LP Sensitivity Analysis

What is the new feasible region? a, e, B, h, d, A and a form feasible region

What is the value of Joint B(X, Y)? B(2/3, 2/3)

Max: 50X + 40Y  Profit
s.t.  1X + 2Y <= 6  (1) Production time in minutes
     5X + 3Y <= 15  (2) Raw materials in units
     2X +  Y >= 2  (3) Customer v demand
     X + 2Y >= 2  (4) Customer w demand
     X, Y >= 0  (5) Non negativity

What are OBJ values at joints e, h and B?

OBJ value at joint h(2, 0) = 50(2) = 100
OBJ value at joint e(0, 2) = 40(2) = 80
OBJ value at joint B(2/3, 2/3) = 50(2/3) + 40(2/3) = 60

OBJ value at joint C(0, 0) feasible? No,
Why? Because C(0, 0) is outside of feasible region

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7
Max: $50X + 40Y$ Profit

s.t.  
1. $X + 2Y \leq 6$ (1) Production time in minutes
2. $5X + 3Y \leq 15$ (2) Raw materials in units
3. $2X + Y \geq 2$ (3) Customer v demand
4. $X + 2Y \geq 2$ (4) Customer w demand
5. $X, Y \geq 0$ (5) Non negativity
Max: \( 50X + 40Y \) Profit
s.t.  
1. \( 1X + 2Y \leq 6 \)  
2. \( 5X + 3Y \leq 15 \)  
3. \( 2X + Y \geq 2 \)  
4. \( X + 2Y \geq 2 \)  
5. \( X, Y \geq 0 \)

### Slacks:

1. Slack for a constraint = value of (LHS – RHS)
2. Slack for binding constraints = zero
3. Slack for not binding constraint > zero
4. Slack for non negative decision variable = amount it exceeded its lower bounds of zero
If OFC of \( x \) > OFC of \( y \), then the \( |\text{slope}| = 1.25 > 1 \), if \( x \) ↑ 1, \( y \) ↑ 1.25.

\[
\text{slope} (b) = \frac{(y_1 - y_2)}{(x_1 - x_2)} = \frac{(2.5 - 0)}{(0 - 2)} = -1.25
\]

The optimal solution (OS) at Point A(12/7, 15/7) will NOT change and the feasible region will be the same:

as OFC of \( x \) ↑ within 50+16 \( \frac{2}{3} \), OFLline turns clock-wise

as OFC of \( x \) ↓ within 50–30, OFLline turns counter-clock-wise

as OFC of \( y \) ↑ within 40+60, OFLline turns counter-clock-wise

as OFC of \( y \) ↓ within 40–10, OFLline turns clock-wise

Update \( \text{OFV} = x\text{OFC} \times 12/7 + 40 \ast 15/7 \) or \( 50 \ast 12/7 + y\text{OFC} \ast 15/7 \)

as xOFC or yOFC at its limit, it may lead to alternative optimal solution (OS) s

SA would NOT be applicable when xOFC is outside xOFCR or yOFC is outside yOFCR

xOFC & yOFC are profit margins, can you see what are optimal solutions when xOFC or yOFC outside of its OFCR?
Answer Reports for xOFC=10

<table>
<thead>
<tr>
<th>Target Cell (Max)</th>
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<tbody>
<tr>
<td>Cell</td>
<td>Name</td>
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<tr>
<td>$D$4</td>
<td>Unit Profits Total Profit</td>
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<tbody>
<tr>
<td>Cell</td>
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<tr>
<td>$B$3</td>
<td>Units to make X</td>
</tr>
<tr>
<td>$C$3</td>
<td>Units to make Y</td>
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<table>
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<tbody>
<tr>
<td>Cell</td>
<td>Name</td>
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<tr>
<td>$D$7</td>
<td>Production Minutes Used</td>
</tr>
<tr>
<td>$D$8</td>
<td>Raw Material Units Used</td>
</tr>
<tr>
<td>$D$9</td>
<td>Customer v Demand Used</td>
</tr>
<tr>
<td>$D$10</td>
<td>Customer w Demand Used</td>
</tr>
</tbody>
</table>

Max: 10X + 40Y Profit  
s.t. 1X + 2Y <= 6 (1)  
5X + 3Y <= 15 (2)  
2X + 2Y >= 2 (3)  
2X + Y >= 2 (4)  
X, Y >= 0 (5)

Sensitivity Reports for xOFC=10

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<tr>
<td>$D$10</td>
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</tbody>
</table>

The Reduced Cost for each variable equals to the per-unit amount that the variable contributes to the objective function value, minus the per-unit value of the resources it consumes at their shadow prices. For example:

\[
\begin{align*}
\text{ConstC Shadow Price X would be profitable (X > 0)} & \quad \text{if xOFC } \uparrow \text{ }10, \text{ or } \\
& \quad \text{xOFC is at least 20. read p.148-9,}
\end{align*}
\]

Fig. 4.10 & Key Points for more details
LP Sensitivity Analysis –
Equ (1) RHS Value Has Increased

Max: \( 50X + 40Y \) Profit
s.t.  
1. \( 1X + 2Y <= 6 \) (1) Production time in minutes
2. \( 5X + 3Y <= 15 \) (2) Raw materials in units
3. \( 2X + Y >= 2 \) (3) Customer demand v
4. \( X + 2Y >= 2 \) (4) Customer demand w
5. \( X, Y >= 0 \) (5) Non negative

Draw Constraint Line (1):

- If RHS (1) = 7, Line(1) moves up, OS moves up OFV value moves up = 178 \( 4/7 \) = 171 \( 3/7 \) + 7 \( 1/7 \)*1
- If RHS (1) = 10, Line (1) & Line (2) joint at Point c OS is at Point c(0, 5) OFV value = 200 = 171 \( 3/7 \) + 4*7 \( 1/7 \) = 40*5
- If RHS (1) = 12, forms new feasible region OS changed, new Line (1) becomes redundant OFV is still at Point c(0,5) = 200

Remarks:
- RHS=7: Let X=0, Y=3.5 or (0, 3.5) & let Y=0, X=7 or (7, 0)
- RHS=10: Let X=0, Y=5 or (0, 5) & let Y=0, X=10 or (10,0)
- RHS=12, Let X=0, Y=6 or (0,6) & let Y=0, X=12 or (12,0)

Degeneracy: if RHS 0 has allowable \( \uparrow \), SA may change, read page 151 on sec 4.5.12 for details
SA of Const. Coef. Can be done with the Reduced Cost computation: to make Reduced cost > 0 for Max or < 0 for Min

Degeneracy: if RHSR has 0 allowable , SA may change, read page 151 on sec 4.5.12 for details. Equ (4) RHS value =6
LP Sensitivity Analysis: Change of RHS Values of Constraint (1)

Max: $50X + 40Y$ Profit
s.t. $1X + 2Y \leq 6$ (1) Production time in minutes
$5X + 3Y \leq 15$ (2) Raw materials in units
$2X + Y \geq 2$ (3) Customer demand v
$X + 2Y \geq 2$ (4) Customer demand w
$X, Y \geq 0$ (5) Non negative

RHS values within RHSR: changes the optimal solution point $(X, Y)$ and the optimal OBJ value by $50X + 40Y = 171 \frac{4}{7} \pm \text{Shadow Price} \times \text{Incremental } \pm \text{of RHS value}$

(1) RHSR = $(6 - 3, 6 + 4)$, Optimal solution $(X, Y)$ changes as RHS varies, thus the change in the optimal OBJ value. The updated $50X + 40Y = 171 \frac{4}{7} \pm 7 \frac{1}{7} \times \text{Incremental } \pm \text{of RHS value}$.

These two situations are best depicted by the graph below with two dot lines each referring to the Constraint (1) at its lower or upper limits. We may view the situation as if Line (1) slides parallel from its lower position passing through point c(3, 0) with the OBJ line sliding to its maximum possible position along the way and its value of $50x + 40y = 50 \times 3 = 150$ at the point d(3, 0), to its upper limit position passing through the point c(0, 5) with the objective function value of $50X + 40Y = 200$.

The Shadow Price of a constraint is the amount of the objective function value to increase or decrease due to one unit of change in the RHS value of that constraint.

The shadow price for any nonbinding constraint is always zero, because RHS value is more than needed.

The shadow price of Const (1) of $7 \frac{1}{7}$ means if the RHS value of Const (1) increases or decreases by 1 unit within the allowable range $(3, 10)$, the objective function value will increase or decrease by $7 \frac{1}{7}$, respectively, i.e., if the RHS value of Const (1) increases by 3 units, still within the allowable range of $(3, 10)$, the optimal objective function value will increase by $3 \times 7 \frac{1}{7} = 21 \frac{3}{7}$. However, the optimal vertex $(X, Y)$ has changed and needs to be calculated through Solver.

<table>
<thead>
<tr>
<th>Constraints</th>
<th>Final Value</th>
<th>Shadow Price</th>
<th>Constraint R.H.S.</th>
<th>Allowable Increase</th>
<th>Allowable Decrease</th>
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<tbody>
<tr>
<td>SDS7 Production Minutes Used</td>
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<td>6</td>
<td>4</td>
<td>3</td>
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<tr>
<td>SDS8 Raw Material Units Used</td>
<td>15</td>
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<td>5</td>
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<tr>
<td>SDS9 Customer v Demand Used</td>
<td>5</td>
<td>2</td>
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<td>1E+30</td>
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</tr>
<tr>
<td>SDS10 Customer w Demand Used</td>
<td>6</td>
<td>2</td>
<td>4</td>
<td>1E+30</td>
<td></td>
</tr>
</tbody>
</table>
LP Sensitivity Analysis: Change of RHS Values of Constraint (2)

Max: 50X + 40Y  Profit
s.t. 1X + 2Y <= 6  (1) Production time in minutes
     5X + 3Y <= 15  (2) Raw materials in units
     2X + Y >= 2   (3) Customer demand v
     X + 2Y >= 2   (4) Customer demand w
     X, Y >= 0     (5) Non negative

Draw Constraint Line (2),

If RHS (2) = 25
If RHS (2) = 30
If RHS (2) = 35
If RHS (2) = 10
If RHS (2) = 9
If RHS (2) = 5

Remarks:
LP Sensitivity Analysis: Change of RHS Values of Constraint (3)

Max: 50X + 40Y Profit
s.t. 1X + 2Y <= 6 (1) Production time in minutes
     5X + 3Y <= 15 (2) Raw materials in units
     2X + Y >= 2 (3) Customer demand v
     X + 2Y >= 2 (4) Customer demand w
     X, Y >= 0 (5) Non negative

Draw Constraint Line (3):

RHS (3) Increased:
If RHS (3) = 2
If RHS (3) = 6
If RHS (3) = 8

RHS (3) Decreased:
Special Cases in LP Modeling

- Alternative optimal solutions – more than one optimal vertex exist.
- Redundant constraints (R)
Special Cases in LP Modeling

Unbounded Solutions

Max: $50X + 40Y$  Profit
s.t.  $1X + 2Y >= 6$  (1) Production time in minutes
      $5X + 3Y >= 15$  (2) Raw materials in units
      $2X + Y >= 2$    (3) Customer demand v
      $X + 2Y >= 2$    (4) Customer demand w
      $X, Y >= 0$      (5) Non negative
MIN: 50X + 40Y  
Production Cost
s.t.  
1X + 2Y <= 6  (1) Production time in minutes
5X + 3Y <= 15  (2) Raw materials in units
2X + Y >= 2   (3) Customer demand v
X + 2Y <= 2   (4) Customer demand w
X, Y >= 0    (5) Non negative
MIN: $50X + 40Y$ Production Cost

s.t.  
1. $X + 2Y \leq 6$ (1) Production time in minutes  
2. $5X + 3Y \leq 15$ (2) Raw materials in units  
3. $2X + Y \geq 2$ (3) Customer demand v  
4. $X + 2Y \geq 2$ (4) Customer demand w  
5. $X, Y \geq 0$ (5) Non negative
Visualizing Optimal Solution as OBJ Coefficients of X Changed

Max: 50X + 40Y  Profit
s.t.  1X + 2Y <= 6  (1) Production time in minutes
      5X + 3Y <= 15  (2) Raw materials in units
      2X + Y >=  2  (3) Customer demand v
      X + 2Y >=  2  (4) Customer demand w
      X, Y >= 0     (5) Non negative

Optimal Max OBJ value at joint
Optimal solution A(12/7, 15/7) = 171 3/7

OBJ Max: 50X + 40Y = 171 3/7

Find Optimal Solutions & OBJ Values,

If OBJ coef of X = 35,
If OBJ coef of X = 20,
If OBJ coef of X = 5,

Remarks:
If OBJ coef of X = 60,
If OBJ coef of X = 66 2/3,
If OBJ coef of X = 75,
Visualizing Optimal Solution as OBJ Coefficient of Y Changed

Max: $50X + 40Y$ Profit
s.t.  
 1. $X + 2Y \leq 6$ (1) Production time in minutes  
 2. $5X + 3Y \leq 15$ (2) Raw materials in units  
 3. $2X + Y \geq 2$ (3) Customer demand v  
 4. $X + 2Y \geq 2$ (4) Customer demand w  
 5. $X, Y \geq 0$ (5) Non negative

Find Optimal Solutions & OBJ Values,

When OFC of Y Decreased:

If OBJ coef of Y = 35,

If OBJ coef of Y = 30,

If OBJ coef of Y = 20,

When OFC of Y Increased:

If OBJ coef of Y = 70,

If OBJ coef of Y = 100,

If OBJ coef of Y = 140,

Optimal Max OBJ value at joint

Optimal solution $A(12/7, 15/7) = 171\ 3/7$

OBJ Max: $50X + 40Y = 171\ 3/7$