

Test 2, Spring 2009

During a normal week, the reference desk of JMU East Library serves students at the rate of one every 6 minutes. Assume the service time is exponentially distributed. It is observed that students arrive at the reference desk in every 10 minutes according to Poisson distribution to join a single waiting line. The library would like to study the current service level at its reference desk using M/M/s model in Q.xls. **You will NOT get much credit if you do not provide details (equations with appropriate notations and procedures) even if you provide correct answers for a question.**

Assume only one reference librarian is working and M/M/1 Queuing model is used for Questions 1 to 7.

1. (8 points) Show the equation and the process to obtain the probability that more than one and less than four students arrive at the reference desk in thirty minutes?
2. (4 points) Show the Excel@ formula to obtain the probability that more than one and less than four students arrive at the reference desk in thirty minutes?
3. (8 points) Show the equation and the process to obtain the probability that a librarian will spend more than 5 minutes to serve a student?
4. (4 points) Show the Excel@ formula or function to be used to compute the probability that a librarian will spend more than 5 minutes to serve a student?
5. (8 points) Show the equation and the process to obtain the probability that there is more than one and less than five students in the reference area?
6. (5 points) Show the equation and the process to obtain the average number of students in the reference area?
7. (5 points) Show the equation and the process to obtain the average amount of time a student spent waiting in the line?

-----END of M/M/1 Model and START of M/M/s Model-----

Assume all of the information is given in Table 1:

- **Two or more librarians are working and M/M/s model is used for Questions 8 to 10**
- **All of time units are in hours and**
- **The arrival rate λ and the service rate μ for M/M/s model may be different than that of M/M/1 model**

	Option 1	Option 2	Option 3	Option 4	Option 5
Arrival rate	6	6	18	18	18
Service rate	10	10	10	10	10
Number of servers	1	2	2	3	4
Utilization	60.00%	30.00%	90.00%	60.00%	45.00%
P(0), probability that the system is empty	0.4	0.5385	0.0526	0.1460	0.1616
Lq, expected queue length	0.9	0.0593	7.6737	0.5321	0.1052
L, expected number in system	1.5	0.6593	9.4737	2.3321	1.9052
Wq, expected time in queue	0.15	0.0099	0.4263	0.0296	0.0058
W, expected total time in system	0.25	0.1099	0.5263	0.1296	0.1058
Pw, Probability that a customer waits	0.6	0.1385	0.8526	0.3547	0.1285
P1	0.2344	0.3274	0.0606	0.2479	0.2798
P2	0.1465	0.1023	0.0568	0.2324	0.2623
P3	0.0916	0.0320	0.0532	0.1452	0.1639
P4	0.0572	0.0100	0.0499	0.0908	0.0768
P5	0.0358	0.0031	0.0468	0.0567	0.0360

8. Assume there are 18 students arrive at the reference desk per hour.
- (8 points) If the library wants the expected time for a student to wait in line before being checked to be no more than 6 minutes, which option(s) should NOT the library take and why? (Show the equations and processes)
 - (8 points) With two reference librarians serve the reference desk, what is the probability that more than one student is waiting in line? (Show the equations and processes)
9. (8 points) if library wants to minimize the hourly total operating cost with the arrival rate λ of 18 students per hour, which option should the library take and why? (Show the equations and processes) Suppose the librarian is paid \$80 per hour and the cost of a student waiting is estimated to be \$0.50 per minute.

10. (7 points) As in the table for the option 2, $P_0 = 0.5385$ for $s = 2$ (two librarians at the reference desk). Use the M/M/s queuing equation $P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \left(\frac{s\mu}{s\mu - \lambda} \right) \right]^{-1}$ to confirm the value of $P_0=0.5385$. (Must show the process)

-----This is the end of JMU Reference area Case. ---Start of COB300.com Case -----

From <http://www.com300.com>.



Heft a cold one in this 22 oz. ceramic stein with gold trim. A great "usable" alternative to the trophy. Make any day Oktoberfest whether with this impressive stein on the shelf or in-hand. Cheers!

- Measures 6.75" tall, 3.75" diameter base, 3" diameter rim
- Dishwasher and microwave safe

COB300.com plans to introduce a new ceramic stein with gold trim

- The fixed cost is \$10,000.
- The variable cost is uniformly distributed between \$3 and \$5 per piece.
- The stein will sell between \$17 and \$20 per piece following the probability distribution in Table 2.
- The demand for the new ceramic stein during the current economic crisis is extremely uncertain. It is also believed that the demand is normally distributed with the mean of 1000 pieces and the standard deviation of 200 pieces.
- If COB300.com overproduces the steins, it is estimated that the inventory holding cost could be \$5 per piece.
- If COB300 has shortages of the steins, it is estimated that the shortage penalty cost could be \$10 per piece.

The management of COB300.com would like to know the mean net profit for the new stein and would also like to know the probability that the project would result in a net profit less than \$1,000 due to the possible weak demand. A spreadsheet is set up for you as in Table 2. You are required to use the information in Table 1 to answer the following questions.

The Total revenue is computed from the pieces sold. The Total cost is the summation of the Fixed cost, Variable cost to make all of the pieces (sold or inventory), Inventory holding cost and Shortage cost. The Net profit is the difference between the Total revenue and the Total cost.

11. (3 points each) For **Trial No. 1** in Table 2, you are to use the random numbers given above each variable to answer the corresponding questions:

a. Show the equation and use it to computer the value of the variable cost? Put the value in C14.

b. Show the Excel@ formula to be used in Cell C14 to obtain the value of the variable cost?

12. (3 points each) For **Trial No. 2** in Table 2, you are to use the values of given variable cost, sell price and demand to answer the following questions and put the answers into the blank cells in Table 2.
- Show the equation and the process to obtain the number of steins sold in Cell E15?
 - Show the Excel@ formula in Cell E15 to obtain the number of steins sold?
 - Show the equation and the process to obtain the number of Shortages and the Shortage cost in Cells F15 and G15 respectively?
 - Show the Excel@ formula to obtain the number of shortages in Cell F15?
 - Show the equation and the process to obtain the leftover inventory and the inventory cost in Cells H15 and I15 respectively?
 - Show the Excel@ formula to obtain the leftover inventory in Cell H15?

13. (3 points each) For **Trial No. 3** in Table 2, you are to use the values of given variable cost, sell price and demand to answer the following questions. Put the correct answers into blank cells in Table 2.
- Show the equation and the process to obtain the number of steins sold in Cell E16?
 - Show the equation and the process to obtain the number of Shortages and the Shortage cost in Cells F16 and G16 respectively?
 - Show the equation and the process to obtain the leftover inventory and the inventory cost in Cells H16 and I16 respectively?
 - Show the equation and the process for Total revenue, Total cost and Net profit in Cells J16, K16 and L16 respectively?

14. (3 points each) Assume the 1,100 trials starts in the Row 21 and ends in the Row 1120 in Table 3. You could use Trials 1 through 4 in Table 2 to verify your answers. Show the Excel@ formula to be used to obtain the number of net profit that is less than \$1,000 from the last 1,000 trials?

Table 3. COB300.com Simulation Part 2

	A	B	C	D	E	F	G	H	I	J	K	L
19					Pieces	Pieces	Shortage	Pieces in	Inventory	Total		
20	Trial No	Sell price/p	Var Cost/p	Demand	Sold	Shortage	Cost	Inventory	Holding Cost	Revenue	Total cost	Net profit
21	1											
22	2											
23	3											
119	99											
120	100											
121	101											
122	102											
1117	1097											
1118	1098											
1119	1099											
1120	1100											

Table 4. COB300.com Simulation Part 3

	M	N	O	P	Q	R	S
1		<i>Production Quantity</i>		AVG Profit	NP<1000	Standard Deviation	
2			900	895.23	463.88	60.40	
3			1,000	1999.96	280.53	86.08	
4			1,100	2470.18	286.96	101.75	
5			1,200	2344.66	349.24	129.91	
6			1,300	1833.30	417.58	145.37	
7							
8		MonteCarlito	AVG Profit	NP<1000			
9	1	-100	\$ 1,849.43	301			
10	2	Mean	2344.66	349.24			
11	3	Standard error	12.99	1.398			
12	4	Median	\$2,348.00	349			
13	5	Standard dev	129.91	13.98			

Now, you design the simulation with 1,100 trials. You discard the first 100 trials and use only the last 1,000 trials in MonteCarlito replications. As in Table 4 and Figure 1, you obtain the summary of the production quantity and the resulting estimated average net profit, the estimated number of times the net profit is less than \$1,000 and the standard deviation for the mean net profit. Answer the following questions:

- 15.
- a. (2 points) What is the Production quantity that results in the highest average net profit?
 - b. (2 points) What is the estimated average number of times that the net profit is less than \$1,000 with the Production quantity that results in the highest average net profit?
 - c. (6 points) Show the equation and the process to obtain the margin of error, and the 95% confidence interval for the highest mean net profit?
 - d. (3 points) Interpret the meaning of the 95% confidence interval for the highest mean net profit to your manager who does not necessarily know much of statistics?
- 16.
- a. (8 points) Show the equation and the process to obtain the probability, the standard error, the marginal error and the 95% confidence interval for the probability or the proportion of times the net profit less than \$1,000 associated with the highest mean net profit? ($n = 1000$)
 - b. (3 points) Interpret the meaning of the 95% confidence interval for the proportion of net profit less than \$1,000 to your manager who does not necessary know much statistics?