COB291, Management Science, Test 2, Fall 2008

Physicians of JMU health center check up patients during the day at the rate of one every 15 minutes. Assume the check up time is exponentially distributed. It is observed that on the average one patient arrives at the health center in every 20 minutes according to Poisson distribution to join the single waiting line. JMU would like to study the current service level at its health center using M/M/s model in Q.xls. **you will NOT get any credit if you do not provide details (equations with appropriate notations and procedures) even if you provide correct answers for a question.**

Assume only one physician is doing the check – ups and M/M/1 Queuing model is used for Questions 1 to 7.

1. (10 points) What is the probability that more than three and less than six patients arrive at the health center in two hours?

2. (5 points) Recall that the function for the Poisson distribution has the form = POISSON(x, mean, cumulative). Entering the formula = POISSON(2, 2, TRUE) in Excel will return a value of 0.6767. Show the standard equation with the result for the problem and briefly interpret the meaning of it.

3. (10 points) What is the probability that a patient will spend more than 10 minutes for the check up?

4. (5 points) What is the Excel@ formula or function to be used to compute the probability that a patient will spend more than 10 minutes for the check up?

5. (5 points) What is the probability that there is more than one patient in the health center for check -ups?

6. (5 points) What is the expected number of patients in the health center?

7. (10 points) Suppose the physician receives pay of $100 per hour and the cost of a patient waiting is estimated to be $1 per minute, what is the total cost per hour for JMU to operate its health center?

-----END of M/M/1 Model and START of M/M/s Model-------

Assume all of the information is given in Table 1:

- **Two or more physician are doing the check – ups and M/M/s model is used for Questions 8 to 10**
- **All of time units are in hours and**
- **The arrival rate λ and the service rate μ for M/M/s model may be different than that of M/M/1 model**

	Option 1	Option 2	Option 3	Option 4	Option 5
Arrival rate	5	5	15	15	15
Service rate	8	8	8	8	8
Number of servers	**1**	**2**	**2**	**3**	**4**
Utilization	62.50%	31.25%	93.75%	62.50%	46.88%
P(0), probability that the system is empty	0.375	0.5238	0.0323	0.1322	0.1492
Lq, expected queue length	1.0417	0.0676	13.6089	0.6457	0.1276
L, expected number in system	1.6667	0.6926	15.4839	2.5207	2.0026
Wq, expected time in queue	0.2083	0.0135	0.9073	0.043	0.0085
W, expected total time in system	0.3333	0.1385	1.0323	0.168	0.1335
Probability that a customer waits	0.625	0.1488	0.9073	0.3874	0.1447
P1	0.2344	0.3274	0.0606	0.2479	0.2798
P2	0.1465	0.1023	0.0568	0.2324	0.2623
P3	0.0916	0.0320	0.0532	0.1452	0.1639
P4	0.0572	0.0100	0.0499	0.0908	0.0768
P5	0.0358	0.0031	0.0468	0.0567	0.0360

8. (5/10 points) Assume there are 15 patients arrive at the health center per hour.
   a. If JMU wants the expected time for a patient to wait in line before being checked to be no more than 3 minutes, which option(s) should JMU take and why?

   b. With three physicians do check – ups, what is the probability that more than one patient is waiting in line?

9. (12 points) if JMU wants to minimize the hourly total operating cost with the arrival rate λ of 15 patients per hour, which option should the bank take and why?

10. (13 points) As in the table for the option 4, P_0 = 0.1322 for s = 3 (three physicians do the check – ups). Use the M/M/s queuing equation $P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu-\lambda}\right)\right]^{-1}$ to confirm the value of 0.1332.

----------This is the end of JMU Health Center Case. ---Start of Toy Doll Case --------

In preparing for the upcoming holiday season, Marshall Toy Co. designed a new doll called Freddy.

- The fixed cost to produce the doll is $1,000.
- The variable cost, which includes material, labor, and shipping cost, follows the probability distribution as in the Table 2 below from $1 to $3.
- The sell price per doll is uniformly distributed between $4 and $6.
- If Marshall overproduces the dolls, the excess dolls will be sold in January through a distributer who has agreed to pay Marshall a discounted price of only $1 per doll.
- Demand for new toys during the holiday selling season is extremely uncertain. Forecasts are expected a demand of 600 dolls with a standard deviation of 150. Assume the demand follows the normal distribution and we know P(Z < 0.1383) = 0.555 for standardized normal distribution.
- If Marshall has shortages of dolls, they would pay a stiff shortage penalty cost of $5 per doll.

You are asked to do a simulation study to analyze the mean net profit and probability of losses for the sales of the dolls. A spreadsheet is set up for you as given in the Table 2 above. You are required to use the information in Table 2 to answer the following questions.

The Total revenue is the summation of the revenue through the dolls sold during the holiday and the excess revenue after the holiday season. The Total cost is the summation of the Fixed cost, Variable cost and Shortage cost. The Net profit is the difference between the Total revenue and the Total cost.

	A	B	C	D	E	F	G	H	I	J	K	L
1	Test 2 Simulation									Sell price low limit		4
2		Production Quantity		600						Sell price high limit		6
3			Fixed Cost	1000								
4										shortage penalty cost		5
5		Variable Cost		Probability						Excess discount price		1
6			1	0.3								
7			2	0.5						Mean of demand		600
8			3	0.2						Standard devision of demand		150
9												
10		Random number										
11		0.0682	0.7669	0.555			Shortage		Excess	Total		
12	Trial No	Var Cost	Sell price	Demand	Sold	Shortage	Cost	Excess	Revenue	Revenue	Total cost	Net profit
13	1				No need to compute any other values for Trial No. 1							
14	2	2	5.637	458.056								
15	3	3	4.797	737.705								
16	4											-12.856

11. (3 points each) For Trial No. 1 in Table 2, you are to use the random numbers given above each variable to answer the corresponding questions:
    a. What is the value of the variable cost to be used?

    b. What is the Excel formula to be used in Cell B13 to obtain the value of the variable cost?

    c. What is the value of the Sell price to be used?

    d. What is the Excel formula to be used in Cell C13 to obtain the value of the sell price?

e. What is the value of the Demand to be used?

f. What is the Excel formula to be used in Cell D13 to obtain the value of the Demand?

12. (3 points each) For Trial No. 2 in Table 2, you are to use the values of given variable cost, sell price and demand to answer the following questions:
    a. What is the number of dolls sold?

    b. What are the number of Shortages and the Shortage cost?

c. What are the number of Excess and the Excess revenue?

d. What are the Total revenue, Total cost and Net profit?

13. (3 points each) For Trial No. 3 in Table 2, you are to use the values of given variable cost, sell price and demand to answer the following questions:
    a. What is the number of dolls sold?

    b. What are the number of Shortages and the Shortage cost?

    c. What are the number of Excess and the Excess revenue?

d. What are the Total revenue, Total cost and Net profit?

14. (3 points each) Use Trial No. 3 in Row 15 in Table 2, you are to provide Excel formulas to be used to simulate the doll sales 1100 times. You need to be sure the Excel formulas will be correct when they are used for all of the 1100 trials. One way to check them is to verify the formulas with your computations for Trials 2 and 3.
    a. What is Excel formula for the number of dolls sold in Cell E15?

    b. What are Excel formulas for the number of Shortages and the Shortage cost in Cell F15 and G15 respectively?

    c. What are Excel formulas for the number of Excess and the Excess revenue in Cell H15 and I15 respectively?

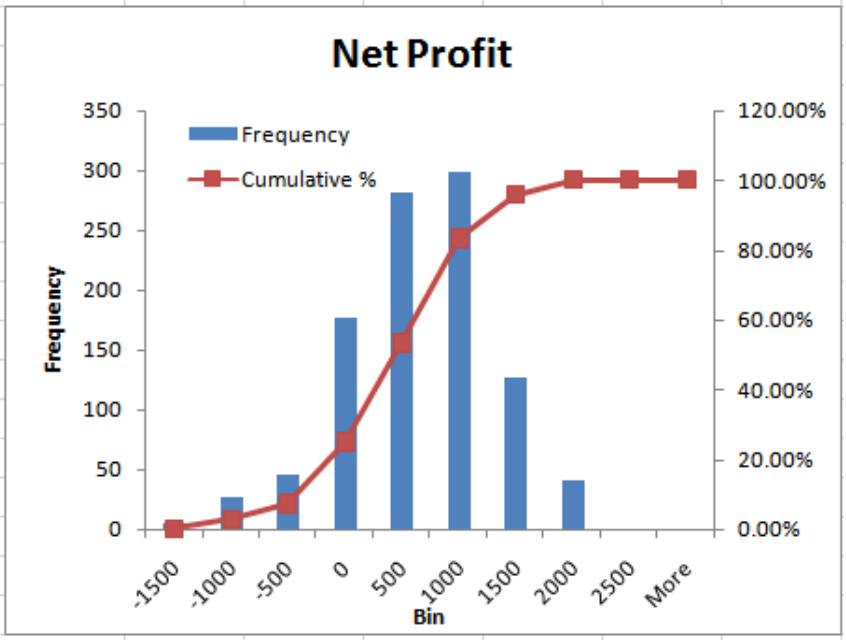
    d. What are Excel formulas for the Total revenue, Total cost and Net profit in Cell J15, K15 and L15 respectively?

15. (3 points each) Assume the 1100 trials starts in the Row 13 and ends in the Row 1112 in Table 2. You could use Trials 1, through 4 in Table 2 to verify your answers.
    a. What is the Excel formula to be used for the number of losses from the last 1000 trials?

    b. What is the Excel formula to be used for the percentage of losses from the last 1,000 trials?

Now, you run the simulation with 1,100 trials. You discard the first 100 trials and use only the last 1,000 trials. As in Table 3 and Figure 1 below, you obtain a frequency table, a histogram and the estimate of the mean net profit of 362.73, the standard error of the mean net profit of 46.78, and the average number of losses of 200.69 from the last 1,000 trials. Answer the following questions:

Net Profit	*Frequency*	*Cumulative %*
-1500	4	0.40%
-1000	26	3.00%
-500	45	7.50%
0	177	25.20%
500	282	53.40%
1000	298	83.20%
1500	127	95.90%
2000	41	100.00%
2500	0	100.00%
More	0	100.00%
	Net profit	No of loss
100		
Mean	362.73	200.69
Standard error	46.78	



16. (5 points each)
    a. What is the margin of error, and what is the 95% confidence interval for the mean net profit?

b. What is the meaning of the 95% confidence interval for the mean net profit?

17. (5 points) What is the probability for the mean net profit to be more than $1,500?

18. (5/5 points)
    a. What is the 95% confidence interval for the proportion of losses? (n = 100)

    b. What is the meaning of the 95% confidence internal for the proportion of losses?