM + SS$		$.5 \times 750000$ ≥ 375000
Higher Risk:	$DS + EV + OP$		$.35 \times 750000$ ≤ 262500
	$AC, DS, EV, MM, OP, SS,$		≥ 0

steps:

1) Put in standardized form
2) Use Excel@Solver for Solution
3) Sensitivity Analysis
   3.1 OFC
   3.2 RHSV

Transportation problem

Distance between a Grove and a plant

Grove	Ocala	Orlando	Leesburg	Total Supply
Mt. Dora	21	50	40	275 000
Eustis	35	30	22	400 000
Clemont	55	20	25	300 000
				975 000 Sum of Supply
Processing Capacity:	200,000	600,000	225,000	1,025,000 Sum of Capacity

May assume a flat rate transportation charge per bushel - mile

Decision variable:	Ocala	Orlando	Leesburg
Mt. Dora	MO_c	MO_r	ML: # of bushels for M to L
Eustis	EO_c	EO_r	EL
Clemont	CO_c	CO_r	CL

min Total Bushel - Miles

$$21\,MO_c + 50\,MO_r + 40\,ML + 35\,EO_c + 30\,EO_r + 22\,EL + 55\,CO_c + 20\,CO_r + 25\,CL$$

s.t.

	LHS	RHS
Supply from Mt. Dora	$MO_c + MO_r + ML$	$= 275000$
Eustis	$EO_c + EO_r + EL$	$= 400000$
Clemont	$CO_c + CO_r + CL$	$= 300000$
Demand in Ocala	$MO_c + EO_c + CO_c$	$\leq 200,000$
Orlando	$MO_r + EO_r + CO_r$	≤ 600000
Leesburg	$ML + EL + CL$	$\leq 225,000$

$MO_c, MO_r, ML, EO_c, EO_r, EL, CO_c, CO_r, CL \geq 0$

Blending Problem – alternative model

Decision Variable:

F_i = # of 1000 pounds of F_i to use in the mix, $i = 1$ to 4

Order Requirements: 8000 pounds, at least 20% corn, 15% grain & 15% minerals

Cost For 1000 pounds	Feed 1	Feed 2	Feed 3	Feed 4
Min	\$250	\$300	\$320	\$150

Min Total Cost:

$$250F_1 + 300F_2 + 320F_3 + 150F_4$$

s.t.

Corn:	$.30F_1 + .05F_2 + .20F_3 + .10F_4 \geq 1.6$	(1600 pds)
Grain:	$.10F_1 + .30F_2 + .15F_3 + .10F_4 \geq 1.2$	(1200 pds)
minerals:	$.20F_1 + .20F_2 + .20F_3 + .30F_4 \geq 1.2$	(1200 pds)
Demand:	$F_1 + F_2 + F_3 + F_4 \geq 8$	(8000 pds)
	$F_1, F_2, F_3, F_4 \geq 0$	

Blending Problem

Decision Variable: F_i = # of Pounds of Feed i to use in the mix, $i = 1$ to 4.

Order Requirements: 8000 pounds, at least 20% Corn, 15% grain, 15% minerals

	Feed 1	Feed 2	Feed 3	Feed 4
Corn	30%	5%	20%	10%
Grain	10%	30%	15%	10%
Minerals	20%	20%	20%	30%
Cost per Pound	\$.25	\$.30	\$.32	\$.15

Total Cost to use Feed 3 only: \$.32 * 8000 = \$2,560.

Min total cost:

$$\$.25F_1 + \$.30F_2 + \$.32F_3 + \$.15F_4$$

s.t.

		LHS	RHS
corn:	$.30F_1 + .05F_2 + .20F_3 + .10F_4$		1600 = $\geq$ 8000 * .20
grain:	$.10F_1 + .30F_2 + .15F_3 + .10F_4$		8000 * .15 $\geq$ 1200
Minerals:	$.20F_1 + .20F_2 + .20F_3 + .30F_4$		$\geq$ 1200
	F_1, F_2, F_3, F_4		≥ 0
Demand:	$F_1 + F_2 + F_3 + F_4$		$= 8000$

Try to use 2000 pounds of each Feed:

	Feed 1	Feed 2	Feed 3	Feed 4	Sum
Corn	30%, 600	5% 100	20% 400	10% 200	1300, 16.25
Grain	10%, 200	30% 600	15% 300	10% 200	1300, 16.25
Minerals	20%, 400	20% 400	20% 400	30% 600	1800, 22.50
Cost per pd.	$.25	$.30	$.32	$.15	
Cost for 2000 pounds					
	$500	$600	$640	$300	Total $2,040.

Production & Inventory planning

Decision variables: P_m = # of units to make in month m, $m = 1$ to 6

Supplemental Variables: B_m = # of units of inventory at Beginning of month m.

	month 1	2	3	4	5	6	7
Unit Prod. Cost:	\$240	\$250	\$265	\$285	\$280	\$260	
Units Demanded:	1000	4500	6000	5500	3500	4000	
Max Production:	4000	3500	4000	4500	4000	3500	
Production in M:	P_1	P_2	P_3	P_4	P_5	P_6	
Unit Inv. Cost	\$240 * .015 \$3.60	\$3.75	\$3.98	\$4.28	\$4.20	\$3.90	
At least Safety Stock:	1500	1500	1500	1500	1500	1500	
$\geq \frac{1}{2}$ max Prod.:	2000	1750	2000	2250	2000	1750	
Beginning Inv.:	B_1 2750 units	B_2	B_3	B_4	B_5	B_6	B_7
Max Inventory:	6000	6000	6000	6000	6000	6000	

Min Total Cost = Production Cost + Inventory Cost

Production Cost: $\$240P_1 + \$250P_2 + \$265P_3 + \$288P_4 + \$280P_5 + \$260P_6$

Inventory Cost: $\$3.60(B_1+B_2)/2 + \$3.75(B_2+B_3)/2 + \$3.98(B_3+B_4)/2 +$
$+ \$4.28(B_4+B_5)/2 + \$4.20(B_5+B_6)/2 + \$3.90(B_6+B_7)/2$

$\geq \frac{1}{2}$ max Prod.   max production

S.T.

$2{,}000 \leq P_1 \leq 4{,}000$ Production in month 1

$1{,}750 \leq P_2 \leq 3{,}500$

$2{,}000 \leq P_3 \leq 4{,}000$

$2{,}250 \leq P_4 \leq 4{,}500$ Production in month 4

$2{,}000 \leq P_5 \leq 4{,}000$

$1{,}750 \leq P_6 \leq 3{,}500$

Ending Inventory = Beginning Inventory (B_m) + Units Produced (P_m) − Units Sold (Demand)

2750 units given

Safety Stock Demand Max Inventory $2500 \le B_1 + P_1 \le 7000$

Ending Inventory for month 1

$$1{,}500 \le B_1 + P_1 - 1{,}000 \le 6{,}000$$
$$1{,}500 \le B_2 + P_2 - 4{,}500 \le 6{,}000 \qquad \le B_2 + P_2 \le$$
$$1{,}500 \le B_3 + P_3 - 6{,}000 \le 6{,}000 \quad \text{or} \quad 7500 \le B_3 + P_3 \le 12000$$
$$1{,}500 \le B_4 + P_4 - 5{,}500 \le 6{,}000 \qquad \le B_4 + P_4 \le$$
$$1{,}500 \le B_5 + P_5 - 3{,}500 \le 6{,}000 \qquad \le B_5 + P_5 \le$$
$$1{,}500 \le B_6 + P_6 - 4{,}000 \le 6{,}000 \qquad \le B_6 + P_6 \le$$

Where =

Beginning Inventory B_m = Ending Inventory B_{m-1}

(2750 units)

$$B_2 = B_1 + P_1 - 1{,}000 \qquad -B_2 + B_1 + P_1 = +1000$$
$$B_3 = B_2 + P_2 - 4{,}500 \qquad -B_3 + B_2 + P_2 = 4500$$
$$B_4 = B_3 + P_3 - 6{,}000$$
$$B_5 = B_4 + P_4 - 5{,}500$$
$$B_6 = B_5 + P_5 - 3{,}500$$
$$B_7 = B_6 + P_6 - 4{,}000$$

P_m $m = 1$ to 6, and B_m, $m = 1$ to 7, $\ge \phi$

Reorganizing

n. OBJ.	P_1	P_2	P_3	P_4	P_5	P_6	B_1	B_2	B_3	B_4	B_5	B_6	B_7
OFC	\$240	\$250	\$265	\$285	\$280	\$260	\$1.80	\$3.675	\$3.865	\$4.13	\$4.24	\$4.05	\$1.95

S.T.

$2000 \le P_1 \le 4000$

$1750 \le P_2 \le 3500$

$2000 \le P_3 \le 4000$

$2250 \le P_4 \le 4500$

$2000 \le P_5 \le 4000$

$1750 \le P_6 \le 3500$

	P_1	P_2	P_3	P_4	P_5	P_6	B_1	B_2	B_3	B_4	B_5	B_6	B_7	RHS
	P_1						B_1							1500+1000 ≥ 2500
		P_2						B_2						≥ 6000
			P_3						B_3					≥ 7500
				P_4						B_4				≥ 7000
					P_5						B_5			≥ 5000
						P_6						B_6		≥ 5500
	P_1						B_1							≤ 7000
		P_2						B_2						≤ 10500
			P_3						B_3					≤ 12000
				P_4						B_4				≤ 11500
					P_5						B_5			≤ 9500
						P_6						B_6		≤ 10000
	P_1						B_1	$-B_2$						$= 1000$
		P_2						B_2	$-B_3$					$= 4500$
			P_3						B_3	$-B_4$				$= 6000$
				P_4						B_4	$-B_5$			$= 5500$
					P_5						B_5	$-B_6$		$= 3500$
						P_6						B_6	$-B_7$	$= 4000$

$P_i, \; B_i, \; i = 1$ to $7, \ge 0$

2750 units

$$\frac{3.6}{2}B_1 + \frac{3.6}{2}B_2 + \frac{3.75}{2}B_2 + \frac{3.75}{2}B_3 + \frac{3.98}{2}B_3 + \frac{3.98}{2}B_4$$

$$\frac{4.28}{2}B_4 + \frac{4.28}{2}B_5 + \frac{4.20}{2}B_5 + \frac{4.20}{2}B_6 + \frac{3.90}{2}B_6 + \frac{3.90}{2}B_7$$

$$\frac{3.6}{2}(2750) + \left(\frac{3.6}{2} + \frac{3.75}{2}\right)B_2 + \frac{(3.75+3.98)}{2}B_3 + \frac{(3.98+4.28)}{2}B_4$$

$$+ \frac{(4.28+4.20)}{2}B_5 + \frac{(4.20+3.90)}{2}B_6 + \frac{3.90}{2}B_7$$

$$4950 + 3.675B_2 + 3.865B_3 + 4.13B_4 + 4.24B_5 + 4.05B_6 + 1.95B_7$$

$$B_1 + P_1 - 1000 = 2750 + P_1 - 1000 = P_1 + 1750$$

Lower limit $P_1 + 1750 \geq 1500$

$P_1 \geq 1500 - 1750 = -250$

$P_1 + 1750 \leq 6000$

Upper limit $P_1 \leq 6000 - 1750 = 4250$

So $-250 \leq P_1 \leq 4250$ Redundant

$$B_1 + P_1 - 1000 \geq 1500$$

$$B_1 + P_1 \geq 1500 + 1000 = 2500$$

Multi-Periods Cash Flow Problem

Total Construction Cost : \$800,000

Scheduled payments:

	\$250,000	End of M_2 or Begin M_3
	\$250,000	End of M_4 or Begin M_5
	\$300,000	End of M_6 or Begin M_7

If \$800000 at Beginning of M_1 in no interest checking, then
Total cash deposit to fund the project is \$800000.

Decision variables:

A_i: the amount of money (\$1000) in investment A_i at Begin M_i, $i = 1$ to 6

B_i: B_i

C_i: C_i

D_i: D_i

Investment	1	2	3	4	5	6	7
	Cash Inflow/Outflow at the Beginning of Month						
A_1	−1 A_1	1.018 A_1					
B_1	−1 B_1	←→	1.035 B_1				
C_1	−1 C_1	←→	←→	1.058 C_1			
D_1	−1 D_1	←→	←→	←→	←→	←→	1.11 D_1
A_2		−1 A_2	1.018 A_2				
A_3			−1 A_3	1.018 A_3			
B_3			−1 B_3	←→	1.035 B_3		
A_4				−1 A_4	1.018 A_4		
C_4				−1 C_4	←→	←→	1.058 C_4
A_5					−1 A_5	1.018 A_5	
B_5					−1 B_5	←→	1.035 B_5
A_6						−1 A_6	1.018 A_6
Req'd Payments (in \$1,000s)	\$0	\$0	−\$250	\$0	−\$250	\$0	−\$300

FIGURE 3.34

Cash-flow summary table for Taco-Viva's investment opportunities

Min Total Initial Investment Beginning of Month 1

$$A_1 + B_1 + C_1 + D_1$$

S.t

Total \$ Amount Matured at Begining of M_i = Total \$ amount Reinvested at Beginning of M_i + Payment Due at Beginning of Month i

Cash Flow at Beginning of M_2 $\quad 1.018A_1 - A_2 = \phi$

— M_3 $\quad 1.035B_1 + 1.018A_2 - A_3 - B_3 = 250$

— M_4 $\quad 1.058C_1 + 1.018A_3 - A_4 - C_4 = \phi$

— M_5 $\quad 1.035B_3 + 1.018A_4 - A_5 - B_5 = 250$

— M_6 $\quad 1.018A_5 - A_6 = \phi$

— M_7 $\quad 1.11D_1 + 1.058C_4 + 1.035B_5 + 1.018A_6 = 300$

$A_i, B_i, C_i, D_i, \geq \phi$ For all i.

Account for Risk

Investment	A,	B,	C,	D
Risk Rating	1,	3,	8,	6

S.T. Weighted average Risk $\leq$ 5

$$\frac{1A_1 + 3B_1 + 8C_1 + 6D_1}{A_1 + B_1 + C_1 + D_1} \leq 5 \qquad \text{month 1}$$

$$\frac{1A_2 + 3B_1 + 8C_1 + 6D_1}{A_2 + B_1 + C_1 + D_1} \leq 5 \qquad \text{month 2}$$

$$\frac{1A_3 + 3B_3 + 8C_1 + 6D_1}{A_3 + B_3 + C_1 + D_1} \leq 5 \qquad \text{month 3}$$

$$\frac{1A_3 + 3B_3 + 8C_4 + 6D_1}{A_4 + B_3 + C_4 + D_1} \leq 5 \qquad \text{month 4}$$

$$\frac{1A_5 + 3B_5 + 8C_4 + 6D_1}{A_5 + B_5 + C_4 + D_1} \leq 5 \qquad \text{month 5}$$

$$\frac{1A_6 + 3B_5 + 8C_4 + 6D_1}{A_6 + B_5 + C_4 + D_1} \leq 5 \qquad \text{month 6}$$

Reorganize into standard form

$1A_1 + 3B_1 + 8C_1 + 6D_1 \leq 5(A_1 + B_1 + C_1 + D_1)$ or

$-4A_1 - 2B_1 + 3C_1 + D_1 \leq \phi$ month 1

$-4A_2 - 2B_1 + 3C_1 - D_1 \leq \phi$ — 2

$-4A_3 - 2B_3 + 3C_1 + D_1 \leq \phi$ — 3

$-4A_4 - 2B_3 + 3C_4 + D_1 \leq \phi$ — 4

$-4A_5 - 2B_5 + 3C_4 + D_1 \leq \phi$ — 5

$-4A_6 - 2B_5 + 3C_4 + D_1 \leq \phi$ — 6