Home Work Simulation 3 Key

(Please follow the COB291 instructions for APS and Mini Project to prepare your report. The lecture and the supplemental reading on BB/Course Document/Simulation and the appendix provide detailed instructions for simulating a waiting line in Excel@ as we discussed in class). Monte Carlito should be used whenever multiple simulation runs are required.

Bank of America in Harrisonburg in Harrisonburg has a single drive-in teller window. Customers arrive at the window about every 10 minutes on average according to a Poisson process or the hourly arrival rate is $\lambda = 6$. It take an average of five minutes (exponentially distributed) to complete each customer order or the hourly service rate is $\mu = 12$. The inter-arrival times can be simulated with an Excel@ formula

$$=-(1/\lambda)*LN(RAND())*60$$

and the service times can be simulated with an Excel@ formula

$$=-60/\mu*LN(RAND())$$

For each of the simulations below, answer the following questions:

- average inter-arrival time
- average service time
- server utilization
- average waiting time
- average in system time
- No. of customers to wait
- Probability of wait

a. Three random numbers are given by =RAND() as 0.5046, 0.2432 and 0.8808 for the arrivals, and another three random numbers are given by =RAND() as 0.2966, 0.6827 and 0.9398 for services, simulate manually the first three customers to arrive at the BoA window for service once the bank opens its door in the morning.

   i. Write down in English and IF, THEN and ELSE statements the logics of single waiting line operations and the relationships among the various time segments. Test your logic carefully with examples. The following table in the lecture might be a useful reference

	A	B	C	D	E	F	G	H
14		Interarrival	Arrival	Service	Waiting	Service	Completion	Time
15	Customer	Time	Time	Start Time	Time	Time	Time	in System
16	1	0.83	0.83	0.83	0.00	1.91	2.74	1.91
17	2	4.55	5.38	5.38	0.00	1.54	6.92	1.54

Arrival (Clock in) time = previous arrival (Clock in) time + current interarrival time

Service start time: if the current arrival (Clock in) time is earlier (smaller) than the last completion (Clock out) time, then Service Start time = current arrival (Clock in) time; Otherwise, if the current arrival (Clock in) time is later (larger) than the last completion (Clock out) time, then Service Start time = the last completion (Clock out) time. For Customer 2 in Cell D17, In Excel@: =IF(C17 <=G16, G16, C17)

Waiting time: Waiting = Service start time – arrival time

Completion (Clock out) time: = Service start time + Service time.

ii. Use Excel@ formulas to realize your logics as the ATM example in the lecture. Again test it carefully.
=-1/6*LN(RAND())*60 for interarrival time
=C11+B12 for arrival (Clock in) time
=IF(C12>G11,C12,G11) for service start time
=D12-C12 for waiting time
=-60/12*LN(RAND()) for service time
=F12+D12 for Completion (Clock out) time
=G12-C12 for Time in system

iii. Use Excel@ formulas to collect performance measures listed above as required and carefully examines the results to make sure the equations are correctly used.

b. Run the simulation for 500 customers, MonteCarlito 100 replications to collect performance measures for the last 400 customers. The analytic model in Queuing indicates an average waiting time of 5 minutes (Wq) per customer. What average waiting time does your simulation model show?

i. Use Excel@ formulas to collect performance measures listed above as required

c. One advantage of using simulation is that a simulation model can be altered easily to reflect other assumptions about the probabilistic inputs. Assume that the service time is more accurately described by a normal probability distribution with a mean of 5 minutes and a standard deviation of 0.2 minutes. Three random numbers are given by =RAND() as 0.5046, 0.2432 and 0.8808 for the arrivals, and another three random numbers are given by =RAND() as 0.2966, 0.6827 and 0.9398 for services, simulate manually the first three customers to arrive at the BoA window for service once the bank opens its door in the morning. What is the impact of this change on the average waiting time? (Excel@ formula =NORMINV(RAND(), mean, std) can be used to generate values of normal distributed random variable)

i. Record the changes made to use normal distribution for service times.
The only change is to use =NORMINV(RAND(), mean, std) as service time formula

ii. Use Excel@ formulas to collect performance measures listed above as required.

d. Run the simulation for 500 customers, MonteCarlito 100 replications to collect performance measures for the last 400 customers. The analytic model in Queuing indicates an average waiting time of 5 minutes (Wq) per customer. Assume that the service time is more accurately described by a normal probability distribution with a mean of 5 minutes and a standard deviation of 0.2 minutes. What is the impact of this change on the average waiting time?

i. Use Excel@ formulas to collect performance measures listed above as required

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	f America M/M/1 Queue Simulation							Arrival rate per hour	6						
2	RN				RN			average interarrival time	9.9245		average inte	9.9649		average i	9.9994
3	0.5046				0.2996			Service Rate per hour	12						
4	0.2432				0.6827			average service time	4.7223		average serv	4.9972		average s	5.0054
5	0.8808				0.9398			server utilization	0.4757		server utiliza	0.5026		server uti	0.5016
6		=C12+B13		=D13-C13		=F13+D13	=G13-C13	average waiting time	4.6522		average wait	2.6662		average v	5.0587
7	=-1/6*LN(RAND())*60		=IF(C13>G12,C13,G12)		=NORMINV(RAND(),5,1.2)			average in system time	9.3745		average in sy	7.6634		average i	10.064
8		Clock in	Start Serve		=-60/12*LN(RAN	Clock out		No. of customers to wait	481		No. of custor	503.38		No. of cu	503.44
9	interarrivalt ime(min)	arrival time	Service Start Time	Waiting time	Service time	Completion time	Time in system	Probability of wait	48.10%						
10	6.8399	6.8399	6.8399	0	6.0265	12.8664	6.0265	=-60/12*LN(RAND())							
11	14.1387	20.9786	20.9786	0	1.9085	22.8871	1.9085	Summary Statistics	M/M/1 b)	M/G/1 c)	Q.xls				
12	1.2692	22.2479	22.8871	0.639252	0.3104	23.1975	0.9497	average interarrival time	9.9994	9.9649	10	λ =	6	hour	
13	14.9161	37.1639	37.1639	0	4.9189	42.0828	4.9189	average service time	5.0054	4.9972	5	μ =	12	hour	
14	40.1495	77.3134	77.3134	0	1.0558	78.3692	1.0558	server utilization ρ or u	0.5016	0.5026	0.5				
15	5.2596	82.5730	82.5730	0	9.2186	91.7916	9.2186	average waiting time Wq	5.0587	2.6662	5.00				
16	6.5009	89.0739	91.7916	2.717723	0.4630	92.2546	3.1807	average in system time W	10.0641	7.6634	10.00				
17	14.9208	103.9947	103.9947	0	2.9054	106.9002	2.9054	No. of customers to wait Lq	503.44	503.38	0.5				
18	6.2861	110.2808	110.2808	0	12.4748	122.7556	12.4748	No. of customers in system L	1.0064		1				
19	3.7926	114.0735	122.7556	8.682162	9.4324	132.1881	18.1146	Probability of wait pw	0.5034	0.5034	0.5				
20	1.6724	115.7458	132.1881	16.44223	0.8512	133.0392	17.2934	MonteCarlito	average ir	average se	server utiliza	average v	aver	No. of customers t	
21	16.1734	131.9193	133.0392	1.11998	6.3653	139.4045	7.4853	-100	10	4.7223	0.4757	4.6522	###	481	
22	1.0789	132.9981	139.4045	6.406396	7.1980	146.6025	13.6044	Mean	10	5.005444	0.50163475	5.05868	10	503.44	
23	14.1820	147.1802	147.1802	0	8.3185	155.4987	8.3185	Standard error	0.030347	0.0150443	0.00212313	0.08011	0.1	2.66508	
24	5.9391	153.1193	155.4987	2.379388	1.4424	156.9411	3.8218	Median	9.986156	5.0121798	0.5023502	5.04177	10	503	
25	0.7026	153.8218	156.9411	3.119281	6.0858	163.0269	9.2051	Standard deviation	0.303466	0.1504435	0.02123128	0.80105	0.9	26.6508	

	A	B	C	D	E	F	G	H	I	J
1	Bank								Arrival rate per hour	6
2		RN				RN			average interarrival time	=AVERAGE(B110:B1109)
3		0.5046				0.2996			Service Rate per hour	12
4		0.2432				0.6827			average service time	=AVERAGE(F110:F1109)
5		0.8808				0.9398			server utilization	=SUM(F110:F1109)/(G1109-G110)
6			=C12+B13		=D13-C13		=F13+D13	=G13-C13	average waiting time	=AVERAGE(E110:E1109)
7		=-1/6*LN(RAND())*60		=IF(C13>G12,C13,G12)		=NORMINV(RAND(),5			average in system time	=AVERAGE(H110:H1109)
8			Clock in	Start Serve		=-60/12*LN(RAND())	Clock out		No. of customers to wait	=COUNTIF(E110:E1109,">0")
9	id	interarrivaltime(min)	arrival time	Service Start Time	Waiting time	Service time	Completio n time	Time in system	Probability of wait	=J8/COUNT(E110:E1109)
10	1	=-1/6*LN(B3)*60	=B10	=C10	=D10-C10	=-60/12*LN(F3)	=F10+D10	=G10-C10	=-60/12*LN(RAND())	
11	2	=-1/6*LN(B4)*60	=C10+B11	=IF(C11>G10,C11,G10)	=D11-C11	=-60/12*LN(F4)	=F11+D11	=G11-C11	Summary Statistics	M/M/1 b)
12	3	=-1/6*LN(B5)*60	=C11+B12	=IF(C12>G11,C12,G11)	=D12-C12	=-60/12*LN(F5)	=F12+D12	=G12-C12	average interarrival time	9.99938326101369
13	4	=-1/6*LN(RAND())*60	=C12+B13	=IF(C13>G12,C13,G12)	=D13-C13	=-60/12*LN(RAND())	=F13+D13	=G13-C13	average service time	5.00544397352349
14	5	=-1/6*LN(RAND())*60	=C13+B14	=IF(C14>G13,C14,G13)	=D14-C14	=-60/12*LN(RAND())	=F14+D14	=G14-C14	server utilization ρ or u	0.501634752824696

e. Run Q.xls program and record the result with the arrival rate of λ = 6 per hour and service rate of μ = 12 per hour, and compare the results from Q.xls with that from your simulation and comment on the findings.

The following table might be used to summarize the results.

Summary Statistics	M/M/1 b)	M/G/1 c)	Q.xls			
average interarrival time	9.9994	9.9649	10	λ =	6	hour
average service time	5.0054	4.9972	5	μ =	12	hour
server utilization ρ or u	0.5016	0.5026	0.5			
average waiting time Wq	5.0587	2.6662	5.00			
average in system time W	10.0641	7.6634	10.00			
No. of customers to wait Lq	503.44	503.38	0.5			
No. of customers in system L	1.0064		1			
Probability of wait pw	0.5034	0.5034	0.5			

You may compute the average queue length and the average number in system with the formulas in Waiting Line chapter from the results in the table for b) and d), and add them to the table.

Among the deliverables are:

1. A table to show your answers to the question a with three customers created with the random variables given in the question a
2. The first and last five rows of results and its formulas
3. Show the changes you made to answer questions c and d, again the first and last five rows of results and its formulas
4. The Table in the answer to the question e.
5. Discussions of the results as required in the question e.