Quiz Queue

UMJ, a mail-order company, receives calls to place orders at an average of 7.5 minutes intervals. UMJ hires one operator and can handle each call in about every 5 minutes on average. The inter-arrival times and service times follow exponential distribution.

1. What are the average arrival and service rates of calls at UMJ?
   a. The average arrival rate \( \lambda \) is 8 per hour and the average service rate \( \mu \) is 12 per hour.
   b. The average arrival rate \( \lambda \) is 7.5 per hour and the average service rate \( \mu \) is 5 per hour.
   c. The average arrival rate \( \lambda \) is 12 per hour and the average service rate \( \mu \) is 8 per hour.
   d. The average arrival rate \( \lambda \) is 5 per hour and the average service rate \( \mu \) is 7.5 per hour.

2. What is the probability of more than 1 call; and less than 5 calls arriving at UMJ in a 30-minute period?
   a. The average arrival rate \( \lambda \) is 4 per 30 minutes, and \( P(2 \leq X \leq 4) = \text{POISSON}(4,4,\text{TRUE}) - \text{POISSON}(2,4,\text{TRUE}) = 0.3907 \)
   b. The average arrival rate \( \lambda \) is 4 per 30 minutes, and \( P(1 < X < 5) = \text{POISSON}(4,4,\text{TRUE}) - \text{POISSON}(1,4,\text{TRUE}) = 0.5373 \)
   c. The average arrival rate \( \lambda \) is 8 per 30 minutes, and \( P(1 < X < 5) = \text{POISSON}(4,8,\text{TRUE}) - \text{POISSON}(1,8,\text{TRUE}) = 0.0966 \)
   d. The average arrival rate \( \lambda \) is 8 per 30 minutes, and \( P(2 \leq X \leq 4) = \text{POISSON}(4,8,\text{TRUE}) - \text{POISSON}(2,8,\text{TRUE}) = 0.085 \)

Suppose UMJ receives calls to place orders at an average rate of 10 per hour and Sarah, the only operator at UMJ, can handle 15 calls per hour on the average. Both the number of calls arrived and the number of calls answered follow Poisson distribution.

3. What are the chances for Sarah take between 3 and 6 minutes to answer a call or \( P(3\text{mins.} \leq X \leq 6 \text{ mins.}) \)?
   a. \( e^{-15\frac{6}{60}} - e^{-15\frac{3}{60}} = \text{EXPONDIST}(6,15,\text{TRUE}) - \text{EXPONDIST}(3,15,\text{TRUE}) = 0.0000 \)
   b. \( e^{-10(6)} - e^{-10(3)} = \text{EXPONDIST}(6,10,\text{TRUE}) - \text{EXPONDIST}(3,10,\text{TRUE}) = 0.0000 \)
   c. \( e^{-15\frac{6}{60}} - e^{-15\frac{3}{60}} = \text{EXPONDIST}(6/60,15,\text{TRUE}) - \text{EXPONDIST}(3/60,15,\text{TRUE}) = 0.2492 \)
   d. \( e^{-15\frac{3}{60}} - e^{-15\frac{6}{60}} = \text{EXPONDIST}(6/60,15,\text{TRUE}) - \text{EXPONDIST}(3/60,15,\text{TRUE}) = 0.2492 \)

4. What is the utilization or the percentage of the time that Sarah is busy answering calls?
   a. \( \lambda/\mu = 66.67\% \)
   b. \( \mu/\lambda = 125\% \)
   c. \( P_0 = \lambda/\mu = 66.67\% \)
   d. \( 1 - P_0 = 1 - \frac{\lambda}{\mu} = 33.33\% \)

5. Which one of the followings is correct for answering the question “What is the average amount of time a customer spent holding on line for Sarah to answer?”
   I. \( \frac{\lambda}{\mu(\mu-\lambda)} \cdot \frac{t_q}{\lambda} = 0.1333 \text{ Hour} = 8 \text{ minutes} \)
   II. \( \frac{\lambda^2}{\mu(\mu-\lambda)} = 1.3333 \)
   III. \( \frac{1}{\mu-\lambda} = 0.2 \text{ Hours} = 12 \text{ minutes} \)
IV. \[ \frac{L_q}{\lambda} = \frac{1.3333}{10} \] \text{Hour} = 8 \text{ minutes}

6. Which one of the followings is correct for answering the question “What is the average number of calls waiting for Sarah to answer?”

   I. \[ \frac{\lambda}{\mu(\mu-\lambda)} = 0.1333 \]
   II. \[ \lambda W_q = 1.3333 \]
   III. \[ W_q + \frac{1}{\mu} = 0.2 \]
   IV. \[ \frac{\lambda^2}{\mu(\mu-\lambda)} = 1.3333 \]

   a. II and IV only
   b. IV only
   c. I and II only
   d. II and III only

7. Which one of the followings is correct for answering the question “How long, on average, does it take for a call to get answered by Sarah?”

   I. \[ \frac{\lambda}{\mu(\mu-\lambda)} = \frac{L_q}{\lambda} = 0.1333 \text{ Hour} = 8 \text{ minutes} \]
   II. \[ W_q + \frac{1}{\mu} = 8 \text{ minutes} + 4 \text{ minutes} = 12 \text{ minutes} \]
   III. \[ \frac{1}{\frac{1}{\mu-\lambda}} = 0.2 \text{ Hours} = 12 \text{ minutes} \]
   IV. \[ L_q + \frac{\lambda}{\mu} = 2 \]

   a. II and IV only
   b. III only
   c. I and II only
   d. II and III only

8. Which one of the followings is correct for answering the question “What is the average number of calls in the system?”

   I. \[ \frac{\lambda}{\mu-\lambda} = \lambda W_q = 1.3333 \]
   II. \[ L_q + \frac{\lambda}{\mu} \]
   III. \[ \lambda W = 2 \]

   a. I, II and III only
   b. II, III and IV only
   c. I, III and IV only
   d. III and IV only
9. Which one of the followings is correct for answering the question “What is the probability that an incoming call will have to hold on line to get answered by Sarah?”
   I. \( \frac{\lambda}{\mu} = 66.67\% \)
   II. \( P(n \geq 1) \)
   III. \( P_0 = 66.67\% \)
   IV. \( 1 - P_0 \)
   a. I and III only
   b. I, II, and IV only
   c. I, III and IV only
   d. I, II and III only

10. Which one of the followings is correct for answering the question “What is the probability there are more than three calls waiting for Sarah to answer?”
    I. \( P(n > 3) = 1 - P(n \leq 3) = 1 - (P_0 + P_1 + P_2 + P_3) \)
    II. \( P(n \geq 4) = 1 - P(n \leq 4) = 1 - (P_0 + P_1 + P_2 + P_3 + P_4) \)
    III. \( P(n \geq 5) = 1 - P(n \leq 4) = 1 - (P_0 + P_1 + P_2 + P_3 + P_4) \)
    IV. \( P_4 = P_0 \left( \frac{\lambda}{\mu} \right)^4 \)
    a. I and IV only
    b. II and IV only
    c. III only
    d. II only

Suppose UMJ receives calls to place orders at an average rate of 10 per hour and Sarah, the only operator at UMJ, can handle 15 calls per hour on the average. Both the number of calls arrived and the number of calls answered follow Poisson distribution. Sarah was paid $15 per hour in wages and benefits. The company estimates that it costs $0.50 per minute in lost sales on average for a customer to hold on line when Sarah is busy answering other calls.

11. What is the hourly average cost for UMJ to operate M/M/1 call center as given above?
    a. Total hourly cost = $15 + $0.50 x 60 = $45.00 per hour
    b. Total hourly cost = $15 + $0.50 x 60 x (L = 2) = $75.00 per hour
    c. Total hourly cost = $0.50 x 60 x (L = 2) = $60.00 per hour
    d. Total hourly cost = $15.00 per hour

(You may use Q.xls to get the answers first for some of the variables for the following questions, but be sure to provide adequate explanations. Q.xls output is given in the following table for reference).

<table>
<thead>
<tr>
<th>Arrival rate</th>
<th>14</th>
<th>14</th>
<th>14</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service rate</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Number of servers</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Utilization</td>
<td>93.33%</td>
<td>46.67%</td>
<td>31.11%</td>
<td>23.33%</td>
</tr>
<tr>
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</tr>
<tr>
<td>P(0), probability that the system is empty</td>
<td>0.0667</td>
<td>0.3636</td>
<td>0.3898</td>
<td>0.3928</td>
</tr>
<tr>
<td>Lq, expected queue length</td>
<td>13.0667</td>
<td>0.2598</td>
<td>0.0346</td>
<td>0.0049</td>
</tr>
<tr>
<td>L, expected number in system</td>
<td>14.0000</td>
<td>1.1932</td>
<td>0.9680</td>
<td>0.9383</td>
</tr>
<tr>
<td>Wq, expected time in queue</td>
<td>0.9333</td>
<td>0.0186</td>
<td>0.0025</td>
<td>0.0004</td>
</tr>
<tr>
<td>W, expected total time in system</td>
<td>1.0000</td>
<td>0.0852</td>
<td>0.0691</td>
<td>0.0670</td>
</tr>
<tr>
<td>Probability that a customer waits</td>
<td>0.9333</td>
<td>0.2970</td>
<td>0.0767</td>
<td>0.0162</td>
</tr>
</tbody>
</table>

12. Suppose that business increases by 40% in Friday afternoon, i.e., the average call arrival rate \( \lambda \) is 14 per hour and Sarah answers 15 calls per hour on the average. What is the average number of calls in the system?

   a. The average number of calls = 13.0667
   b. The average number of calls = 60
   c. The average number of calls = 56
   d. The average number of calls = 14

13. What is the total hourly operating cost for UMJ if only Sarah answers calls with the increased incoming call volume?

   a. Total hourly cost = $15 = $15.00 per hour
   b. Total hourly cost = $15 + $0.50 \times 60 \times (L = 2) = $75.00 per hour
   c. Total hourly cost = $15 + $0.50 \times 60 \times (L = 14) = $435.00 per hour
   d. Total hourly cost = $0.50 \times 60 \times (L = 14) = $415.00 per hour

14. With the increased incoming call volume, what is the total hourly operating cost for UMJ if Eric is hired as the second operator with the same pay of $15 per hour and Eric answers 15 calls per hour as Sarah does?

   a. Total hourly cost = $15 \times 2 = $30.00 per hour
   b. Total hourly cost = $15 \times 2 + $0.50 \times 60 \times (L = 1.1932) = $65.80 per hour
   c. Total hourly cost = $15 \times 2 + $0.50 \times 60 \times (L = 14) = $450.00 per hour
   d. Total hourly cost = $0.50 \times 60 \times (L = 1.1932) = $35.80 per hour

15. What is a call’s average holding time if there is a separate phone line for each operator and we assume that approximately half of the calls join each line to hold? Assume \( \lambda = 7 \) incoming calls per hour for each line, and \( \mu = 15 \) calls per hour for each operator.

   a. = 3.5 minutes
   b. = 7.5 minutes
   c. = 0.876 minutes
   d. = 0.4667 minutes

16. What is the total cost per hour if there is a separate phone line for each operator and we assume that approximately half of the calls join each line? Assume \( \lambda = 7 \) incoming calls per hour for each line, and \( \mu = 15 \) calls per hour for each operator, $15 per hour is paid for each operator and $0.50 per minute for each minute a customer spent to make the call.

   a. = 2 \times $15 = $30.00 per hour
   b. = 2 \times \{$15 + $0.50 \times 60 \times 0.875\} = $82.50 per hour
   c. = 2 \times \{$15 + $0.50 \times 60 \times 1.1932\} = $101.59 per hour
   d. = 2 \times $0.50 \times 60 \times 0.875 = $52.50 per hour
17. Would you recommend UMJ to hire third operator for the situation? Why and Why not? Assume calls are through one single line to feed all of the three operators.
   a. No, Total hourly cost = $15 x 3 + $0.50 x 60 x (L = 0.968) = $74.04
   b. No, Total hourly cost = $15 x 3 + $0.50 x 60 x (L = 1.1932) = $80.80 per hour
   c. Yes, Total hourly cost = $15 x 3 = $45.00
   d. Yes, Total hourly cost = $0.50 x 60 x (L = 0.9680) = $29.04 per hour

18. What is the probability that no any calls in the system for UMJ with three operators to answer calls?
   Assume $\lambda = 14$ calls per hours and $\mu = 15$ calls per hour for each operator and $s = 3$.
   a. 0.0667
   b. 0.3636
   c. 0.3898
   d. 0.0162

20. What is the value of the probability that no any calls in the system ($s=2$)? You should be able to derive the answer from the general equation of M/M/s queue.
   a. 0.0667
   b. 0.3636
   c. 0.4667
   d. 0.2970

21. What is the chance that there is at least one call in the system?
   a. 0.9333
   b. 0.6102
   c. 0.6072
   d. 0.6364

22. What is the average number of calls waiting to be answered?
   a. 13.0667
   b. 0.2598
   c. 14.000
   d. 1.1932

23. What is the average number of calls in the system?
   a. 13.0667
   b. 0.2598
   c. 14.0000
   d. 1.1932

24. What is the average amount of time spent holding on line by customers of UMJ before being answered?
   a. 56.0000 minutes
   b. 60.0000 minutes
   c. 1.1136 minutes
   d. 5.1136 minutes
25. How long, on average, does it take a call to be answered at UMJ?
   a. 56.0000 minutes
   b. 60.0000 minutes
   c. 1.1136 minutes
   d. 5.1136 minutes

26. What is the probability that a call will have to be held on line to be answered at UMJ?
   a. 0.2970
   b. 0.9333
   c. 0.0767
   d. 0.0162

27. Once a call is started to be answered, how long on the average it takes for a call to be answered?
   a. 1 minute
   b. 2 minutes
   c. 3 minutes
   d. 4 minutes

28. What is the probability there are two calls at UMJ?
   a. $= \frac{(\lambda/\mu)^2}{2!} P_0$
   b. $= P_0$
   c. $= P_0 + P_1 + P_2$
   d. $= 1 - P_2$

29. What is the probability there are more than two customers waiting to be served?
   a. $= P(n \geq 3 ) = 1 - P(n \leq 2) = 1 - \{P_0 + P_1 + P_2\}$
   b. $= P(n \geq 4 ) = 1 - P(n \leq 3) = 1 - \{P_0 + P_1 + P_2 + P_3\}$
   c. $= P(n \geq 5 ) = 1 - P(n \leq 4) = 1 - \{P_0 + P_1 + P_2 + P_3 + P_4\}$
   d. $= P_4 = \frac{(\lambda/\mu)^4}{2! 2^2} P_0$