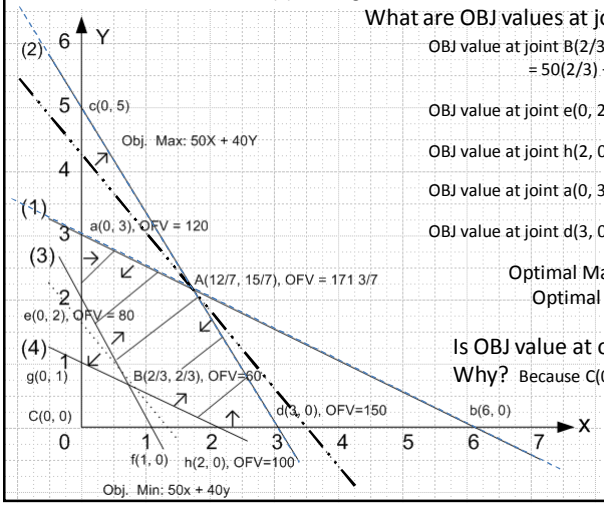
LP Sensitivity Analysis: Solutions and OBJ Values versus Optimal Solution and Optimal OBJ Value

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

What is the new feasible region?
a, e, B, h, d, A and a form feasible region

What is the value of Joint B(X, Y)? B(2/3, 2/3)



What are OBJ values at joints e, ,h and B?

OBJ value at joint B(2/3, 2/3) =
= 50(2/3) + 40(2/3) = 60

OBJ value at joint e(0, 2) = 40(2) = 80

OBJ value at joint h(2, 0) = 50(2) = 100

OBJ value at joint a(0, 3) = 40(3) = 120

OBJ value at joint d(3, 0) = 50(3) = 150

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

Is OBJ value at origin C(0, 0) feasible? No,
Why? Because C(0, 0) is outside of feasible region

Excel@ Solver Reports: Answer, Sensitivity, and Limits to Max OBJ 50X + 40Y

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Microsoft Excel 12.0 Answer Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM
Result: Solver found a solution. All constraints and optimality conditions are satisfied.
Engine: Standard LP Simplex
Solution Time: 00 Seconds
Iterations: 5
Subproblems: 0
Incumbent Solutions: 0

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	171.4285714

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	0	1 5/7
$C$3	Units to make Y	0	2 1/7

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	6	$D$7<=$F$7	Binding	0
$D$8	Raw Material Units Used	15	$D$8<=$F$8	Binding	0
$D$9	Customer v Demand Used	6	$D$9>=$F$9	Not Binding	4
$D$10	Customer w Demand Used	5 4/7	$D$10>=$F$10	Not Binding	3.571428571

Microsoft Excel 12.0Limits Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM

Cell	Name	Target Value
$D$4	Unit Profits Total Profit	171 3/7

Cell	Name	Adjustable Value	Lower Limit	Target Result	Upper Limit	Target Result
$B$3	Units to make X	1 5/7	0	85 5/7	1 5/7	171 3/7
$C$3	Units to make Y	2 1/7	1/7	91 3/7	2 1/7	171 3/7

Let X=0, OFV=0+40*2 1/7 = 85 5/7
Let Y=1/7, OFV=50*1 5/7+40*1/7=91 3/7

Slacks:
1) Slack for a constraint = value of (LHS – RHS)
2) Slack for binding constraints = zero
3) Slack for not binding constraint > zero
4) Slack for non negative decision variable = amount it exceeded its lower bounds of zero

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM

Target Cell (Max)

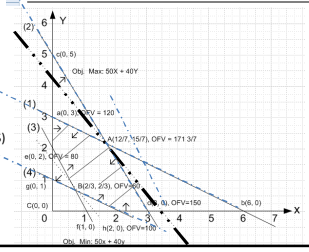
Cell	Name	Final Value
$D$4	Unit Profits Total Profit	171.4285714

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	1 5/7	0	50	16.66666667	30
$C$3	Units to make Y	2 1/7	0	40	60	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	6	7 1/7	6	4	3
$D$8	Raw Material Units Used	15	8 4/7	15	15	6
$D$9	Customer v Demand Used	6	0	2	4	1E+30
$D$10	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30



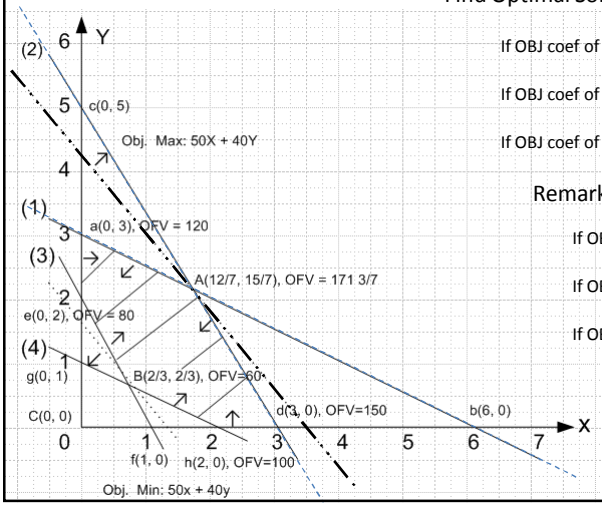
Visualizing Optimal Solution as OBJ Coefficients of X Changed

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

OBJ Max: 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,



If OBJ coef of X = 35,

If OBJ coef of X = 20,

If OBJ coef of X = 5,

Remarks:

If OBJ coef of X = 60,

If OBJ coef of X = 66 2/3,

If OBJ coef of X = 75,

Remarks:

Visualizing Optimal Solution as OBJ Coefficients of X Decreased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

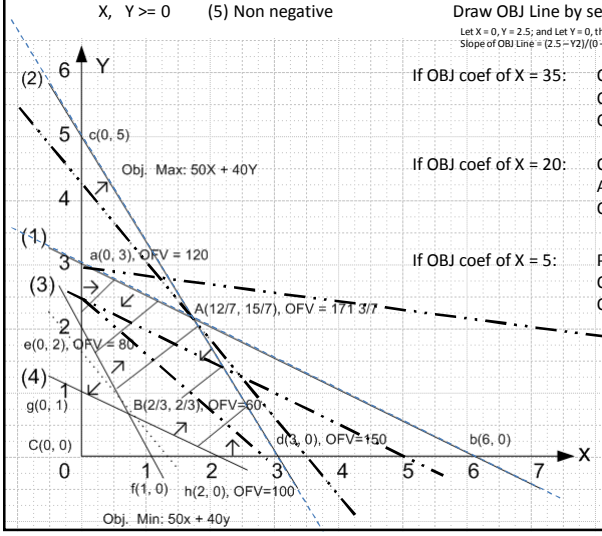
Optimal solution A(12/7, 15/7)

Optimal OFV (Max): 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

Draw OBJ Line by setting: 50X+40Y = 100

Let X = 0, Y = 2.5; and Let Y = 0, then X = 2, 2 6/7, 5, and 20, respectively
Slope of OBJ Line = (2.5 – Y2)/(0 – X2) = - 1.25, - 7/8, -0.5, and – 1/8, respectively



If OBJ coef of X = 35: OBJ Line counter clockwise & flatter,
OS no change at A(12/7, 15/7)
OFV = 35*12/7 + 40*15/7 =145 5/7

If OBJ coef of X = 20: OBJ Line overlap Line (1),
Alternative OSs b/t points a and A on Line (1)
OFV = 20*12/7+40*15/7=20*0+40*3 = 120

If OBJ coef of X = 5: Point A is not OS any more,
OS at Point a
OFV = 5*0 + 40*3 = 120

Remarks: OBJ coef of X can vary within a range, i.e., OBJ Line can turn at OS point A, and swing b/t points d and b to keep point A optimal

Visualizing Optimal Solution as OBJ Coefficients of X Increased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

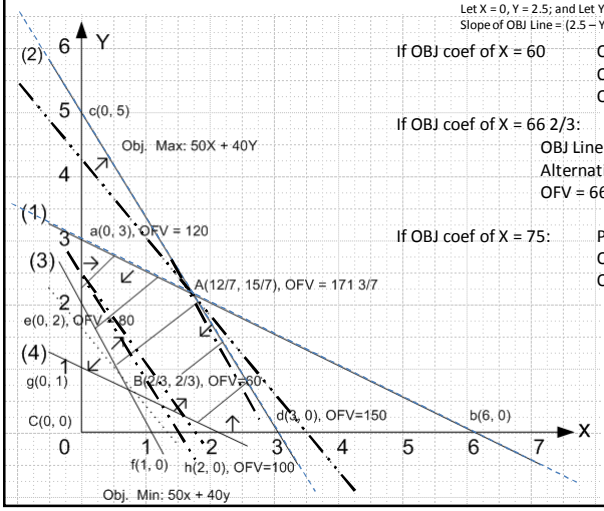
Optimal solution A(12/7, 15/7)

Optimal OFV (Max): 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

Draw OBJ Line by setting: 50X+40Y = 100

Let X = 0, Y = 2.5; and Let Y = 0, then X = 2, 1 2/3, 1.5, and 1 1/3, respectively
Slope of OBJ Line = (2.5 – Y2)/0 – X1) = -1.25, -1.5, - 1.667, and – 1 7/8, respectively



If OBJ coef of X = 60 — OBJ Line turns clockwise & steeper,
OS no change at A(12/7, 15/7)
OFV = 60*12/7 + 40*15/7 =188 4/7

If OBJ coef of X = 66 2/3:
OBJ Line overlap Line (2),
Alternative OSs b/t points A and d on Line (2)
OFV = 66 2/3*12/7+40*15/7=66 2/3*3+40*0 = 200

If OBJ coef of X = 75: — Point A is not OS any more,
OS at Point d(3, 0)
OFV = 75*3 + 40*0 = 225

Remarks: OBJ coef of X can vary within a range, i.e., OBJ Line can turn at OS point A, and swing b/t points d and b to keep point A optimal

LP Sensitivity Analysis of OBJ Coefficents (OFC)

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]LPSolver
Report Created: 03/22/2008 11:52:08 AM

Target Cell (Max)

Cell	Name	Final Value
$D$159	Unit Profits Total Profit	171.4285714

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$158	Units to make X	1 5/7	0	50	16.66666667	30
$C$158	Units to make Y	2 1/7	0	40	60	10

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Sensitivity Analysis of Adjustable Cells for Objective function:

The X coefficient of 50 varies from (50 – 30) to (50 + 16.667), the OBJ line turns at the optimal solution, thus the original optimal solution of (12/7, 15/7) will not change, the feasible region will not change, but the optimal objective function value will change due to the change in X coefficient.

When OBJ function line is overlap with Line (1) at the vertex (12/7, 15/7), it leads to unlimited number of **alternative optimal solutions** from the line segment between vertexes (0, 3) and (12/7, 15/7), where the X coefficient of 50 decreases to its lower limit of 20 = 50 – 30, or the Y coefficient of 40 increases to its upper limit of 100 = 40 + 60.

When the objective function line is overlap with Line (2) at the vertex (12/7, 15/7), it leads to unlimited number of **alternative optimal solutions** from the line segment between vertexes (3, 0) and (12/7, 15/7), where X coefficient of 50 increases to its upper limit of 66 2/3 = 50 + 16 2/3, or Y coefficient of 40 decreases to its lower limit of 30 = 40 - 10.

The Reduced Cost for each variable equals to the per-unit amount that the variable contributes to the objective function value, minus the per-unit value of the resources it consumes at their shadow prices. For example:
Reduced cost for X: 50 – 7 1/7 x 1 – 8 4/7 x 5 = 0 — 1X + 2Y <= 6 (1)
Reduced cost for Y: 40 – 7 1/7 x 2 – 8 4/7 x 3 = 0 — 5X + 3Y <= 15 (2)

Read the pages 148 and 149 for more discussions on the shadow prices and reduced cost.
Read the pages 150 and 151 for discussions on degeneracy and changes in more than on objective coefficients.

Visualizing Optimal Solution as OBJ Coefficient of Y Changed

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

OBJ Max: 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

When OFC of Y Decreased:

If OBJ coef of Y = 35,

If OBJ coef of Y = 30,

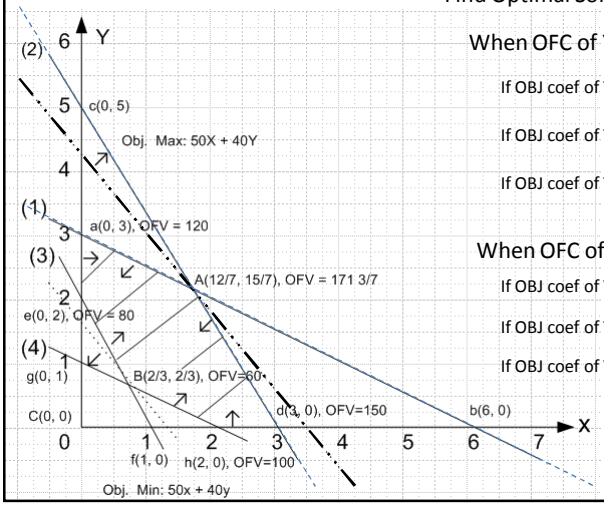
If OBJ coef of Y = 20,

When OFC of Y Increased:

If OBJ coef of Y = 70,

If OBJ coef of Y = 100,

If OBJ coef of Y = 140,



Visualizing Optimal Solution as OBJ Coefficients of Y Decreased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Optimal solution A(12/7, 15/7)

Optimal OFV (Max): 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

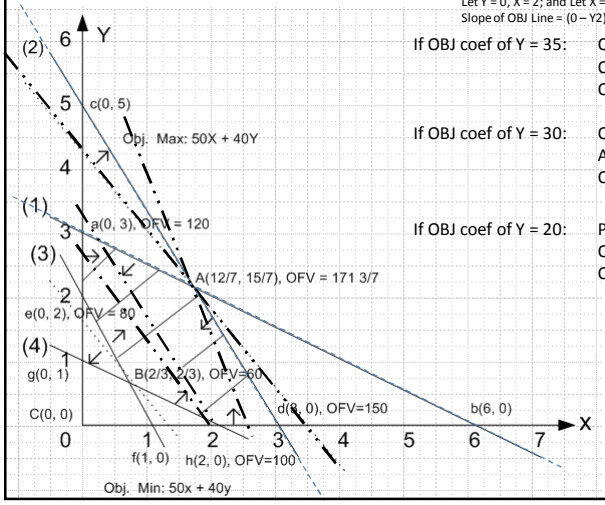
Draw OBJ Line by setting: 50X+40Y = 100

Let Y = 0, X = 2; and Let X = 0, then Y = 2 6/7, 3 1/3 and 5, respectively
Slope of OBJ Line = (0 – Y2)/(2 –0) = - 1 3/7, - 1 2/3, and – 2, respectively

If OBJ coef of Y = 35: OBJ Line counter clockwise & flatter,
OS no change at A(12/7, 15/7)
OFV = 50*12/7 + 35*15/7 =160 5/7

If OBJ coef of Y = 30: OBJ Line overlap Line (1),
Alternative OSs b/t points a and A on Line (1)
OFV = 50*12/7+30*15/7=50*3+30*0 = 150

If OBJ coef of Y = 20: Point A is not OS any more,
OS at Point d
OFV = 50*3 + 20*0 = 150



Remarks: OBJ coef of Y can vary within a range, i.e., OBJ Line can turn at OS point A, and swing b/t points c and a to keep point A optimal

Visualizing Optimal Solution as OBJ Coefficients of Y Increased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

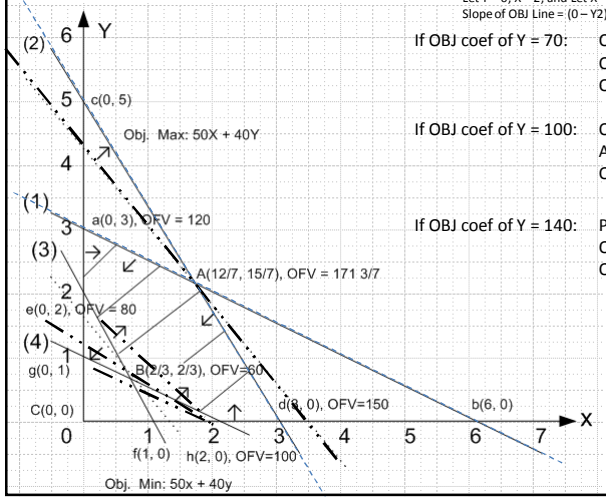
Optimal solution A(12/7, 15/7)

Optimal OFV (Max): 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

Draw OBJ Line by setting: 50X+40Y = 100

Let Y = 0, X = 2; and Let X = 0, then Y = 1 3/7, 1 and 5/7, respectively
Slope of OBJ Line = (0 – Y2)/(2 – 0) = - 5/7, - ½, and – 1/3, respectively



If OBJ coef of Y = 70: OBJ Line counter clockwise & flatter,
OS no change at A(12/7, 15/7)
OFV = 50*12/7 + 70*15/7 =235 5/7

If OBJ coef of Y = 100: OBJ Line overlap Line (1),
Alternative OSs b/t points a and A on Line (1)
OFV = 50*12/7+100*15/7=50*0+100*3 = 300

If OBJ coef of Y = 140: Point A is not OS any more,
OS at Point a
OFV = 50*0 + 140*3 = 420

Remarks: OBJ coef of Y can vary within a range, i.e., OBJ Line can turn at OS point A, and swing b/t points c and a to keep point A optimal

LP Sensitivity Analysis of OBJ Coefficents of Y

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

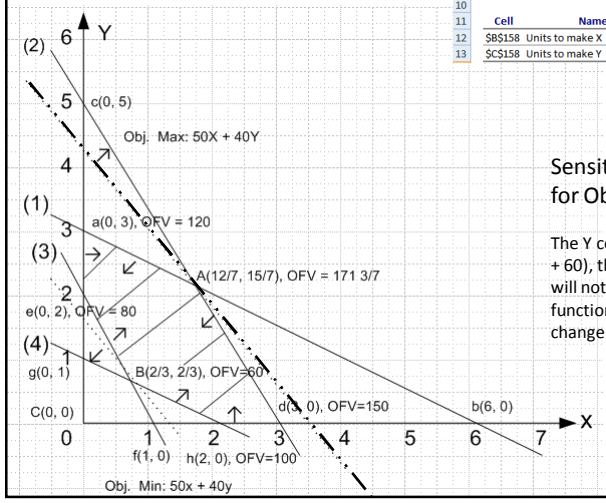
Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]LPSolver
Report Created: 03/22/2008 11:52:08 AM

Target Cell (Max)

Cell	Name	Final Value
$D$159	Unit Profits Total Profit	171.4285714

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$158	Units to make X	1 5/7	0	50	16.66666667	30
$C$158	Units to make Y	2 1/7	0	40	60	10



Sensitivity Analysis of Adjustable Cells for Objective function:

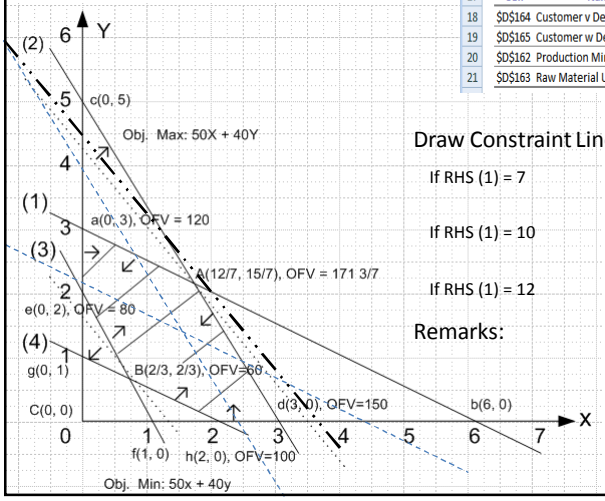
The Y coefficient of 40 varies from (40 – 10) to (40 + 60), the optimal solution at Point A(12/7, 15/7) will not change, but the optimal objective function value (OFV) will change due to the change in Y coefficient.

LP Sensitivity Analysis –

Change of RHS Values of Constraint (1)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

	A	B	C	D	E	F	G	H
1	Microsoft Excel 12.0 Sensitivity Report							
2	Worksheet: [LP.xlsx]LPSolver							
3	Report Created: 03/22/2008 11:52:08 AM							
15	Constraints							
16				Final	Shadow	Constraint	Allowable	Allowable
17		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
18		$D$164	Customer v Demand Used	6	0	2	4	1E+30
19		$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
20		$D$162	Production Minutes Used	6	7 1/7	6	4	3
21		$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (1),

If RHS (1) = 7

If RHS (1) = 5

If RHS (1) = 10

If RHS (1) = 3

If RHS (1) = 12

If RHS (1) = 2

Remarks:

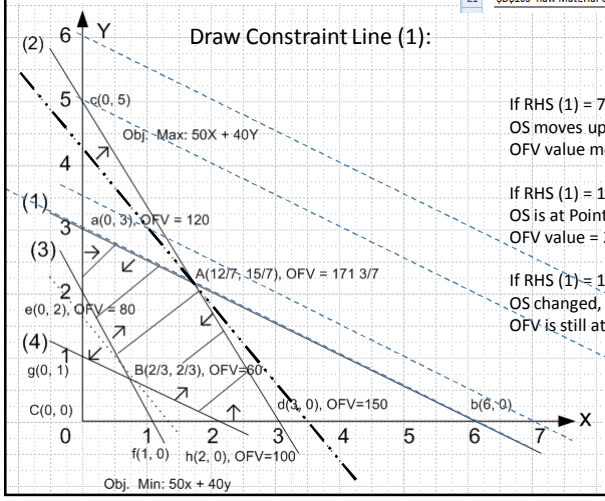
Remarks:

LP Sensitivity Analysis –

RHS Values of Constraint (1) Increased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

15	Constraints							
16				Final	Shadow	Constraint	Allowable	Allowable
17		Cell	Name	Value	Price	R.H. Side	Increase	Decrease
18		$D$164	Customer v Demand Used	6	0	2	4	1E+30
19		$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
20		$D$162	Production Minutes Used	6	7 1/7	6	4	3
21		$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (1):

If RHS (1) = 7, Line(1) moves up,
OS moves up
OFV value moves up = 178 4/7 = 171 3/7 + 7 1/7*1

If RHS (1) = 10, Line (1) & Line (2) joint at Point c
OS is at Point c(0, 5)
OFV value = 200 = 171 3/7 + 4*7 1/7=40*5

If RHS (1) = 12, forms new feasible region
OS changed, new Line (1) becomes redundant
OFV is still at Point c(0,5) = 200

Remarks:

RHS=7: Let X=0, Y=3.5 or (0, 3.5) & let Y=0, X=7 or (7, 0)
RHS=10: Let X=0, Y=5 or (0, 5) & let Y=0, X=10 or (10,0)
RHS=12, Let X=0, Y=6 or (0,6) & let Y=0, X=12 or (12,0)

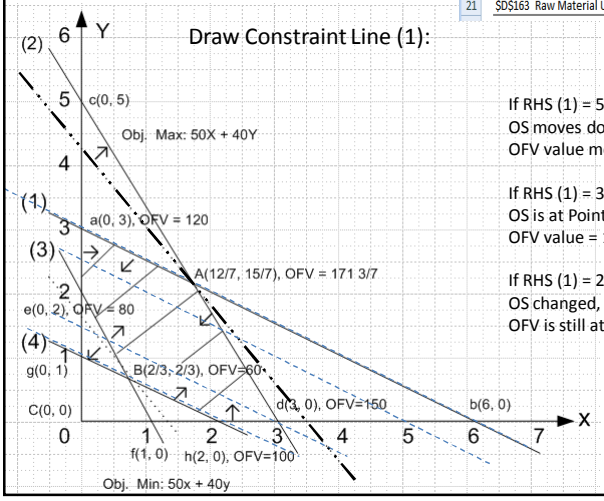
LP Sensitivity Analysis –

RHS Values of Constraint (1) Decreased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6



If RHS (1) = 5, Line(1) moves down,
OS moves down
OFV value moves down = 164 2/7 = 171 3/7 - 7 1/7*1

If RHS (1) = 3, Line (1) & Line (2) joint at Point d
OS is at Point d(3, 0)
OFV value = 150 = 171 3/7- 3*7 1/7=50*3

If RHS (1) = 2, forms new feasible region on Line (4)
OS changed, Line (2) becomes redundant
OFV is still at Point h(2,0) = 100

Remarks:

RHS=5: Let X=0, Y=2.5 or (0, 2.5) & let Y=0, X=5 or (5, 0)
RHS=3: Let X=0, Y=1.5 or (0, 1.5) & let Y=0, X=3 or (3,0)
RHS=2, Let X=0, Y=1 or (0,1) & let Y=0, X=2 or (2,0)

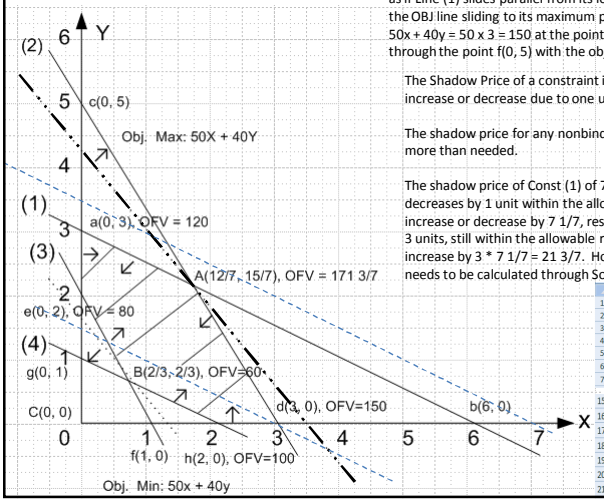
LP Sensitivity Analysis: Change of RHS Values of Constraint (1)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Changing RHS values within the given ranges lead to the changes in the optimal solution point (X, Y) and thus the value of the optimal OBJ value.

RHS of constraint (1) varies from the value of (6 – 3) to the value of (6 + 4), Optimal solution is still the vertex of the lines (1) and (2), but the values of Optimal solution or vertex (X, Y) are changed, thus the change in the value of the optimal OBJ value.

These two situations are best depicted by the graph below with two dot lines each referring to the Constraint (1) at its lower or upper limits. We may view the situation as if Line (1) slides parallel from its lower position passing through point c(3, 0) with the OBJ line sliding to its maximum possible position along the way and its value of 50x + 40y = 50 x 3 = 150 at the point c(3, 0), to its upper limit position passing through the point f(0, 5) with the objective function value of 50X + 40Y = 200.



The Shadow Price of a constraint is the amount of the objective function value to increase or decrease due to one unit of change in the RHS value of that constraint.

The shadow price for any nonbinding constraint is always zero, because RHS value is more than needed.

The shadow price of Const (1) of 7 1/7 means if the RHS value of Const (1) increases or decreases by 1 unit within the allowable range (3, 10), the objective function value will increase or decrease by 7 1/7, respectively, i.e., if the RHS value of Const (1) increases by 3 units, still within the allowable range of (3, 10), the optimal objective function value will increase by 3 * 7 1/7 = 21 3/7. However, the optimal vertex (X, Y) has changed and needs to be calculated through Solver

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]LPSolver
Report Created: 03/22/2008 11:52:08 AM

Target Cell (Max)

Cell	Name	Final Value
$D$159	Unit Profits Total Profit	171.4285714

Constraints

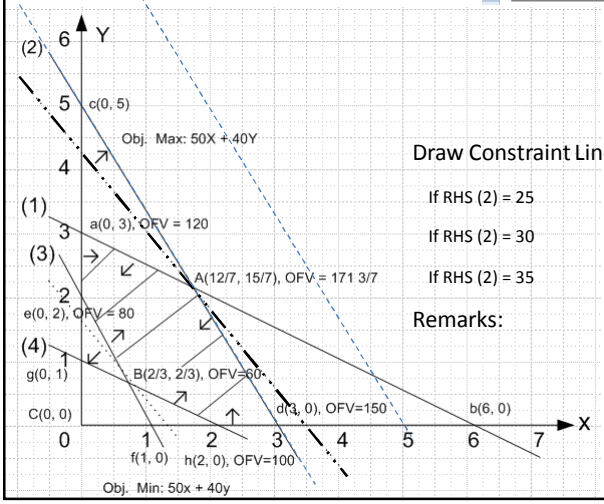
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6

LP Sensitivity Analysis: Change of RHS Values of Constraint (2)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

15 Constraints

	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
18	$D$164	Customer v Demand Used	6	0	2	4	1E+30
19	$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
20	$D$162	Production Minutes Used	6	7 1/7	6	4	3
21	$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (2),

If RHS (2) = 25

If RHS (2) = 30

If RHS (2) = 35

Remarks:

If RHS (2) = 10

If RHS (2) = 9

If RHS (2) = 5

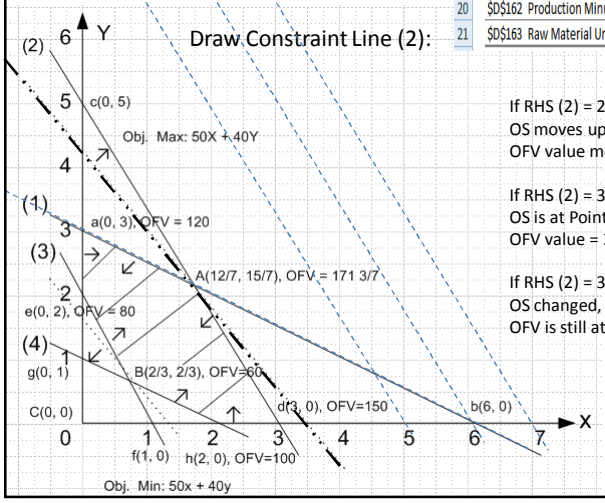
Remarks:

LP Sensitivity Analysis –
RHS Values of Constraint (2) Increased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

15 Constraints

	Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
18	$D$164	Customer v Demand Used	6	0	2	4	1E+30
19	$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
20	$D$162	Production Minutes Used	6	7 1/7	6	4	3
21	$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (2):

If RHS (2) = 25, Line(1) moves up,
OS moves up to Point (4 4/7, 5/7)
OFV value moves up = 257 1/7 = 171 3/7 + 8 4/7*10

If RHS (2) = 30, Line (1) & Line (2) joint at Point b
OS is at Point b(6, 0)
OFV value = 300 = 171 3/7 + 8 4/7*15 = 50*6

If RHS (2) = 35, forms new feasible region
OS changed, new Line (2) becomes redundant
OFV is still at Point b(6,0) = 300

Remarks:

RHS=25: Let X=0, Y=8 1/3 or (0, 8 1/3) & let Y=0, X=5 or (5, 0)
RHS=30: Let X=0, Y=10 or (0, 10) & let Y=0, X=6 or (6,0)
RHS=35, Let X=0, Y=11 2/3 or (0,11 2/3) & let Y=0, X=7 or (7,0)

LP Sensitivity Analysis –

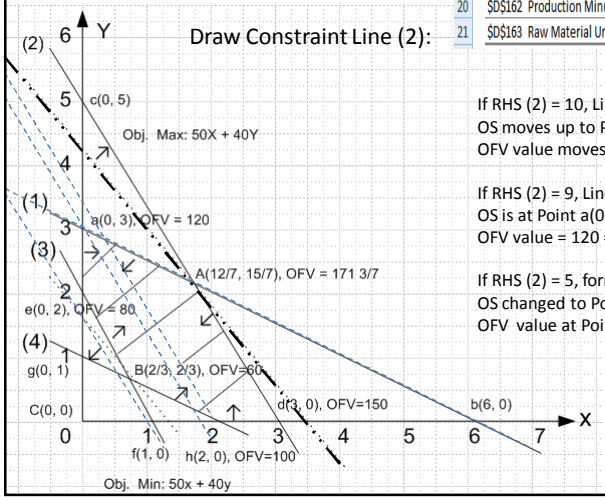
RHS Values of Constraint (2) Decreased

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6

Draw Constraint Line (2):



If RHS (2) = 10, Line(1) moves down,
OS moves up to Point (2/7, 2 6/7)
OFV value moves up = 128 4/7 = 171 3/7 - 8 4/7*5

If RHS (2) = 9, Line (1) & Line (2) joint at Point a
OS is at Point a(0, 3)
OFV value = 120 = 171 3/7 - 8 4/7*6 = 50*0+40*3

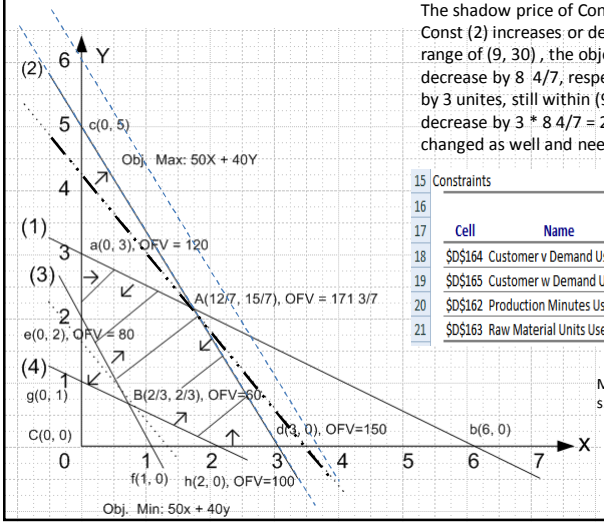
If RHS (2) = 5, forms new feasible region
OS changed to Point(1,0), new Line (2) becomes redundant
OFV value at Point(1,0) = 50

Remarks:

RHS=10: Let X=0, Y=3 1/3 or (0, 3 1/3) & let Y=0, X=2 or (2, 0)
RHS=9 Let X=0, Y=3 or (0, 3) & let Y=0, X=1 4/5 or (1 4/5,0)
RHS=5, Let X=0, Y=1 2/3 or (0,1 2/3) & let Y=0, X=1 or (1,0)

LP Sensitivity Analysis: Change of RHS Values of Constraint (2)

RHS of Const (2) varies from the value of (15 – 6) to the value of (15 + 15), the optimal solution is still the intersect of Lines (1) and (2), but the values of the optimal solution or vertex (x, y) are changed, thus the change in the value of the optimal objective function value from 120 at the point a(0, 3) to 300 at the point e(6, 0). These two situations are best depicted by the graph below with two dot lines each referring to Const (2) at its lower or upper limits.



The shadow price of Const (2) of 8 4/7 means if the RHS value of Const (2) increases or decreases by 1 unit within the allowable range of (9, 30) , the objective function value will increase or decrease by 8 4/7, respectively. If RHS value of Const (2) deceases by 3 unites, still within (9, 30), the objective function value will decrease by 3 * 8 4/7 = 25 5/7. However, the optimal vertex (X, Y) is changed as well and needs to be computed again with Solver.

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6

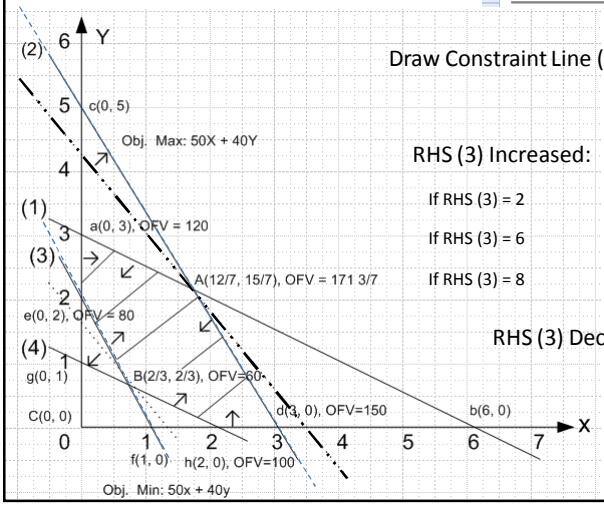
Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

LP Sensitivity Analysis: Change of RHS Values of Constraint (3)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (3):

RHS (3) Increased:

If RHS (3) = 2

If RHS (3) = 6

If RHS (3) = 8

RHS (3) Decreased:

LP Sensitivity Analysis: Change of RHS Values of Constraint (3)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Microsoft Excel 12.0 Answer Report

Constraints

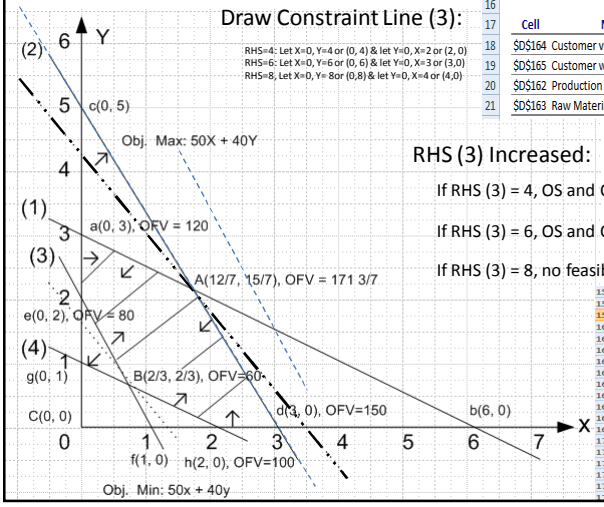
Cell	Name	Cell Value	Formula	Status	Slack
$D$164	Customer v Demand Used	6	$D$164>=$F$164	Not Binding	4
$D$165	Customer w Demand Used	5 4/7	$D$165>=$F$165	Not Binding	3.571428571
$D$162	Production Minutes Used	6	$D$162<=$F$162	Binding	0
$D$163	Raw Material Units Used	15	$D$163<=$F$163	Binding	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6

Draw Constraint Line (3):

RHS=4: Let X=0, Y=4 or (0, 4) & let Y=0, X=2 or (2, 0)
RHS=6: Let X=0, Y=6 or (0, 6) & let Y=0, X=3 or (3,0)
RHS=8, Let X=0, Y= 8or (0,8) & let Y=0, X=4 or (4,0)



RHS (3) Increased:

RHS (3) Decreased:

If RHS (3) = 4, OS and OFV no change = 171 3/7

If RHS (3) = 6, OS and OFV no change = 171 3/7

If RHS (3) = 8, no feasible region

	X	Y	
Units to make	0	3	Total Profit
Unit Profits	$50	$40	120

Constraints	X	Y	LHS Used		RHS Available
Production Minutes	1	2	6	<=	6
Raw Material Units	5	3	9	<=	15
Customer v Demand	1	2	6	>=	8
Customer w Deman	2	1	3	>=	2

Solver Results
Solver could not find a feasible solution.
Keep Solver Solution
Restore Original Values
Reports: Feasibility, Feasibility-Bounds
Return to Solver Parameters Dialog
Outline Reports
OK | Cancel | Save Scenario... | Help

LP Sensitivity Analysis: Change of RHS Values of Constraint (4)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Microsoft Excel 12.0 Answer Report

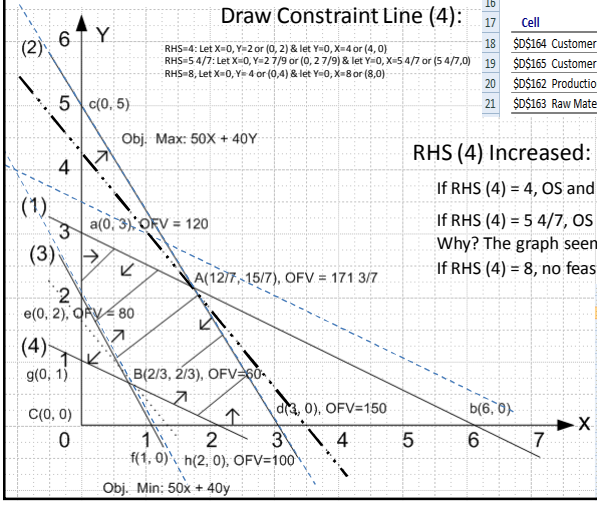
Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$164	Customer v Demand Used	6	$D$164>=$F$164	Not Binding	4
$D$165	Customer w Demand Used	5 4/7	$D$165>=$F$165	Not Binding	3.571428571
$D$162	Production Minutes Used	6	$D$162<=$F$162	Binding	0
$D$163	Raw Material Units Used	15	$D$163<=$F$163	Binding	0

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6

Draw Constraint Line (4):



RHS (4) Increased:

If RHS (4) = 4, OS and OFV no change = 171 3/7

If RHS (4) = 5 4/7, OS and OFV no change = 171 3/7,
Why? The graph seems NOT support the result.

If RHS (4) = 8, no feasible region

RHS (4) Decreased:

		X	Y		
Units to make		0	3	Total Profit	
Unit Profits		$50	$40	120	
				LHS	RHS
Constraints		X	Y	Used	Available
Production Minutes		1	2	6	<= 6
Raw Material Units		5	3	9	<= 15
Customer v Demand		1	2	6	>= 8
Customer w Deman		2	1	3	>= 2

Solver Results
Solver could not find a feasible solution.
Keep Solver Solution
Restore Original Values
Reports: Feasibility, Feasibility-Bounds
Return to Solver Parameters Dialog
Outline Reports
OK | Cancel | Save Scenario... | Help

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]LPSolver
Report Created: 03/22/2008 21:31:33 PM

Target Cell (Max)

Cell	Name	Final Value
$D$159	Unit Profits Total Profit	171.4285714

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$158	Units to make X	1 5/7	0	50	16.66666667	30
$C$158	Units to make Y	2 1/7	0	40	60	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$164	Customer v Demand Used	6	0	2	4	1E+30
$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
$D$162	Production Minutes Used	6	7 1/7	6	4	3
$D$163	Raw Material Units Used	15	8 4/7	15	15	6

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

		X	Y		
Units to make		1 5/7	1 3/7	Total Profit	
Unit Profits		$50	$40	164 2/7	
				LHS	RHS
Constraints		X	Y	Used	Available
Production Minutes		1	2	5	<= 6
Raw Material Units		5	3	15	<= 15
Customer v Demand		1	2	5	>= 2
Customer w Deman		2	1	5 5/7	>= 5 5/7

		X	Y		
Units to make		3	0	Total Profit	
Unit Profits		$50	$40	150	
				LHS	RHS
Constraints		X	Y	Used	Available
Production Minutes		1	2	3	<= 6
Raw Material Units		5	3	15	<= 15
Customer v Demand		1	2	3	>= 2
Customer w Deman		2	1	6	>= 6

Solver Results
Solver found a solution. All constraints and optimality conditions are satisfied.
Keep Solver Solution
Restore Original Values
Reports: Answer, Sensitivity, Limits
Return to Solver Parameters Dialog
Outline Reports
OK | Cancel | Save Scenario... | Help

		X	Y		
Units to make		3	0	Total Profit	
Unit Profits		$50	$40	150	
				LHS	RHS
Constraints		X	Y	Used	Available
Production Minutes		1	2	3	<= 6
Raw Material Units		5	3	15	<= 15
Customer v Demand		1	2	3	>= 2
Customer w Deman		2	1	6	>= 6 1/7

Solver Results
Solver could not find a feasible solution.
Keep Solver Solution
Restore Original Values
Reports: Feasibility, Feasibility-Bounds
Return to Solver Parameters Dialog
Outline Reports
OK | Cancel | Save Scenario... | Help

Excel@ Solver Reports: Answer, Sensitivity, and Limits to Max OBJ 50X + 40Y

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Microsoft Excel 12.0 Answer Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM
Result: Solver found a solution. All constraints and optimality conditions are satisfied.
Engine: Standard LP Simplex
Solution Time: 00 Seconds
Iterations: 5
Subproblems: 0
Incumbent Solutions: 0

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	171.4285714

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	0	1 5/7
$C$3	Units to make Y	0	2 1/7

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	6	$D$7<=$F$7	Binding	0
$D$8	Raw Material Units Used	15	$D$8<=$F$8	Binding	0
$D$9	Customer v Demand Used	6	$D$9>=$F$9	Not Binding	2
$D$10	Customer w Demand Used	5 4/7	$D$10>=$F$10	Not Binding	3.571428571

Microsoft Excel 12.0Limits Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM

Cell	Name	Target Value
$D$4	Unit Profits Total Profit	171 3/7

Cell	Name	Adjustable Value	Lower Limit	Target Result	Upper Limit	Target Result
$B$3	Units to make X	1 5/7	0	85 5/7	1 5/7	171 3/7
$C$3	Units to make Y	2 1/7	1/7	91 3/7	2 1/7	171 3/7

Let X=0, OFV=0+40*2 1/7 = 85 5/7
Let Y=1/7, OFV=50*1 5/7+40*1/7=91 3/7

Microsoft Excel 12.0 Sensitivity Report
Worksheet: [LP.xlsx]Ex1
Report Created: 03/16/2008 16:33:08 PM

Target Cell (Max)

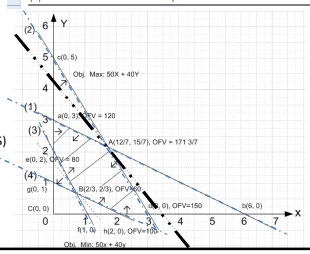
Cell	Name	Final Value
$D$4	Unit Profits Total Profit	171.4285714

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	1 5/7	0	50	16.66666667	30
$C$3	Units to make Y	2 1/7	0	40	60	10

Constraints

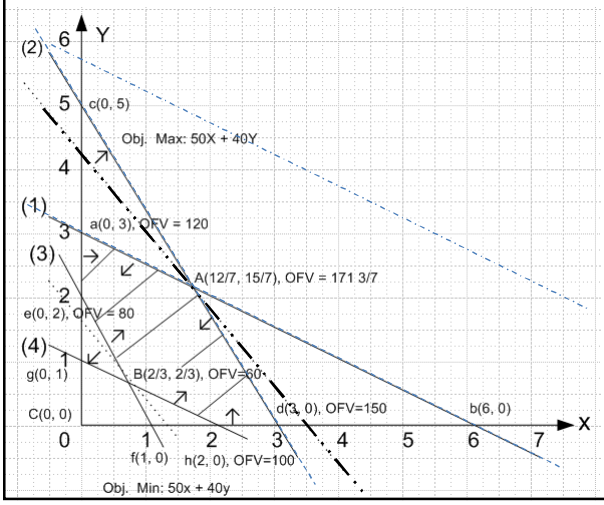
Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	6	7 1/7	6	4	3
$D$8	Raw Material Units Used	15	8 4/7	15	15	6
$D$9	Customer v Demand Used	6	0	2	4	1E+30
$D$10	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30



Slacks:
1) Slack for a constraint = value of (LHS – RHS)
2) Slack for binding constraints = zero
3) Slack for not binding constraint > zero
4) Slack for non negative decision variable = amount it exceeded its lower bounds of zero

Special Cases in LP Modeling

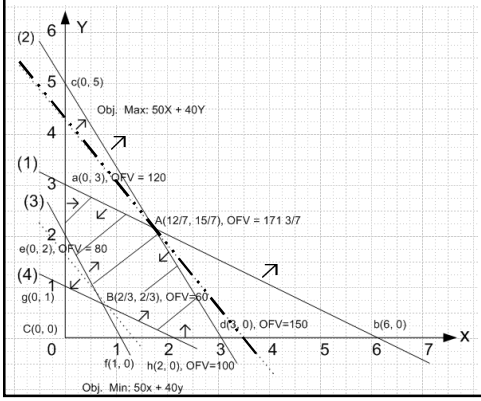
- Alternative optimal solutions – more than one optimal vertex exist.
- Redundant constraints (R)



Special Cases in LP Modeling

Unbounded Solutions

Max: 50X + 40Y Profit
s.t. 1X + 2Y >= 6 (1) Production time in minutes
5X + 3Y >= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative



Infeasible solutions