Solution to LP Modeling in Chapter 3 Ragsdale Revised 5th Edition, 2007

12. A furniture manufacturer produces two types of tables (country and contemporary) using three types of machines. The time required to produce the tables on each machine is given in the following table.

Machine	Country	Contemporary	Total Machine Time Available per Week
Router	1.5	2.0	1,000
Sander	3.0	4.5	2,000
Polisher	2.5	1.5	1,500

Country tables sell for $350 and contemporary tables sell for $450. Management has determined that at least 20% of the tables made should be country and at least 30% should be contemporary. How many of each type of table should the company produce if it wants to maximize its revenue?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?
d. How will your spreadsheet model differ if there are 25 types of tables and 15 machine processes involved in manufacturing them?

12. a. X_1 = Number of country tables to produce

X_2 = Number of contemporary tables to produce

MAX $350 X_1 + 450 X_2$

ST $1.5 X_1 + 2 X_2 \leq 1{,}000$

$3 X_1 + 4.5 X_2 \leq 2{,}000$

$2.5 X_1 + 1.5 X_2 \leq 1{,}500$

$X_1/(X_1 + X_2) \geq 0.20$ (implement as $X_1 \geq 0.2*(X_1 + X_2)$)

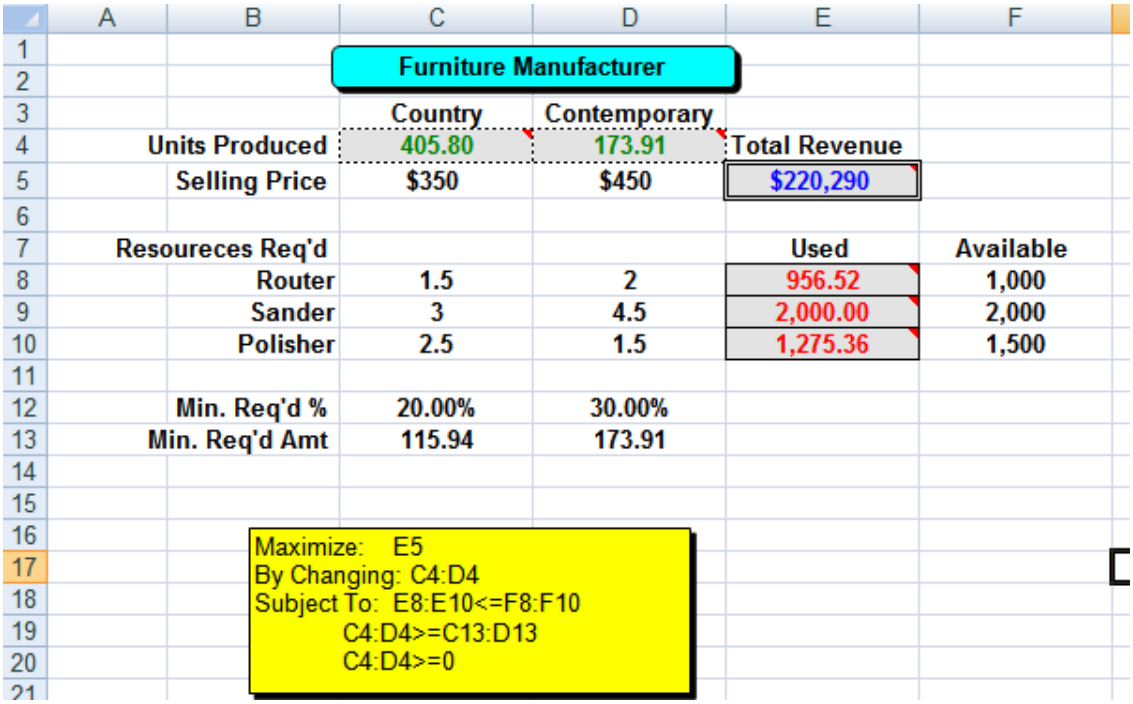
$X_2/(X_1 + X_2) \geq 0.30$ (implement as $X_2 \geq 0.3*(X_1 + X_2)$)

$X_i \geq 0$

Many students attempt to implement the ratio constraints in their original form; resulting in a division by zero error at the null solution and a message from Solver that the model is not linear. The algebraic equivalence of the alternate form of these constraints (given parenthetically above) should be noted.

b. See file: Prb3_12.xls

c. $X_1 = 405.80$, $X_2 = 173.91$, Maximum revenue = $220,290



	A	B	C	D	E	F
1						
2			Furniture Manufacturer			
3			Country	Contemporary		
4		Units Produced	405.80	173.91	Total Revenue	
5		Selling Price	$350	$450	$220,290	
6						
7		Resoureces Req'd			Used	Available
8		Router	1.5	2	956.52	1,000
9		Sander	3	4.5	2,000.00	2,000
10		Polisher	2.5	1.5	1,275.36	1,500
11						
12		Min. Req'd %	20.00%	30.00%		
13		Min. Req'd Amt	115.94	173.91		
14						
15						
16						
17						
18						
19						
20						
21						

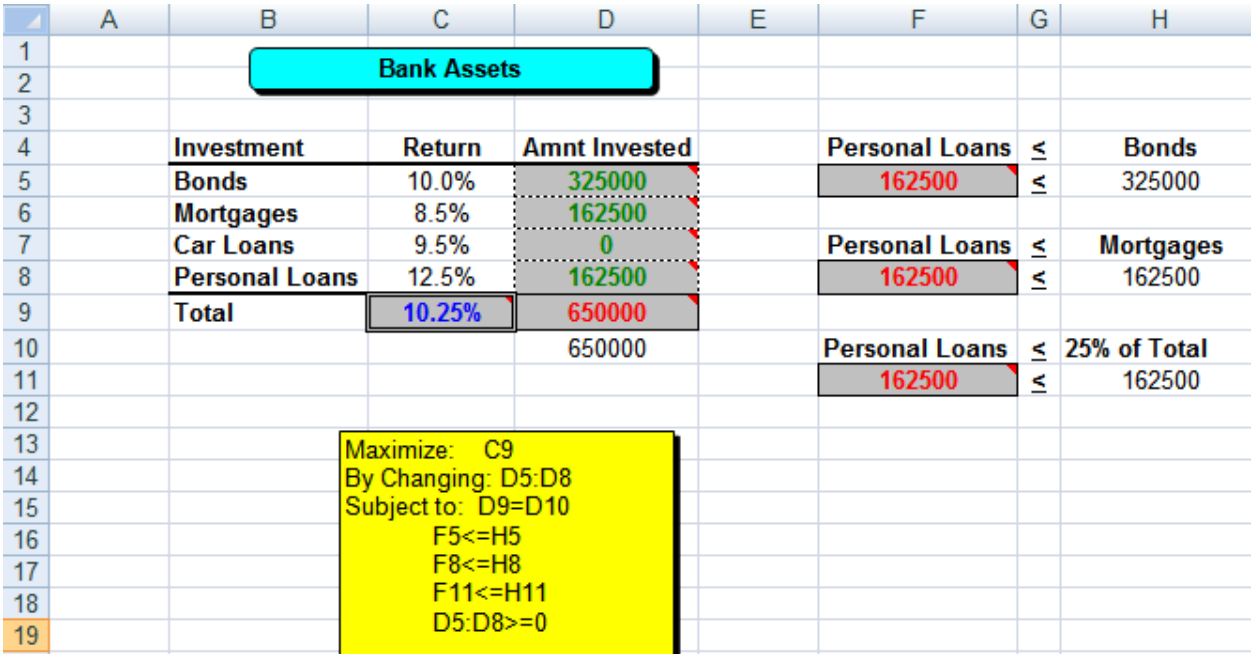
Maximize: E5
By Changing: C4:D4
Subject To: E8:E10<=F8:F10
C4:D4>=C13:D13
C4:D4>=0

17. A bank has $650,000 in assets to allocate among investments in bonds, home mortgages, car loans, and personal loans. Bonds are expected to produce a return of 10%, mortgages 8.5%, car loans 9.5%, and personal loans 12.5%. To make sure the portfolio is not too risky, the bank wants to restrict personal loans to no more than the 25% of the total portfolio. The bank also wants to ensure that more money is invested in mortgages than in personal loans. It also wants to invest more in bonds than personal loans.
    a. Formulate an LP model for this problem with the objective of maximizing the expected return on the portfolio.
    b. Implement your model in a spreadsheet and solve it.
    c. What it the optimal solution?

17. a. X_1 = Amount invested in Bonds
X_2 = Amount invested in Mortgages
X_3 = Amount invested in Car loans
X_4 = Amount invested in Personal Loans

MAX $10 X_1 + 8.5 X_2 + 9.5 X_3 + 12.5 X_4$
ST $X_1 + X_2 + X_3 + X_4 = 650{,}000$
$X_4 \le .25*(650000)$
$X_4 \le X_2$
$X_4 \le X_1$
$X_1, X_2, X_3, X_4 \ge 0$

b. See file: Prb3_17.xls
c. X_1 = 325,000, X_2 = 162,500, X_3 = 0, X_4 = 162,500, Maximum return = 10.25%



19. A trust officer at the Blacksburg National Bank needs to determine how to invest $100,000 in the following collection of bonds to maximize the annual return.

Bond	Annual Return	Maturity	Risk	Tax-Free
A	9.5%	Long	High	Yes
B	8.0%	Short	Low	Yes
C	9.0%	Long	Low	No
D	9.0%	Long	High	Yes
E	9.0%	Short	High	No

The officer wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. At least 30% of the funds should go into tax-free investments and at least 40% of the total annual return should be tax-free.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

19. a. A = amount to invest in bond A
B = amount to invest in bond B
C = amount to invest in bond C
D = amount to invest in bond D
E = amount to invest in bond E

MAX 0.095A + 0.08B + 0.09C + 0.09D + 0.09E
ST A + B + C + D + E = 100,000
B + E ≥ 50,000
A + D + E ≤ 50,000
A + B + D ≥ 30,000
0.095A + 0.08B + 0.09D ≥ 0.4* (0.095A + 0.08B + 0.09C + 0.09D + 0.09E)
A, B, C, D, E ≥ 0

b. See file Prb3_19.xls
c. A=20,339, B=20,339, C=29,661, D=0 , E=29,661
Maximum return = $8,898 (or 8.898%)

	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
26	By changing: D6:D20																			
27	Subject to: G21:N21=G22:N22																			
28	P21:W21<=0																			
29	D6:D20>=0																			
30																				
31																				
32		1	0	0	0	0	0	0	0	1	0	0	0	1	0	1				
33		A	A	A	A	A	A	A	A	B	B	B	B	C	C	D				
34		1	2	3	4	5	6	7	8	1	3	5	7	1	4	1				
35	Amount	$0.000	$0.000	$0.000	$0.000	$11.321	$0.000	$4.945	$1.354	$31.306	$35.688	$29.364	$14.531	$0.000	$0.000	$7.341	$38.647			
36	Return	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	6.0%	14.0%	14.0%	14.0%	14.0%	18.0%	18.0%	65.0%	38.64673065			
37																	LHS		RHS	
38	1	-1								-1				-1		-1	Surplus Funds:	<=>	Req'd Payments:	
39	2	1.06	-1							<---->				<---->		<---->	0.0	=	$0	
40	3		1.06	-1						1.14	-1			<---->		<---->	0.0	=	$0	
41	4			1.06	-1						<---->			1.18	-1	<---->	0.0	=	$0	
42	5				1.06	-1					1.14	-1			<---->	<---->	0.0	=	$0	
43	6					1.06	-1					<---->			<---->	<---->	12.0	=	$12	
44	7						1.06	-1				1.14	-1		1.18	<---->	14.0	=	$14	
45	8							1.06	-1				<---->			1.65	16.0	=	$16	
46	9								1.06				1.14				18.0	=	$18	
47	1	-3.0								-1.0				2.0		4.0	-1.9	<=	0	
48	2		-3.0							-1.0				2.0		4.0	-1.9	<=	0	
49	3			-3.0							-1.0			2.0		4.0	-6.3	<=	0	
50	4				-3.0						-1.0				2.0	4.0	-6.3	<=	0	
51	5					-3.0						-1.0			2.0	4.0	-34.0	<=	0	
52	6						-3.0					-1.0			2.0	4.0	0.0	<=	0	
53	7							-3.0					-1.0			4.0	0.0	<=	0	
54	8								-3.0				-1.0				-18.6	<=	0	
55																				

	A	B	C	D	E	F	G	H	I	J	K
1			Blacksburg National Bank								
2										Tax-Free	
3	Bond	Amount	Yield	Maturity		Risk		Tax-Free		Return	
4	A	$20,339	9.5%	Long	0	High	1	Yes	1	0.095	
5	B	$20,339	8.0%	Short	1	Low	0	Yes	1	0.08	
6	C	$29,661	9.0%	Long	0	Low	0	No	0	0	
7	D	$0	9.0%	Long	0	High	1	Yes	1	0.09	
8	E	$29,661	9.0%	Short	1	High	1	No	0	0	
9		$100,000	$8,898		$50,000		$50,000		$40,678	$3,559	
10		$100,000		>=	$50,000	<=	$50,000	>=	$30,000	$3,559	
11										40%	min
12											
13			Maximize: C9								
14			By Changing: B4:B8								
15			Subject to: B9=B10								
16			E9>=E10								
17			G9<=G10								
18			I9:J9>=I10:J10								
19			B4:B8>=0								

22. Tarmac Chemical Corporation produces a special chemical compound—called CHEMIX—that is used extensively in high school chemistry classes. This compound must contain at least 20% sulfur, at least 30% iron oxide, and at least 30% but no

more than 45% potassium. Tarmac's marketing department has estimated that it will need at least 600 pounds of this compound to meet the expected demand during the coming school session. Tarmac can buy three compounds to mix together to produce CHEMIX. The makeup of these compounds is shown in the following table.

Compound	Sulfur	Iron Oxide	Potassium
1	20%	60%	20%
2	40%	30%	30%
3	10%	40%	50%

Compounds 1, 2, and 3 cost \$5.00, \$5.25, and \$5.50 per pound, respectively. Tarmac wants to use an LP model to determine the least costly way of producing enough CHEMIX to meet the demand expected for the coming year.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

Tarmac Chemical Corporation

Compound	Sulfur	Iron Oxide	Potassium	Cost	% in Mix
1	20%	60%	20%	\$5.00	57.14%
2	40%	30%	30%	\$5.25	14.29%
3	10%	40%	50%	\$5.50	28.57%
Actual	20%	50%	30%	\$5.18	100.00%
Minimum	20%	30%	30%		
Maximum	*na*	*na*	45%		

Minimize: F7
By Changing: G4:G6
Subject To: G7=1
C7:E7>=C8:E8
E7<=E9
G4:G6>=0

28. Kentwood Electronics manufactures three components for stereo systems: CD players, tape decks, and stereo tuners. The wholesale price and manufacturing cost of each item are shown in the following table.

	Wholesale Manufacturing	
Component	Price	Cost
CD Player	\$150	\$75
Tape Deck	\$85	\$35
Stereo Tuner	\$70	\$30

Each CD player produced requires 3 hours of assembly; each tape deck requires 2 hours of assembly; and each tuner requires 1 hour of assembly. The marketing department has indicated that it can sell no more than 150,000 CD players, 100,000 tape decks, and 90,000 stereo tuners. However, the demand is expected to be at least 50,000 units of each item, and Kentwood wants to meet this demand. If Kentwood has 400,000 hours of assembly time available, how many CD players, tape decks, and stereo tuners should it produce to maximize profits while meeting the minimum demand figures?

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

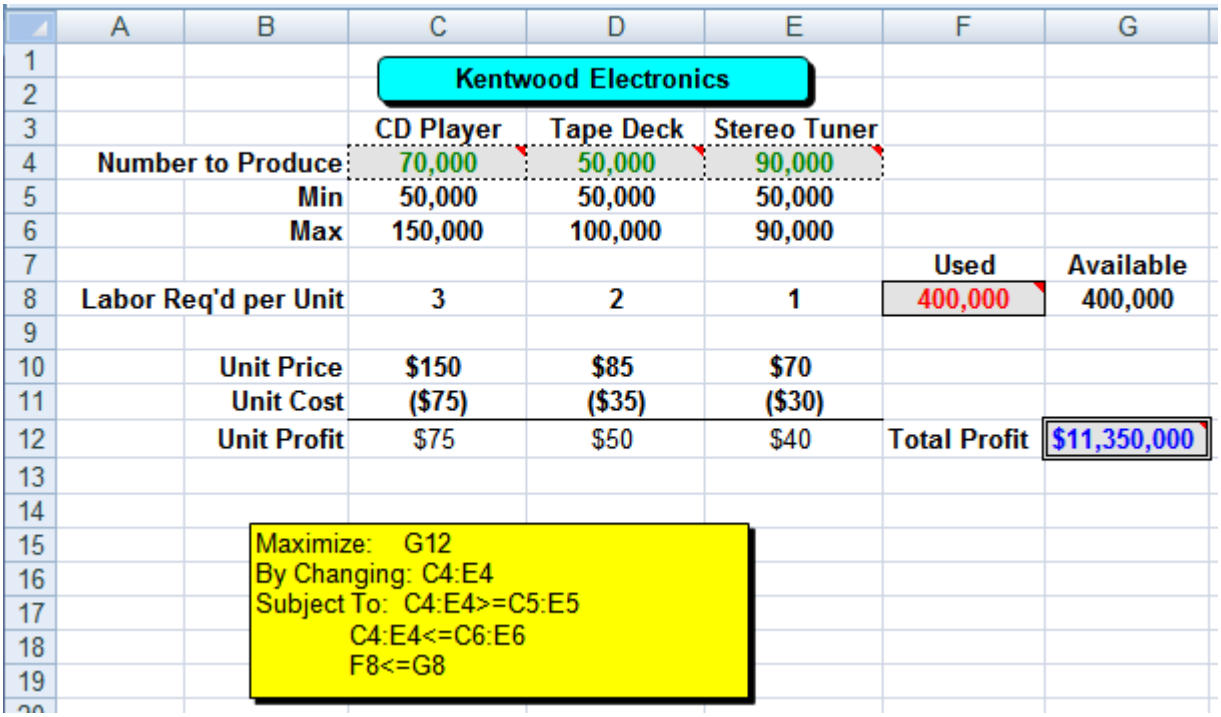
The officer wants to invest at least 50% of the money in short-term issues and no more than 50% in high-risk issues. At least 30% of the funds should go into tax-free investments and at least 40% of the total annual return should be tax-free.

a. Formulate an LP model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?

28. a. X_1 = number of CD players to produce
X_2 = number of tape decks to produce
X_3 = number of stereo tuners to produce

MAX $75 X_1 + 50 X_2 + 40 X_3$
ST $3 X_1 + 2 X_2 + 1 X_3 \leq 400{,}000$
$50{,}000 \leq X_1 \leq 150{,}000$
$50{,}000 \leq X_2 \leq 100{,}000$
$50{,}000 \leq X_3 \leq 90{,}000$

b. See file: Prb3_28.xls
c. X_1= 70,000, X_2 = 50,000, X_3 = 90,000
Maximum profit = $11,350,000



	A	B	C	D	E	F	G
1							
2			Kentwood Electronics				
3			CD Player	Tape Deck	Stereo Tuner		
4		Number to Produce	70,000	50,000	90,000		
5		Min	50,000	50,000	50,000		
6		Max	150,000	100,000	90,000		
7						Used	Available
8		Labor Req'd per Unit	3	2	1	400,000	400,000
9							
10		Unit Price	$150	$85	$70		
11		Unit Cost	($75)	($35)	($30)		
12		Unit Profit	$75	$50	$40	Total Profit	$11,350,000
13							
14							
15			Maximize: G12				
16			By Changing: C4:E4				
17			Subject To: C4:E4>=C5:E5				
18			C4:E4<=C6:E6				
19			F8<=G8				

30. The Sentry Lock Corporation manufactures a popular commercial security lock at plants in Macon, Louisville, Detroit, and Phoenix. The per unit cost of production at each plant is $35.50, $37.50, $39.00, and $36.25, respectively, and the annual production capacity at each plant is 18,000, 15,000, 25,000, and 20,000, respectively. Sentry's locks are sold to retailers through wholesale distributors in seven cities across the United States. The unit cost of shipping from each plant to each distributor is summarized in the following table along with the forecasted demand from each distributor for the coming year.

	Unit Shipping Cost to Distributor in						
Plants	**Tacoma**	**San Diego**	**Dallas**	**Denver**	**St. Louis**	**Tampa**	**Baltimore**
Macon	$2.50	$2.75	$1.75	$2.00	$2.10	$1.80	$1.65
Louisville	$1.85	$1.90	$1.50	$1.60	$1.00	$1.90	$1.85
Detroit	$2.30	$2.25	$1.85	$1.25	$1.50	$2.25	$2.00
Phoenix	$1.90	$0.90	$1.60	$1.75	$2.00	$2.50	$2.65
Demand	8,500	14,500	13,500	12,600	18,000	15,000	9,000

Sentry wants to determine the least expensive way of manufacturing and shipping locks from their plants to the distributors. Because the total demand from distributors exceeds the total production capacity for all the plants, Sentry realizes it will not be able to satisfy all the demand for its product, but wants to make sure each distributor will have the opportunity to fill at least 80% of the orders they receive.

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution?

Sentry Lock Corp.

Plants	Tacoma	San Diego	Dallas	Denver	St. Louis	Tampa	Baltimore	Unit Production Cost
			Unit Shipping Costs to Distribution Center in:					
Macon	$2.50	$2.75	$1.75	$2.00	$2.10	$1.80	$1.65	$35.50
Louisville	$1.85	$1.90	$1.50	$1.60	$1.00	$1.90	$1.85	$37.50
Detroit	$2.30	$2.25	$1.85	$1.25	$1.50	$2.25	$2.00	$39.00
Phoenix	$1.90	$0.90	$1.60	$1.75	$2.00	$2.50	$2.65	$36.25

Minimize: B23
By chaning: B15:H18
Subject to: I15:I18=J15:J18
B19:H19<=B20:H20
B19:H19>=B21:H21
B15:H18>=0

Plants	Tacoma	San Diego	Dallas	Denver	St. Louis	Tampa	Baltimore	Total Produced	Capacity
			Quantity Shipped to Distribution Cente						
Macon	0	0	0	0	0	12000	6000	18000	18000
Louisville	600	0	0	0	14400	0	0	15000	15000
Detroit	400	0	10800	12600	0	0	1200	25000	25000
Phoenix	5800	14200	0	0	0	0	0	20000	20000
Total Shipped	6800	14200	10800	12600	14400	12000	7200		
Demand	8500	14500	13500	12600	18000	15000	9000		
Min Shipped	6800	11600	10800	10080	14400	12000	7200		

Total Cost	$3,011,360

34. a. P_i = Number of units to produce in month *i*

I_i = Inventory held at the end of month *i*

MIN $49 X_1 + 45 X_2 + 46 X_3 + 47 X_4 - 1.5 (120 + 2I_1 + 2I_2 + 2I_3 + I_4)/2$

ST $I_1 = 120 + P_1 - 420$

$I_2 = I_1 + P_2 - 580$

$I_3 = I_2 + P_3 - 310$

$I_4 = I_3 + P_4 - 540$

$400 \le P_1 \le 500$

$400 \le P_2 \le 520$

$400 \le P_3 \le 450$

$400 \le P_4 \le 550$

$I_i \ge 50$

Note that the I_i can be computed directly from the P_i. Therefore, the P_i are the only decision variables (changing cells) in the model. The I_i can be computed in the spreadsheet using the conditions imposed by the first four constraints. Therefore, it is not necessary to indicate these as constraints cells for Solver. However, note that lower bounds of 50 must be indicated for these cells.

b. See file: Prb3_34.xls

c. $X_1 = 410$, $X_2 = 520$, $X_3 = 400$, $X_4 = 450$, $I_1 = 110$, $I_2 = 50$, $I_3 = 140$, $I_4 = 50$

Minimum cost = $83,617

39. a. MIN $A_{12} + B_{13} + C_{14} + D_{18}$

ST $1.06A_{12} - A_{23} = 0$

$1.06A_{23} + 1.14B_{13} - A_{34} - B_{35} = 0$

$1.06A_{34} + 1.18C_{14} - A_{45} - C_{47} = 0$

$1.06A_{45} + 1.14B_{35} - A_{56} - B_{57} = 0$

$1.06A_{56} - A_{67} = 12$

$1.06A_{67} + 1.14B_{57} + 1.18C_{47} - A_{78} - B_{79} = 14$

$1.06A_{78} + 1.65D_{18} - A_{89} = 16$

$1.06A_{89} + 1.14B_{79} = 18$

$A_{ij}, B_{ij}, C_{ij}, D_{ij} \ge 0$

b. See file Prb3_39.xls

c. A_{56} = $11,321, A_{89} = $16,981, B_{13} = $18,161, B_{35} = $20,703, B_{57} = $12,281, D_{18} = $19,989

Minimum investment = $38,149

40. a. Same as in problem 38 above with the following additional constraints:

$-3A_{12} - 1B_{13} + 2C_{14} + 4D_{18} \le 0$

$-3A_{23} - 1B_{13} + 2C_{14} + 4D_{18} \le 0$

$-3A_{34} - 1B_{35} + 2C_{14} + 4D_{18} \le 0$

$-3A_{45} - 1B_{35} + 2C_{47} + 4D_{18} \le 0$

$-3A_{56} - 1B_{57} + 2C_{47} + 4D_{18} \le 0$

$-3A_{67} - 1B_{57} + 2C_{47} + 4D_{18} \le 0$

$-3A_{78} - 1B_{79} + 4D_{18} \le 0$

$-3A_{89} - 1B_{79} \le 0$

b. See file Prb3_40.xls

c. A_{56} = $11,321, A_{78} = $4,945, A_{89} = $1,345, B_{13} = $31,306, B_{35} = $35,688, B_{57} = $29,364, B_{79} = $14,531, D_{18} = $7,341

Minimum investment = $38,647