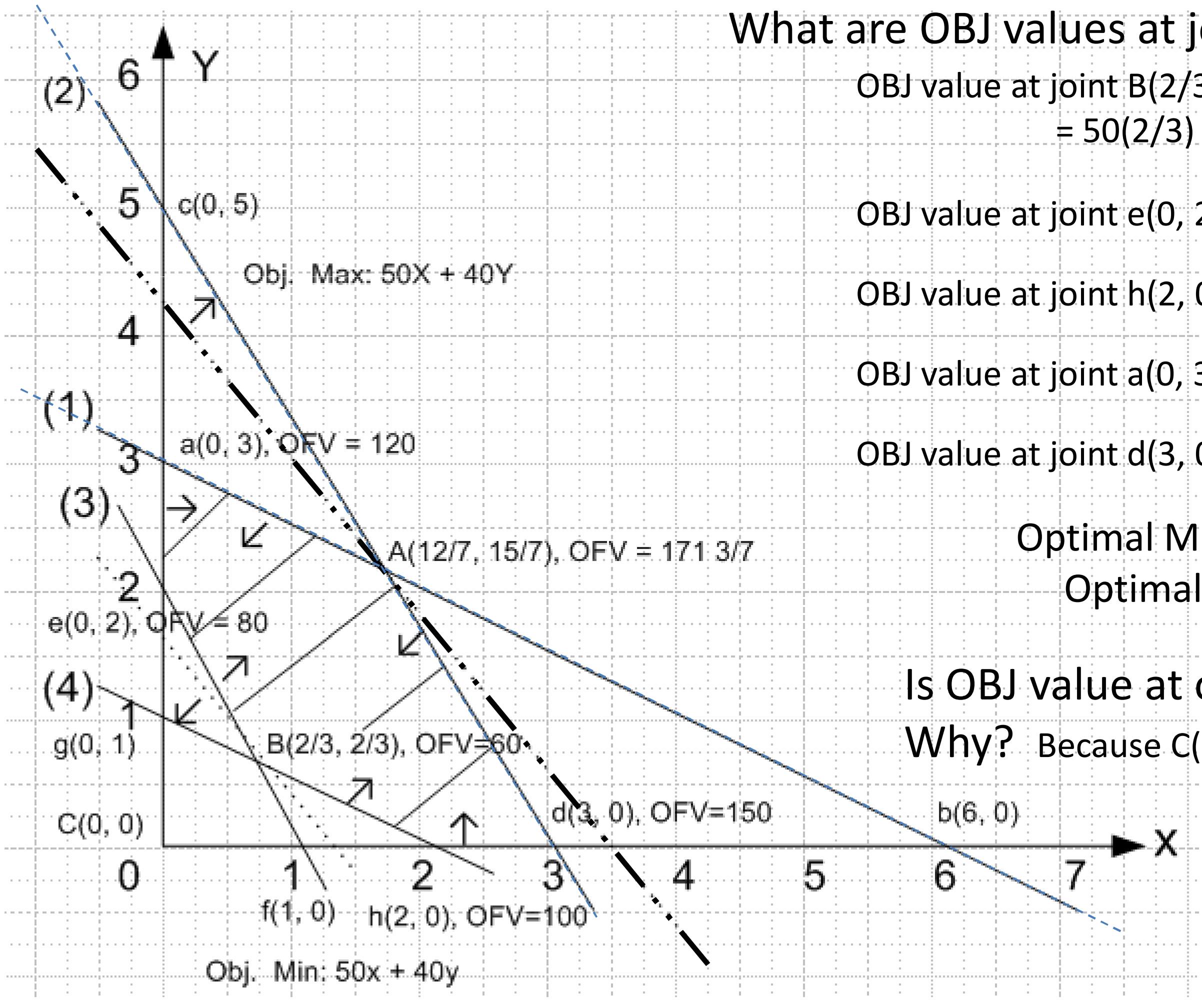
LP Sensitivity Analysis

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer v demand
X + 2Y >= 2 (4) Customer w demand
X, Y >= 0 (5) Non negativity

What is the new feasible region?
a, e, B, h, d, A and a form feasible region

What is the value of Joint B(X, Y)? B(2/3, 2/3)



What are OBJ values at joints e, ,h and B?

OBJ value at joint B(2/3, 2/3) =
= 50(2/3) + 40(2/3) = 60

OBJ value at joint e(0, 2) = 40(2) = 80

OBJ value at joint h(2, 0) = 50(2) = 100

OBJ value at joint a(0, 3) = 40(3) = 120

OBJ value at joint d(3, 0) = 50(3) = 150

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

Is OBJ value at origin C(0, 0) feasible? No,
Why? Because C(0, 0) is outside of feasible region

Max: $50X + 40Y$ Profit

s.t. $1X + 2Y <= 6$ (1) Production time in minutes

$5X + 3Y <= 15$ (2) Raw materials in units

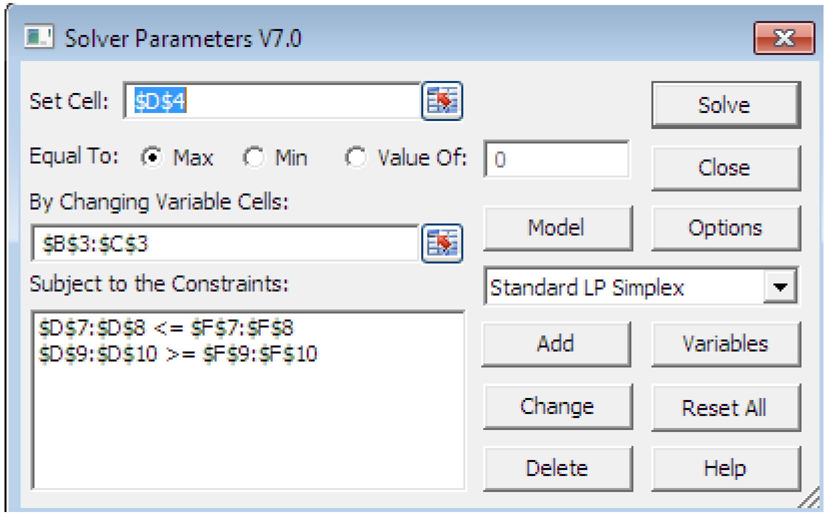
$2X + Y >= 2$ (3) Customer v demand

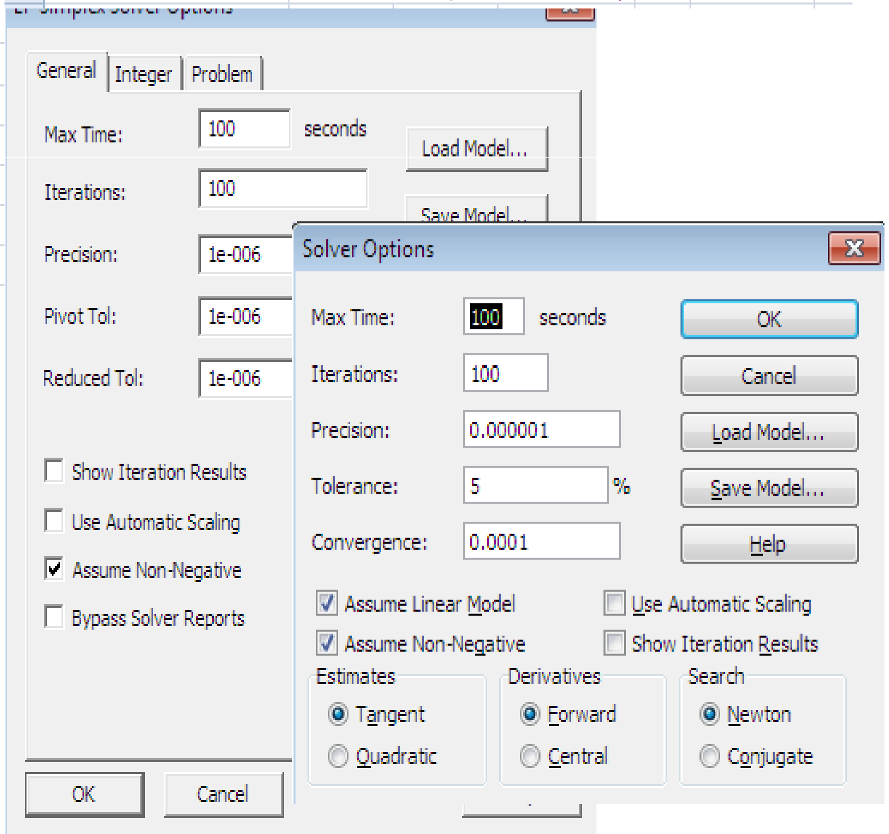
$X + 2Y >= 2$ (4) Customer w demand

$X, Y >= 0$ (5) Non negativity

	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C$3)		
3	Units to make	0	0	Total Profit		
4	Unit Profits	$50	$40	0		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	0	<=	6
8	Raw Material Units	5	3	0	<=	15
9	Customer v Demand	2	1	0	>=	2
10	Customer w Demand	1	2	0	>=	2
11		=SUMPRODUCT(B7:C7,$B$3:$C$3)				

	A	B	C	D	E	F
1	Maximize Profit					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C		
3	Units to make	1 5/7	2 1/7	Total Profit		
4	Unit Profits	$50	$40	171 3/7		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	6	<=	6
8	Raw Material Units	5	3	15	<=	15
9	Customer v Demand	2	1	5 4/7	>=	2
10	Customer w Demand	1	2	6	>=	2





Max: $50X + 40Y$ Profit

s.t. $1X + 2Y <= 6$ (1)

$5X + 3Y <= 15$ (2)

$2X + Y >= 2$ (3)

$X + 2Y >= 2$ (4)

$X, Y >= 0$ (5)

Microsoft Excel 12.0 Sensitivity Report

Worksheet: [Copy of LP.xlsx]Ex1

Report Created: 02/27/2010 18:27:03 PM

Target Cell (Max)

Cell	Name	Final Value
$D$4	Unit Profits Total Profit	171 3/7

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	1 5/7	0	50	16 2/3	30
$C$3	Units to make Y	2 1/7	0	40	60	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	6	7 1/7	6	4	3
$D$8	Raw Material Units Used	15	8 4/7	15	15	6
$D$9	Customer v Demand Used	5 4/7	0	2	3 4/7	1E+30
$D$10	Customer w Demand Used	6	0	2	4	1E+30

Microsoft Excel 12.0 Answer Report

Worksheet: [Copy of LP.xlsx]Ex1

Report Created: 02/27/2010 18:27:03 PM

Result: Solver found a solution. All const

Engine: Standard LP Simplex

Solution Time: 00 Seconds

Iterations: 5

Subproblems: 0

Incumbent Solutions: 0

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	171 3/7

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	0	1 5/7
$C$3	Units to make Y	0	2 1/7

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	6	$D$7<=$F$7	Binding	0
$D$8	Raw Material Units Used	15	$D$8<=$F$8	Binding	0
$D$9	Customer v Demand Used	5 4/7	$D$9>=$F$9	Not Binding	3 4/7
$D$10	Customer w Demand Used	6	$D$10>=$F$10	Not Binding	4

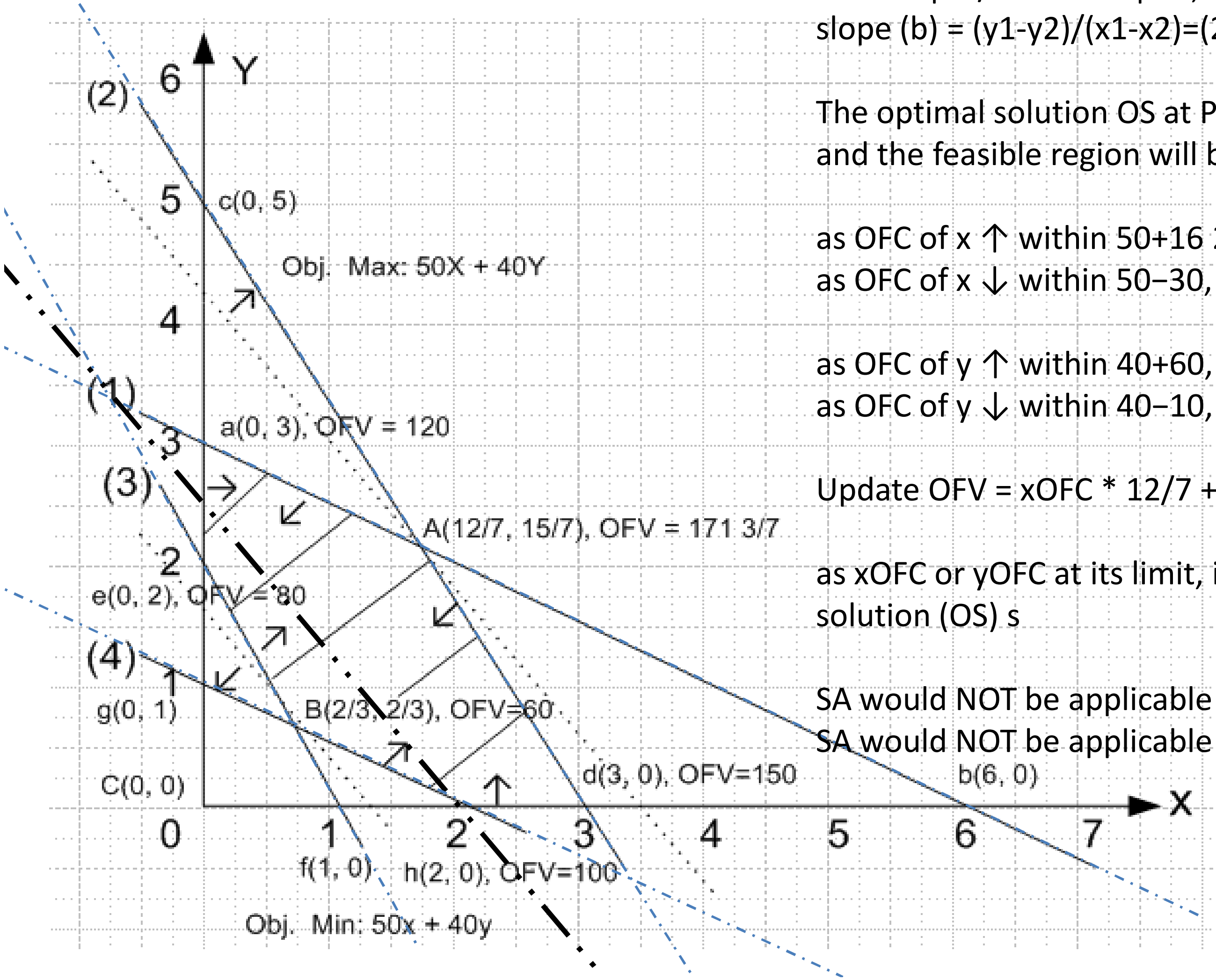
Slacks:

1) Slack for a constraint = value of (LHS – RHS)
2) Slack for binding constraints = zero
3) Slack for not binding constraint > zero
4) Slack for non negative decision variable = amount it exceeded its lower bounds of zero

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	1 5/7	0	50	16 2/3	30
$C$3	Units to make Y	2 1/7	0	40	60	10

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1)
5X + 3Y <= 15 (2)
2X + Y >= 2 (3)
X + 2Y >= 2 (4)
X, Y >= 0 (5)



If OFC of x > OFC of y, then the |slope|= >1
=|50/40= −1.25| >1, If x ↑ 1, →, |y| ↑ 1.25
slope (b) = (y1-y2)/(x1-x2)=(2.5−0)/(0−2) = −1.25

The optimal solution OS at Point A(12/7, 15/7) will NOT change and the feasible region will be the same:

as OFC of x ↑ within 50+16 2/3, OFLine turns clock-wise
as OFC of x ↓ within 50−30, OFLine turns counter-clock-wise

as OFC of y ↑ within 40+60, OFLine turns counter-clock-wise
as OFC of y ↓ within 40−10, OFLine turns clock-wise

Update OFV = xOFC * 12/7 + 40* 15/7 or 50*12/7 + yOFC*15/7

as xOFC or yOFC at its limit, it may lead to alternative optimal solution (OS) s

SA would NOT be applicable when xOFC is outside xOFCR or
SA would NOT be applicable when yOFC is outside yOFCR

xOFC & yOFC are profit margins, can you see what are optimal solutions when xOFC or yOFC outside of its OFCR?

Answer Reports for xOFC=10

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	102.8571429	120

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	1 5/7	0
$C$3	Units to make Y	2 1/7	3

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	6	$D$7<=$F$7	Binding	0
$D$8	Raw Material Units Used	9	$D$8<=$F$8	Not Binding	6
$D$9	Customer v Demand Used	3	$D$9>=$F$9	Not Binding	1
$D$10	Customer w Demand Used	6	$D$10>=$F$10	Not Binding	4

The Reduced Cost for each variable equals to the per-unit amount that the variable contributes to the objective function value, minus the per-unit value of the resources it consumes at their shadow prices. For example:

xConstC Shadow Price

$$10 - \begin{pmatrix}1\\5\\2\\1\end{pmatrix} \times \begin{pmatrix}20\\0\\0\\0\end{pmatrix} = 10 - 20 = -10$$

$$40 - \begin{pmatrix}2\\3\\1\\2\end{pmatrix} \times \begin{pmatrix}20\\0\\0\\0\end{pmatrix} = 40 - 40 = 0$$

X would be profitable (X > 0)
if xOFC ↑ $10, or
xOFC is at least 20. read p.148-9, Fig. 4.10 & Key Points for more details

Max: 10X + 40Y Profit

s.t. 1X + 2Y <= 6 (1)

5X + 3Y <= 15 (2)

2X + Y >= 2 (3)

X + 2Y >= 2 (4)

X, Y >= 0 (5)

	A	B	C	D
1	Maximize Profit			
2		X	Y	=SUMPRODUC
3	Units to make	1 5/7	2 1/7	Total Profit
4	Unit Profits	$10	$40	102 6/7

Sensitivity Reports for xOFC=10

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	0	-10	10	10	1E+30
$C$3	Units to make Y	3	0	40	1E+30	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	6	20	6	4	2
$D$8	Raw Material Units Used	9	0	15	1E+30	6
$D$9	Customer v Demand Used	3	0	2	1	1E+30
$D$10	Customer w Demand Used	6	0	2	4	1E+30

LP Sensitivity Analysis –

Equ (1) RHS Value Has Increased

Max: 50X + 40Y Profit

s.t. 1X + 2Y <= 6 (1) Production time in minutes

5X + 3Y <= 15 (2) Raw materials in units

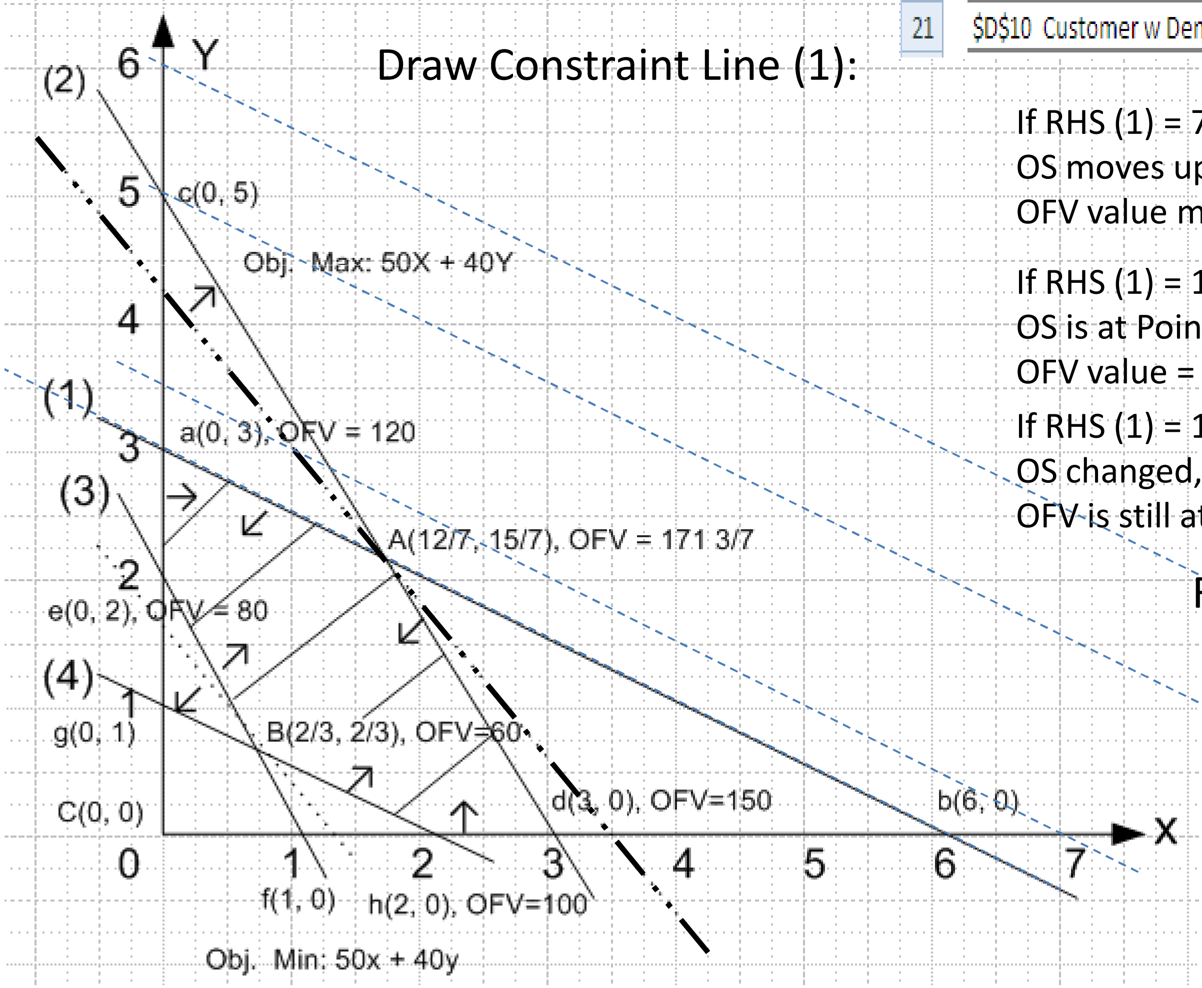
2X + Y >= 2 (3) Customer demand v

X + 2Y >= 2 (4) Customer demand w

X, Y >= 0 (5) Non negative

15	Constraints						
16			Final	Shadow	Constraint	Allowable	Allowable
17	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
18	\$D\$7	Production Minutes Used	6	7 1/7	6	4	3
19	\$D\$8	Raw Material Units Used	15	8 4/7	15	15	6
20	\$D\$9	Customer v Demand Used	5 4/7	0	2	3 4/7	1E+30
21	\$D\$10	Customer w Demand Used	6	0	2	4	1E+30

Draw Constraint Line (1):



If RHS (1) = 7, Line(1) moves up,
OS moves up
OFV value moves up = 178 4/7 = 171 3/7 + 7 1/7*1

If RHS (1) = 10, Line (1) & Line (2) joint at Point c
OS is at Point c(0, 5)
OFV value = 200 = 171 3/7 + 4*7 1/7=40*5

If RHS (1) = 12, forms new feasible region
OS changed, new Line (1) becomes redundant
OFV is still at Point c(0,5) = 200

Remarks:

RHS=7: Let X=0, Y=3.5 or (0, 3.5) & let Y=0, X=7 or (7, 0)
RHS=10: Let X=0, Y=5 or (0, 5) & let Y=0, X=10 or (10,0)
RHS=12, Let X=0, Y=6 or (0,6) & let Y=0, X=12 or (12,0)

Degeneracy: if RHSR has 0 allowable ↕, SA may change, read page 151 on sec 4.5.12 for details

Microsoft Excel 12.0 Sensitivity Report

Worksheet: [Copy of LP.xlsx]Ex1

Report Created: 02/27/2010 22:53:53 PM

Target Cell (Max)

Cell	Name	Final Value
$D$4	Unit Profits Total Profit	171 3/7

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	1 5/7	0	50	16 2/3	30
$C$3	Units to make Y	2 1/7	0	40	60	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	6	7 1/7	6	4	0
$D$8	Raw Material Units Used	15	8 4/7	15	15	6
$D$9	Customer v Demand Used	5 4/7	0	2	3 4/7	1E+30
$D$10	Customer w Demand Used	6	0	6	0	1E+30

Degeneracy: if RHSR has 0 allowable $\updownarrow$, SA may change, read page 151 on sec 4.5.12 for details. Equ (4) RHS value =6

SA of Const. Coef. Can be done with the Reduced Cost computation: to make Reduced cost > 0 for Max or < 0 for Min

LP Sensitivity Analysis: Change of RHS Values of Constraint (1)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

RHS values within RHSR: changes the optimal solution point (X, Y) and the optimal OBJ value by 50X + 40Y = 171 4/7 ± Shadow Price × Incremental ± of RHS value

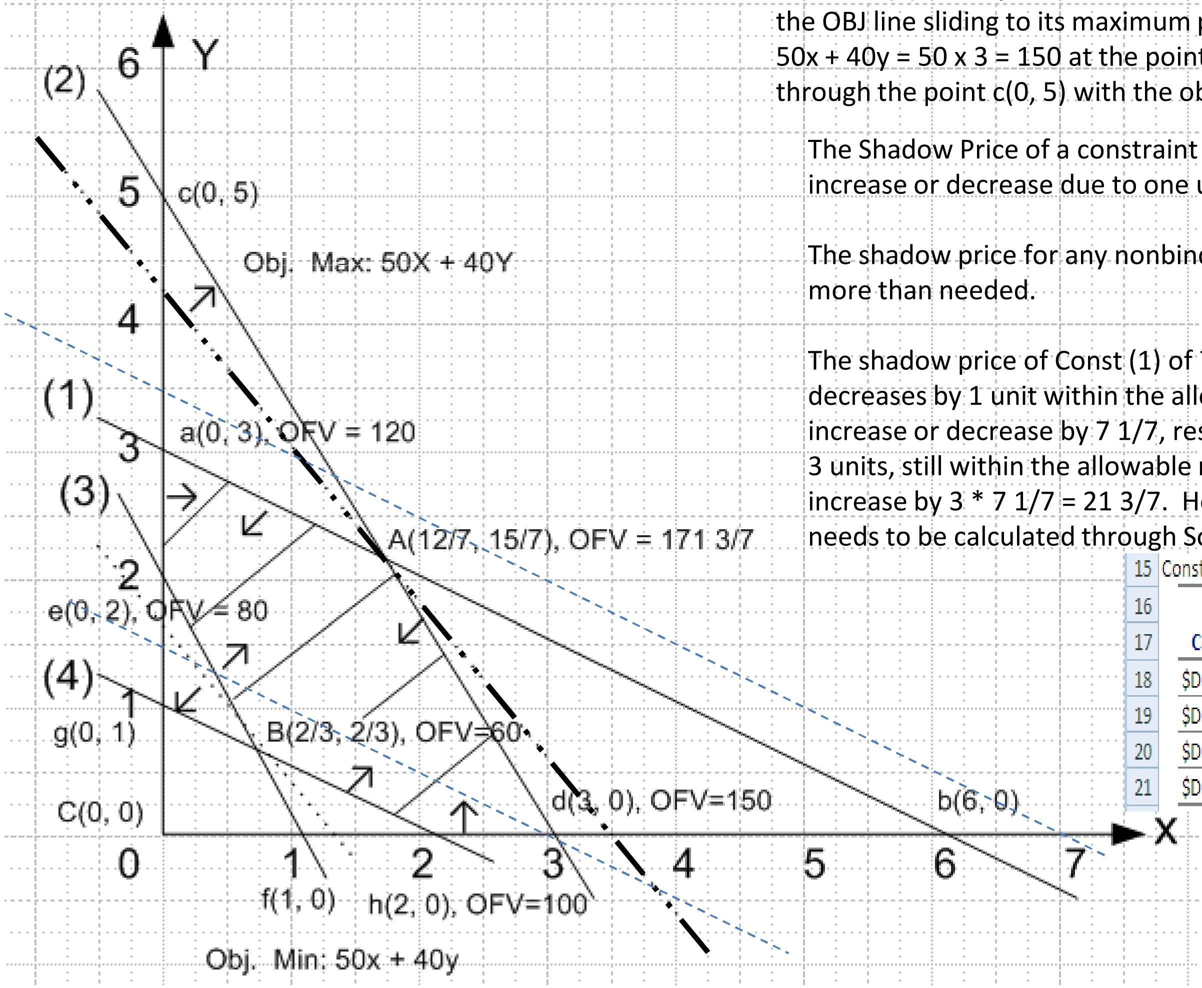
(1)RHSR = (6 – 3, 6 + 4), Optimal solution (X, Y) changes as RHS varies, thus the change in the optimal OBJ value. The updated 50X + 40Y = 171 4/7 ± 7 1/7 × Incremental ± of RHS value.

These two situations are best depicted by the graph below with two dot lines each referring to the Constraint (1) at its lower or upper limits. We may view the situation as if Line (1) slides parallel from its lower position passing through point c(3, 0) with the OBJ line sliding to its maximum possible position along the way and its value of 50x + 40y = 50 x 3 = 150 at the point d(3, 0), to its upper limit position passing through the point c(0, 5) with the objective function value of 50X + 40Y = 200.

The Shadow Price of a constraint is the amount of the objective function value to increase or decrease due to one unit of change in the RHS value of that constraint.

The shadow price for any nonbinding constraint is always zero, because RHS value is more than needed.

The shadow price of Const (1) of 7 1/7 means if the RHS value of Const (1) increases or decreases by 1 unit within the allowable range (3, 10), the objective function value will increase or decrease by 7 1/7, respectively, i.e., if the RHS value of Const (1) increases by 3 units, still within the allowable range of (3, 10), the optimal objective function value will increase by 3 * 7 1/7 = 21 3/7. However, the optimal vertex (X, Y) has changed and needs to be calculated through Solver



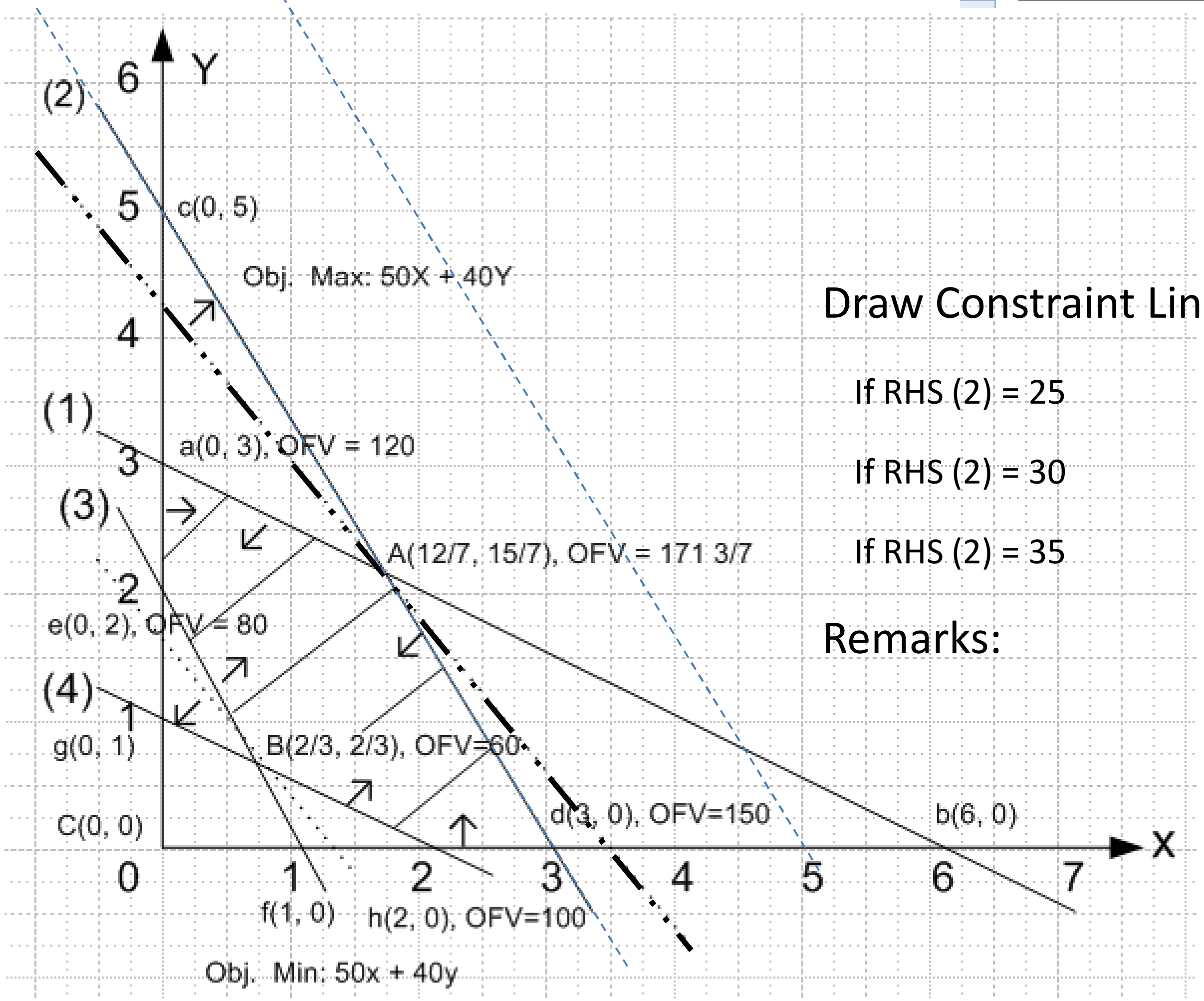
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$D\$7	Production Minutes Used	6	7 1/7	6	4	3
\$D\$8	Raw Material Units Used	15	8 4/7	15	15	6
\$D\$9	Customer v Demand Used	5 4/7	0	2	3 4/7	1E+30
\$D\$10	Customer w Demand Used	6	0	2	4	1E+30

LP Sensitivity Analysis: Change of RHS Values of Constraint (2)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

15	Constraints						
16			Final	Shadow	Constraint	Allowable	Allowable
17	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
18	$D$164	Customer v Demand Used	6	0	2	4	1E+30
19	$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
20	$D$162	Production Minutes Used	6	7 1/7	6	4	3
21	$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (2),

If RHS (2) = 25

If RHS (2) = 30

If RHS (2) = 35

Remarks:

If RHS (2) = 10

If RHS (2) = 9

If RHS (2) = 5

Remarks:

LP Sensitivity Analysis: Change of RHS Values of Constraint (3)

Max: 50X + 40Y Profit

s.t. 1X + 2Y <= 6 (1) Production time in minutes

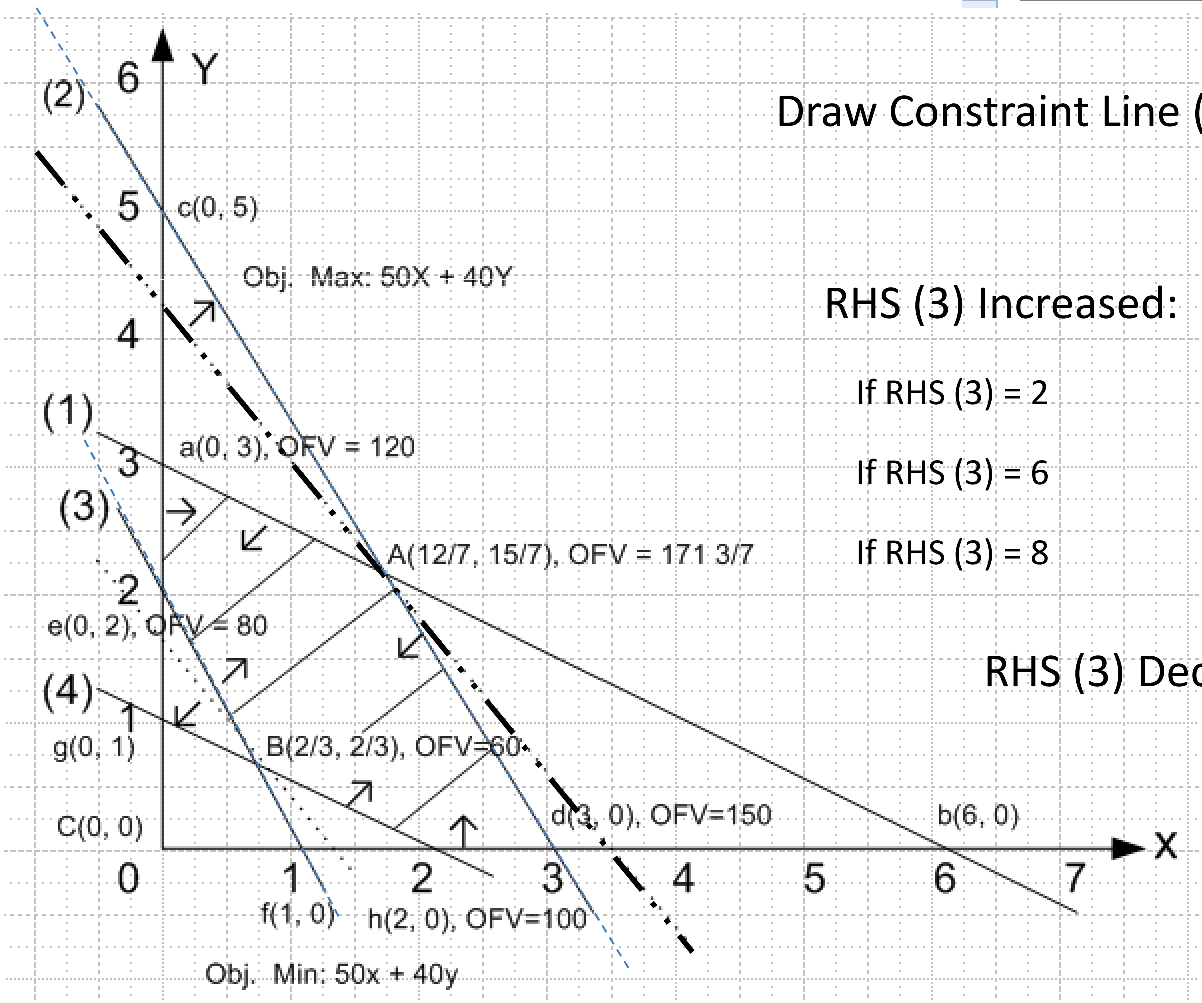
5X + 3Y <= 15 (2) Raw materials in units

2X + Y >= 2 (3) Customer demand v

X + 2Y >= 2 (4) Customer demand w

X, Y >= 0 (5) Non negative

15	Constraints						
16			Final	Shadow	Constraint	Allowable	Allowable
17	Cell	Name	Value	Price	R.H. Side	Increase	Decrease
18	$D$164	Customer v Demand Used	6	0	2	4	1E+30
19	$D$165	Customer w Demand Used	5 4/7	0	2	3.571428571	1E+30
20	$D$162	Production Minutes Used	6	7 1/7	6	4	3
21	$D$163	Raw Material Units Used	15	8 4/7	15	15	6



Draw Constraint Line (3):

RHS (3) Increased:

If RHS (3) = 2

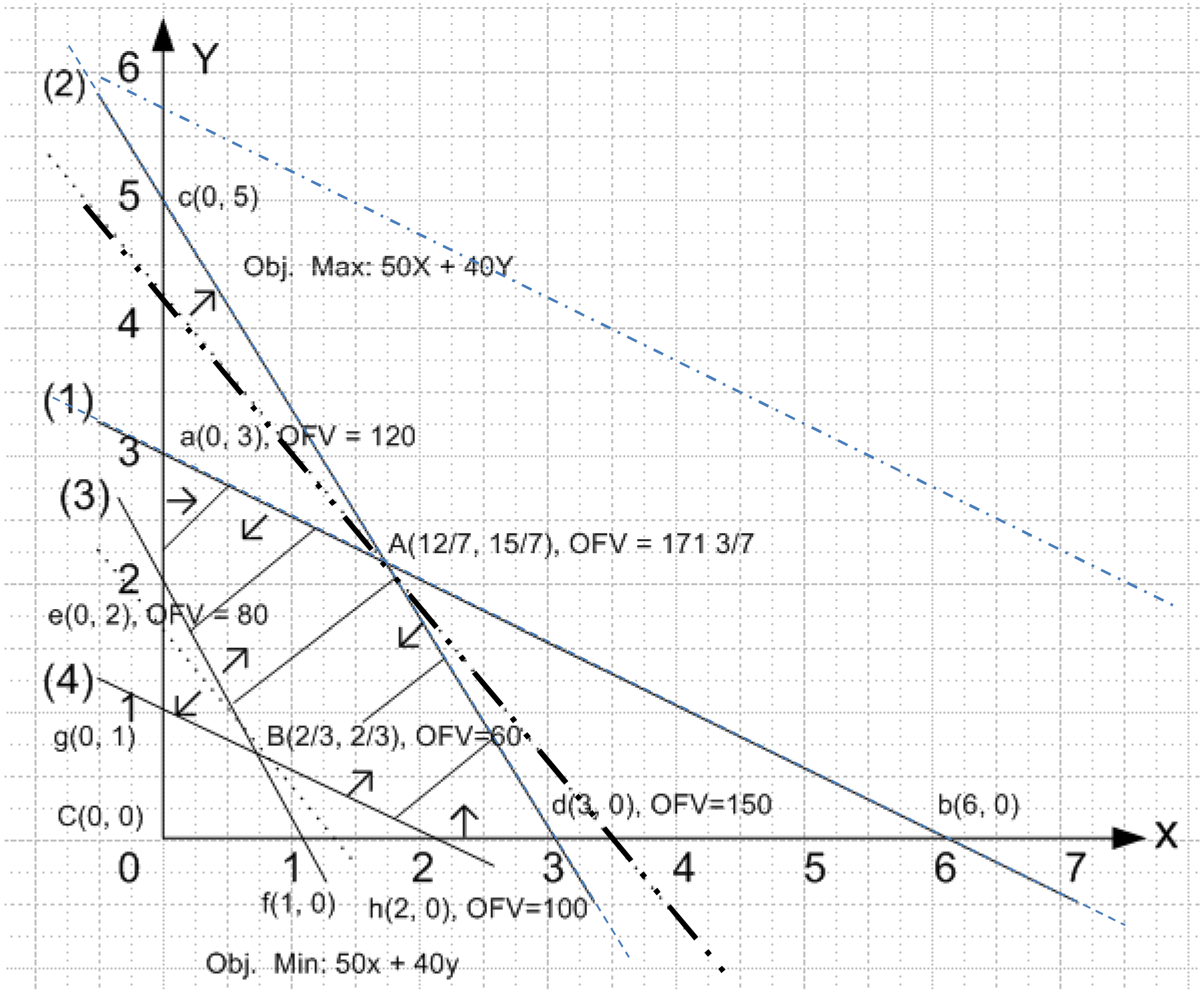
If RHS (3) = 6

If RHS (3) = 8

RHS (3) Decreased:

Special Cases in LP Modeling

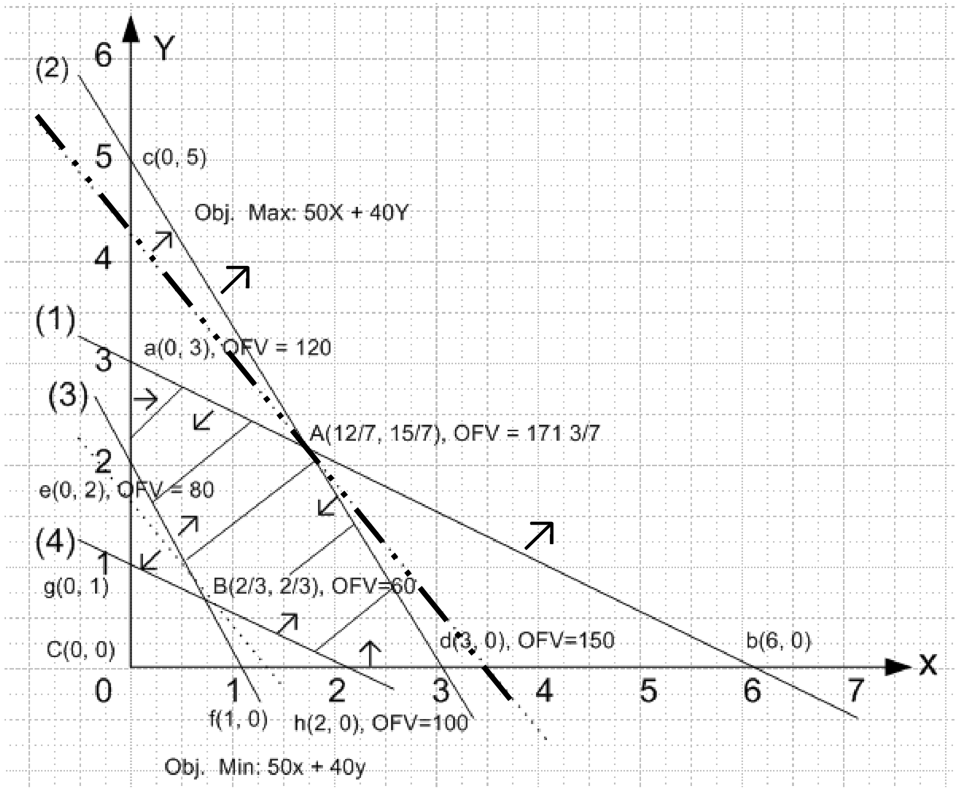
- Alternative optimal solutions – more than one optimal vertex exist.
- Redundant constraints (R)



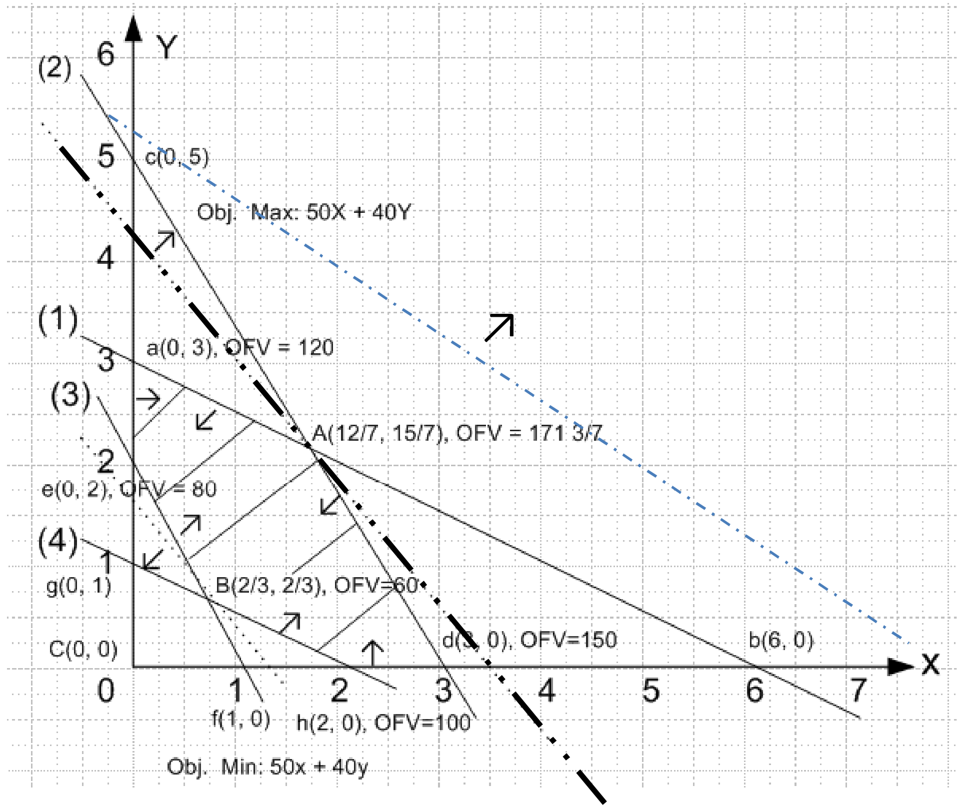
Special Cases in LP Modeling

Unbounded Solutions

Max: 50X + 40Y Profit
s.t. 1X + 2Y >= 6 (1) Production time in minutes
5X + 3Y >= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative



Infeasible solutions



MIN: 50X + 40Y Production Cost

s.t. 1X + 2Y <= 6 (1) Production time in minutes

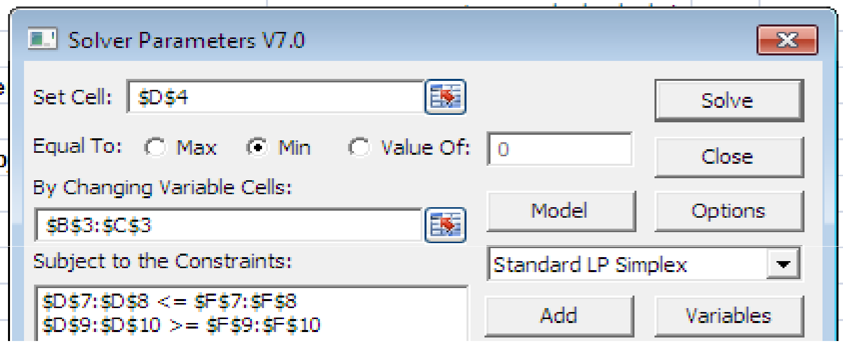
5X + 3Y <= 15 (2) Raw materials in units

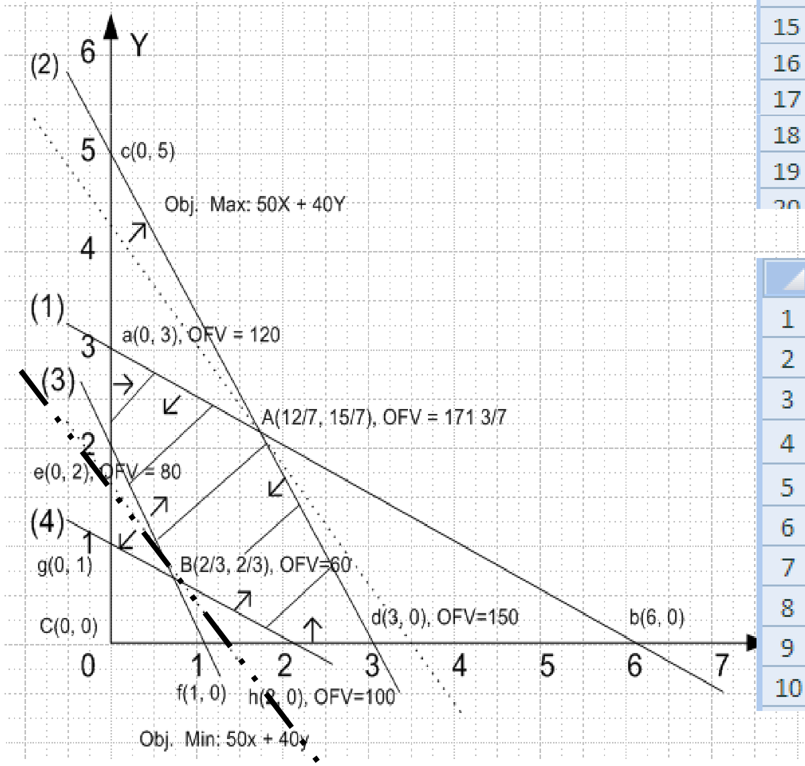
2X + Y >= 2 (3) Customer demand v

X + 2Y >= 2 (4) Customer demand w

X, Y >= 0 (5) Non negative

	A	B	C	D	E	F
1	Min Product Cost					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:C		
3	Units to make	0	0	Total Profit		
4	Unit Profits	$50	$40	0		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	0	<=	6
8	Raw Material Units	5	3	0	<=	15
9	Customer v Demand	2	1	0	>=	2
10	Customer w Demand	1	2	0	>=	2
11						
12						
13	Se					
14						
15	Ob					
16						
17						
18						
19						





	A	B	C	D	E	F
1	Min Product Cost					
2		X	Y	=SUMPRODUCT(B4:C4,$B$3:$C		
3	Units to make	2/3	2/3	Total Profit		
4	Unit Profits	$50	$40	60		
5				LHS		RHS
6	Constraints	X	Y	Used		Available
7	Production Minutes	1	2	2	<=	6
8	Raw Material Units	5	3	5 1/3	<=	15
9	Customer v Demand	2	1	2	>=	2
10	Customer w Demand	1	2	2	>=	2

Target Cell (Min)

Cell	Name	Original Value	Final Value
$D$4	Unit Profits Total Profit	0	60

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$3	Units to make X	0	2/3
$C$3	Units to make Y	0	2/3

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Production Minutes Used	2	$D$7<=$F$7	Not Binding	4
$D$8	Raw Material Units Used	5 1/3	$D$8<=$F$8	Not Binding	9 2/3
$D$9	Customer v Demand Used	2	$D$9>=$F$9	Binding	0
$D$10	Customer w Demand Used	2	$D$10>=$F$10	Binding	0

MIN: $50X + 40Y$ Production Cost

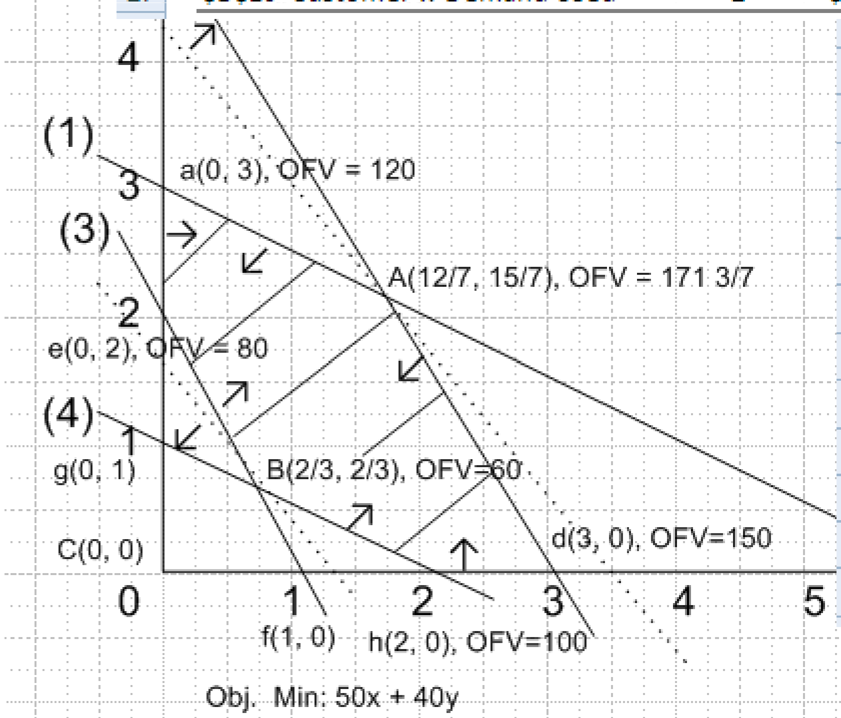
s.t. $1X + 2Y <= 6$ (1) Production time in minutes

$5X + 3Y <= 15$ (2) Raw materials in units

$2X + Y >= 2$ (3) Customer demand v

$X + 2Y >= 2$ (4) Customer demand w

$X, Y >= 0$ (5) Non negative



Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$3	Units to make X	2/3	0	50	30	30
$C$3	Units to make Y	2/3	0	40	60	15

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Production Minutes Used	2	0	6	1E+30	4
$D$8	Raw Material Units Used	5 1/3	0	15	1E+30	9 2/3
$D$9	Customer v Demand Used	2	20	2	2	1
$D$10	Customer w Demand Used	2	10	2	2	1

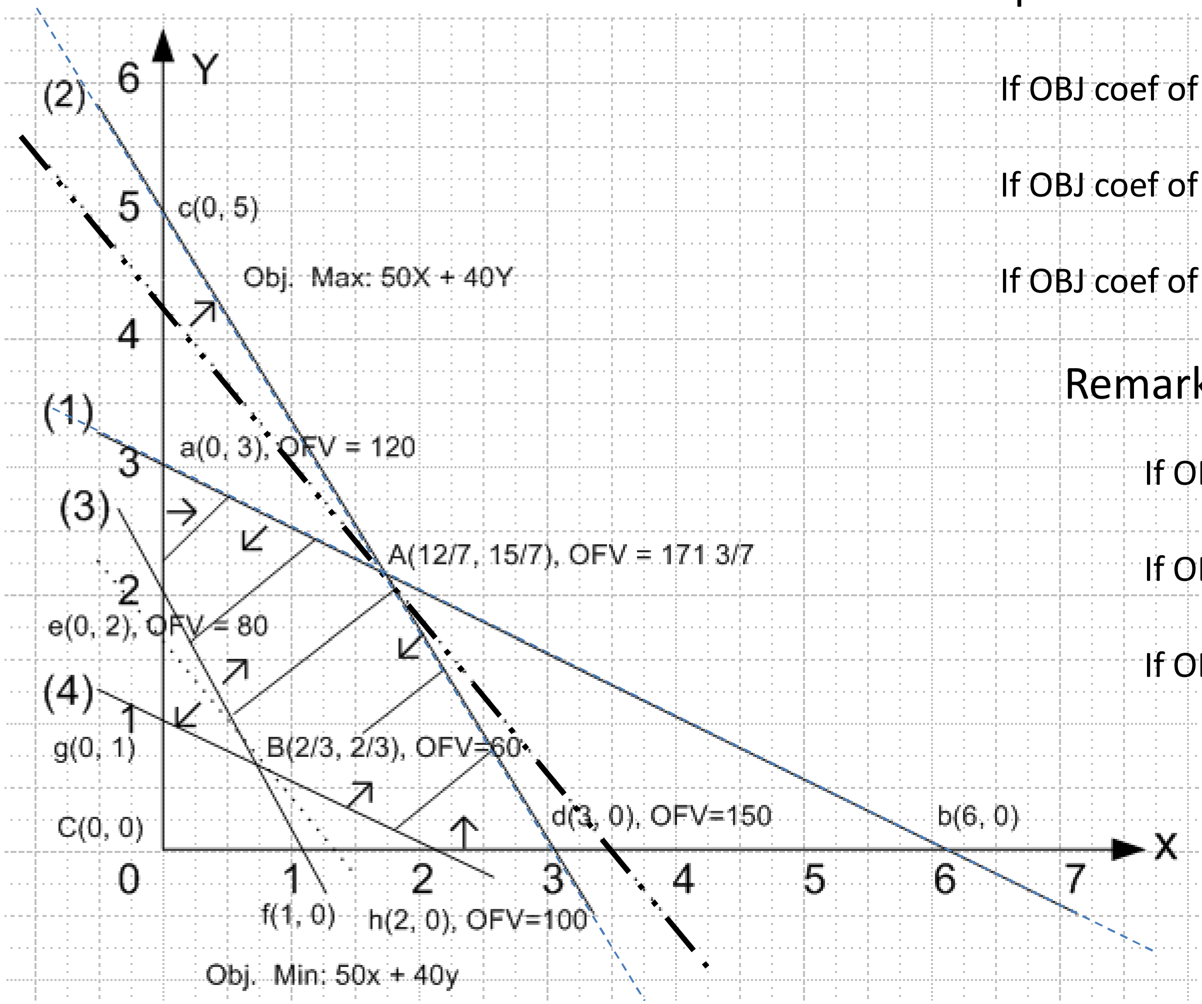
Visualizing Optimal Solution as OBJ Coefficients of X Changed

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
5X + 3Y <= 15 (2) Raw materials in units
2X + Y >= 2 (3) Customer demand v
X + 2Y >= 2 (4) Customer demand w
X, Y >= 0 (5) Non negative

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

OBJ Max: 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,



If OBJ coef of X = 35,

If OBJ coef of X = 20,

If OBJ coef of X = 5,

Remarks:

If OBJ coef of X = 60,

If OBJ coef of X = 66 2/3,

If OBJ coef of X = 75,

Remarks:

Visualizing Optimal Solution as OBJ Coefficient of Y Changed

Max: 50X + 40Y   Profit

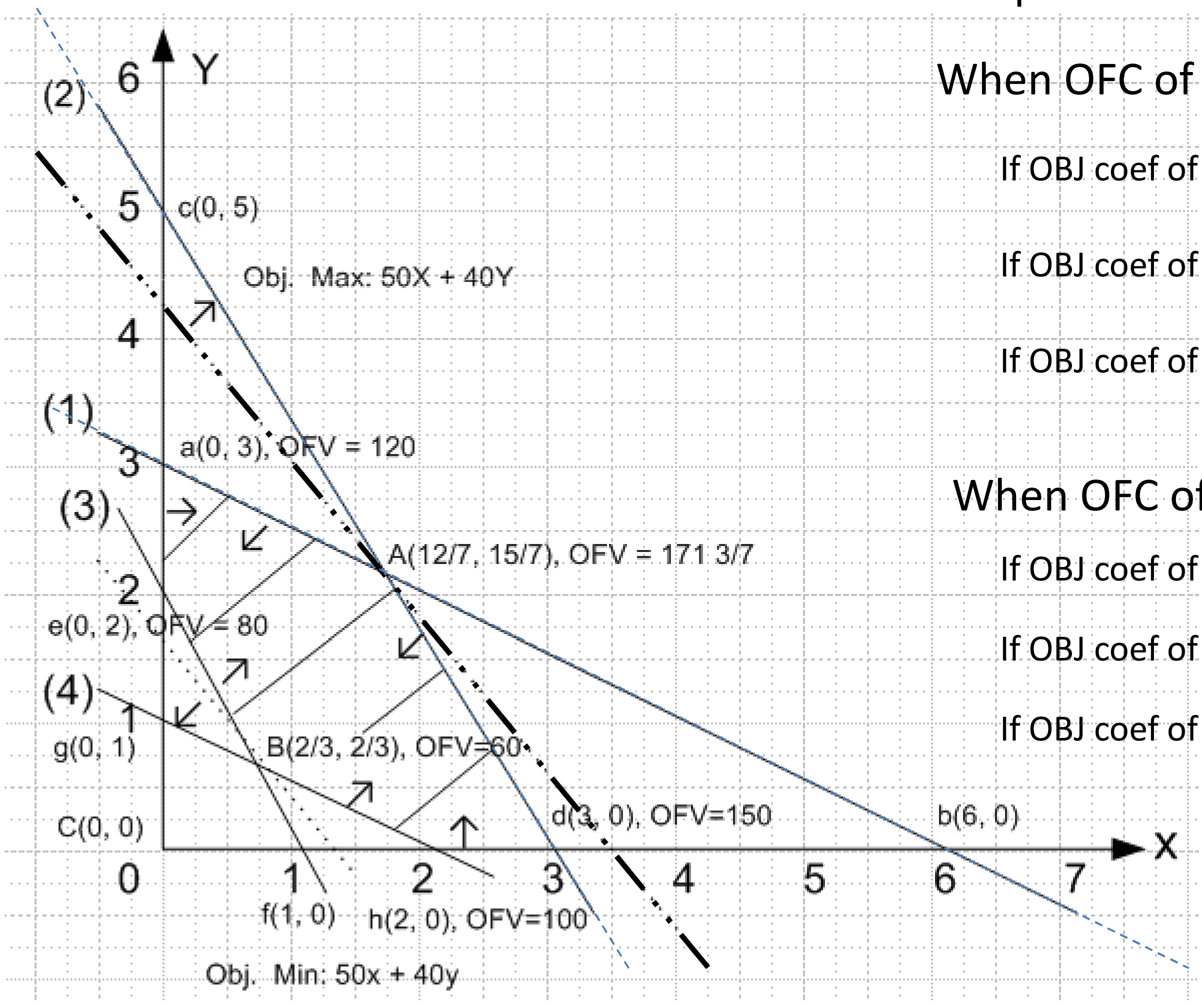
s.t.   1X + 2Y <= 6   (1) Production time in minutes

5X + 3Y <= 15   (2) Raw materials in units

2X +   Y >= 2   (3) Customer demand v

X + 2Y >= 2   (4) Customer demand w

X,   Y >= 0   (5) Non negative



Optimal Max OBJ value at joint

Optimal solution A(12/7, 15/7) = 171 3/7

OBJ Max: 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

When OFC of Y Decreased:

If OBJ coef of Y = 35,

If OBJ coef of Y = 30,

If OBJ coef of Y = 20,

When OFC of Y Increased:

If OBJ coef of Y = 70,

If OBJ coef of Y = 100,

If OBJ coef of Y = 140,