

**DO NOT TURN TO THE NEXT PAGE UNTIL YOU ARE INSTRUCTED TO DO SO!**

The following exam consists of 6 problems for a total of 204 points and will be graded as 200 points. Please keep three decimal points in all of your computations. To get credits, you are required to provide computational details such as ***providing equations, models, procedures, Excel@ formulas and explanations*** for related questions. You will have 75 minutes to complete the test. This means that you have, on average, about 2 minutes for each question.

1. Write down (PRINT) your name in the blank provided on the top right corner on this page.
2. This test is closed book and closed notes. You may bring ONE 8" x 11" note card or equation sheet (One side only). You can write or type anything on the card or sheet. You have to hand it in along with your test paper.
3. Partial credits are available and unsupported answers will not receive any credit even though they may be correct.
4. You are expected to remain in the classroom until you have turned in your test paper. You could not get your test paper back once you turn it in.
5. You must have your own calculator for the test. You cannot use pocket PC, cell phone, PDA, or anything like these for your test. You cannot share calculator with anyone.
6. If you believe that someone sitting near you is cheating, raise your hand and quietly inform a teacher of this. We'll keep an eye peeled, and your anonymity will be respected.
7. If any question seems unclear or ambiguous to you, raise your hand, and your teacher will attempt to clarify it. Or just put it on your test booklet, I will look at it when I grade the test.

Pledge: On my honor as a JMU student, I pledge that I have neither given nor received unauthorized assistance on this examination.

Signature \_\_\_\_\_

1. (42 points) In order to Minimize the total production cost, you decide to use the following linear programming model to determine how many products of X and Y to make:

$$M: 5X + 5Y$$

Subject to:

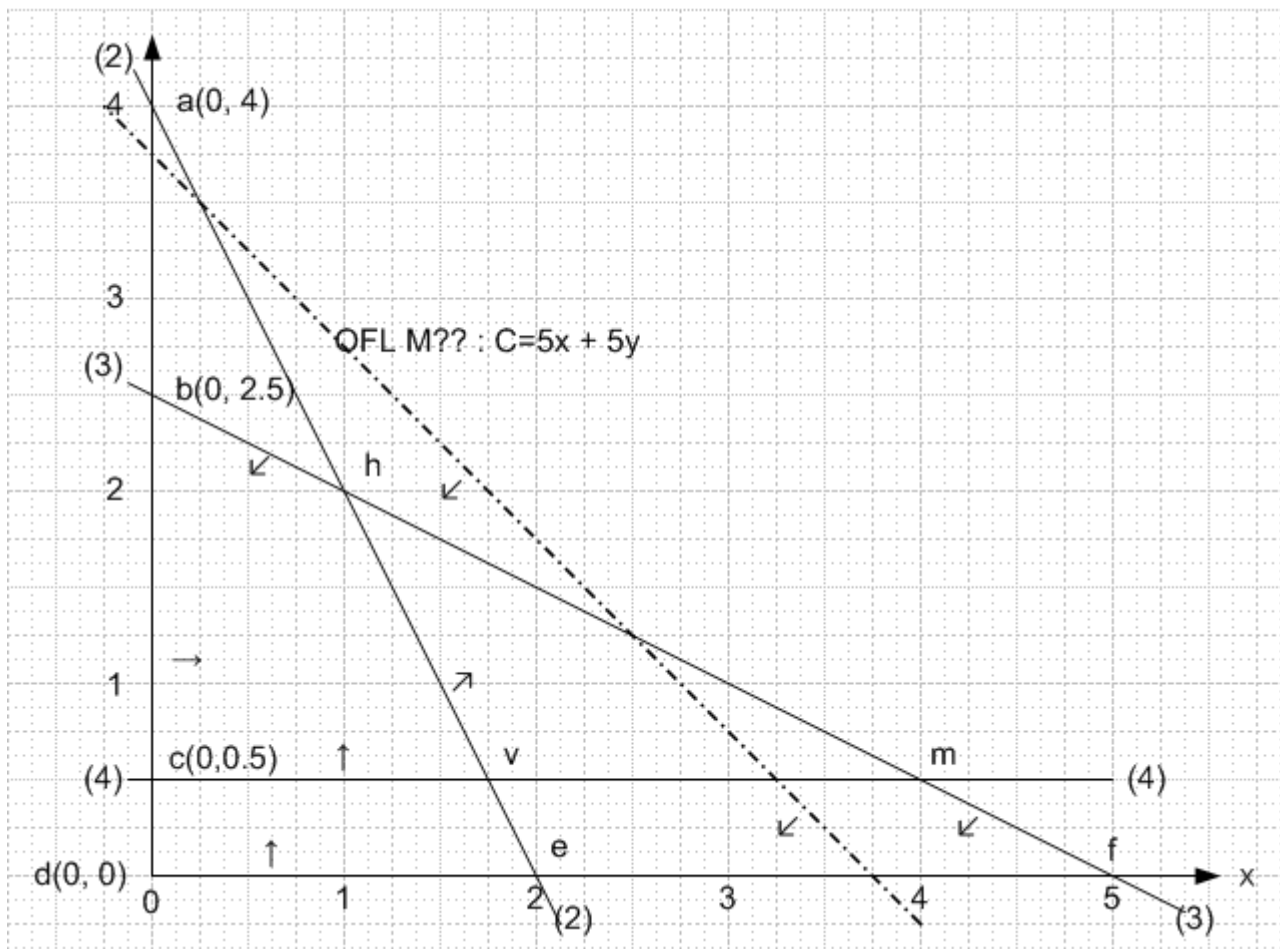
- $2X + 1Y \geq 4$  (2) (Customer A Demand)
- $1Y \geq 0.5$  (4) (Product Y Requirement)
- $X, Y \geq 0$  (5) (Non negativity for Products x and y)

- a. (2 pts) What is the objective of the LP model based on the arrow in the graph?
  - i. Maximization
  - ii. Minimization
  - iii. Can't determine
  
- b. (2 pts) Shade the feasible region of the LP model on the graph.
  
- c. (2 pts) Select an optimal point, if one exists. If one does not exist, indicate why not on your graph. If there are alternative optimal solutions, indicate these points on your graph.  
 Point a,      Point b,      Point h,      Point v,      Point e,      Others\_\_\_\_\_
  
- d. (2 pts) What is the optimal solution if one exists? If it does not exist, why not?
  
  
- e. (4 pts) Solve this LP model graphically using the method from class. Be sure to show and label the optimal objective line and the values of the optimal solution, if it exists.
  
  
- f. (2 pts) What is the optimal objective function value, if it exists?
  
  
- g. (6 pts) Now the constraint 3 (the number of hours available) is added to the problem as indicated with the line (3) on the graph, what is the equation for the constraint 3?
  
  
  
  
- h. (2 pts) Which of the following signs should be used in the equation 3?  
 <      <=      =      >      >=      <>      Others\_\_\_\_\_

i. (6 pts) Show the details to find the intersect h of Equations (2) and (3).

j. (2 pts) Shade the new feasible region if it is different than the one before.

k. (6 pts) Now a constraint (1) with the equation  $x - y \leq 2$  is added to the problem. Draw the constraint line for the equation 1 for this LP model on the graph and shade the new feasible region if it is different than the one without it.



l. (2 pts) Is there any redundant constraint in this LP model? If yes, which one it is? If no, why not?

m. (4 pts) Compute and interpret the slack amount in the constraint (3) in terms understandable to the management who is not familiar with linear programming models.

2. (2 pts) Can a linear programming model have both alternative optimal solutions and be unbounded.

Circle either TRUE or FALSE

3. (45 points) Coors Brewing Company has received an order for 1500 gallons of 3 percent beer (that is, 3 percent alcoholic content). This is a custom order because Coors does not produce a 3 percent product. They do brew the following products.

Product	Percent Alcohol	Cost per Gallon
Light	0.042	0.55
Banguet	0.05	0.65
Blue Moon	0.054	0.8
Keystone Ice	0.059	0.75
Water	0	0.01

There are 500 gallons of each of these products on hand. Rather than brewing a 3 percent beer from scratch, the brewmaster has decided to mix existing stocks, perhaps with some water (0 percent alcoholic content), to satisfy this small order in the shortest possible time, hoping that the taste will be adequate. In case the taste is had and he has to throw the mixture out, he would like to minimize the cost of the mix.

- a. (10 pts) Write the objective function with its goal to minimize the total cost of the mix?
- b. (23 pts) Identify the constraints by name and write out each of the constraints:
- c. (12 pts) Suppose that in addition to the requirements given above, the brewmaster would like to limit the alcoholic content from Keystone Ice to no more than 50 percent of the total alcoholic content. Write out the constraint.

4. (40 points) As part of the settlement for a class action lawsuit, Enron must provide sufficient cash to make the following annual payments (in thousands of dollars).

Year	1	2	3	4
Payment	190	215	240	285

The annual payments must be made at the end of each year. The judge will approve an amount that, along with earnings on its investment, will cover the annual payments. Investment of the funds will be limited to money market savings (at 4% annually) and government securities A and B, at rates currently quoted in *The Wall Street Journal*. Enron wants to develop a plan to minimize its initial investments in order to make the annual payments by investing in the following securities (par value = \$1000). Funds not invested in these securities will be placed in savings.

Security	Rate (%)	Years to Maturity	Risk Ratings
S	4.0	1	2
A	6.75	3	5
B	5.125	4	7

The plan will be submitted to the judge and, if approved, Enron will be required to pay a trustee the amount that will be required to fund the plan.

- a. (10 pts) Define decision variables clearly:
  
- b. (5 pts) Write the objective function with the goal:
  
- c. (20 pts) Write the constraints:
  
- d. (5 pts) Suppose the investments available to Enron have the levels of risk associated with them as given in the table: If the judge wants the weighted average risk level of the investments in the year 2 (from the beginning to the end of the year 2) to not exceed 4, write an additional constraint for the problem.

5. (40 points) A winery has the following capacity to produce an exclusive dinner wine at either of its two vineyards at the indicated costs. Four Italian restaurants around the country are interested in purchasing this wine.

Vineyard	Capacity (bottles)	Cost per Bottle	Shipping Costs per Bottle Restaurant				
			Vineyard	1	2	3	4
1	3,500	\$23	1	\$7	\$8	\$13	\$9
2	3,100	\$25	2	\$12	\$6	\$8	\$7
Price per Bottle				\$69	\$67	\$70	\$66
Demand (Bottles)				1,800	2,300	1,250	1,750

Because the wine is exclusive, they all want to buy as much as they need but will take whatever they can get. The maximum amounts required by the restaurants and the prices they are willing to pay are summarized in the table. The costs of shipping a bottle from the vineyards to the restaurants are summarized in the table. The winery needs to determine the production and shipping plan that allows it to maximize its profits on this wine.

The LP Model and Optimal Solution are given here. A detailed LP model is on the next page.

	A	B	C	D	E	F	G	H
1			<b>Winery</b>					
2			<b>Shipping Costs per Bottle Restaurant</b>					
3								
4								<b>Production</b>
5		<b>Vineyard</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>		<b>Cost</b>
6		1	\$7	\$8	\$13	\$9		\$23
7		2	\$12	\$6	\$8	\$7		\$25
8		<b>Price per Bottle</b>	\$69	\$67	\$70	\$66		
9								
10								
11								
12								
13		<b>Vineyard</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>		
14		1	\$39	\$36	\$34	\$34		
15		2	\$32	\$36	\$37	\$34	<b>Total Profit</b>	<b>\$241,750</b>
16								
17								
18								
19								
20								
21								
22								
23								
24								
25								
26								
27								
28								
29								

Maximize: H14  
 By Changing: C19:F20  
 Subject to: G19:G20<=H19:H20  
 C21:F21<=C22:F22  
 C19:F20>=0  
 (Alternate optima exist!)

The decision variables are defined as:  $V_{iRj}$  for  $i = 1$  and  $2$  and  $j = 1$  to  $4$  represents the number of bottles to ship from vineyard  $i$  to the restaurant  $j$ .

The profit per bottle equals to the price per bottle minus the production and shipping costs per bottle, i.e. the profit for a bottle of wine shipped from the vineyard 1 to the restaurant 1 is \$69 - \$23 - \$7 = \$39. The objective function is to maximize the total profit:

MAX: 39 V1R1 + 36 V1R2 + 34 V1R3 + 34 V1R4 + 32 V2R1 + 36 V2R2 + 37 V2R3 + 34 V2R4 (in H4\$14)  
 Subject to:

- Vineyard 1 Capacity: V1R1 + V1R2 + V1R3 +V1R4 = 3500 bottles (in \$H\$19)
- Vineyard 2 Capacity: V2R1 + V2R2 + V2R3 +V2R4 = 3100 bottles (in \$H\$20)
- Restaurant 1 Demand: V1R1 + V2R1 <= 1800 bottles (in \$C\$22)
- Restaurant 2 Demand: V1R2 + V2R2 <= 2300 bottles (in \$D\$22)
- Restaurant 3 Demand: V1R3 + V2R3 <= 1250 bottles (in \$E\$22)
- Restaurant 4 Demand: V1R4 + V2R4 <= 1750 bottles (in \$F\$22)
- Non Negativity: All of ViRj >= 0 for i=1 to 2 and j=1 to 4.

The Answer Report and the Sensitivity Report are shown as follows:

	A	B	C	D	E	F	G
12	Target Cell (Max)						
13		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
14	\$H\$14	Total Profit		0	241750		
15							
16							
17	Adjustable Cells						
18		<b>Cell</b>	<b>Name</b>	<b>Original Value</b>	<b>Final Value</b>		
19	\$C\$19	Restaurant		0	1,800		
20	\$D\$19	Profit per Bottle		0	1,700		
21	\$E\$19			0	0		
22	\$F\$19			0	0		
23	\$C\$20	Restaurant		0	0		
24	\$D\$20	Profit per Bottle		0	600		
25	\$E\$20			0	1,250		
26	\$F\$20			0	1,250		
27							
28	Constraints						
29		<b>Cell</b>	<b>Name</b>	<b>Cell Value</b>	<b>Formula</b>	<b>Status</b>	<b>Slack</b>
30	\$C\$21	Received Restaurant		1,800	\$C\$21<=\$C\$22	Binding	0
31	\$D\$21	Received Profit per Bottle		2,300	\$D\$21<=\$D\$22	Binding	0
32	\$E\$21	Received		1,250	\$E\$21<=\$E\$22	Binding	0
33	\$F\$21	Received		1,250	\$F\$21<=\$F\$22	Not Binding	500
34	\$G\$19	Produce		3,500	\$G\$19<=\$H\$19	Binding	0
35	\$G\$20	Produce		3,100	\$G\$20<=\$H\$20	Binding	0
36	\$C\$19	Restaurant		1,800	\$C\$19>=0	Not Binding	1800
37	\$D\$19	Profit per Bottle		1,700	\$D\$19>=0	Not Binding	1700
38	\$E\$19			0	\$E\$19>=0	Binding	0
39	\$F\$19			0	\$F\$19>=0	Binding	0
40	\$C\$20	Restaurant		0	\$C\$20>=0	Binding	0
41	\$D\$20	Profit per Bottle		600	\$D\$20>=0	Not Binding	600
42	\$E\$20			1,250	\$E\$20>=0	Not Binding	1250
43	\$F\$20			1,250	\$F\$20>=0	Not Binding	1250

	A	B	C	D	E	F	G	H
1	<b>Microsoft Excel 12.0 Sensitivity Report</b>							
2	<b>Worksheet: [Prb03_32.xls]Sales Plan</b>							
3	<b>Report Created: 04/19/2009 10:53:09 AM</b>							
4								
5	Target Cell (Max)							
6	<b>Cell</b>		<b>Name</b>	<b>Final Value</b>				
7	\$H\$14		Total Profit	241750				
8								
9	Adjustable Cells							
10	<b>Cell</b>		<b>Name</b>	<b>Final Value</b>	<b>Reduced Cost</b>	<b>Objective Coefficient</b>	<b>Allowable Increase</b>	<b>Allowable Decrease</b>
11	\$C\$19		Restaurant	1,800	0	39	1E+30	5
12	\$D\$19		Profit per Bottle	1,700	0	36	5	0
13	\$E\$19			0	-3	34	3	1E+30
14	\$F\$19			0	0	34	0	1E+30
15	\$C\$20		Restaurant	0		32	7	1E+30
16	\$D\$20		Profit per Bottle	600	0	36	0	2
17	\$E\$20			1,250	0	37	1E+30	3
18	\$F\$20			1,250	0	34	2	0
19								
20								
21	Constraints							
22	<b>Cell</b>		<b>Name</b>	<b>Final Value</b>	<b>Shadow Price</b>	<b>Constraint R.H. Side</b>	<b>Allowable Increase</b>	<b>Allowable Decrease</b>
23	\$C\$21		Received Restaurant	1,800	5	1800	1250	500
24	\$D\$21		Received Profit per Bottle	2,300	2	2300	1250	500
25	\$E\$21		Received	1,250	3	1250	1250	500
26	\$F\$21		Received	1,250	0	1750	1E+30	500
27	\$G\$19		Produce	3,500	34	3500	500	1250
28	\$G\$20		Produce	3,100	34	3100	500	1250
29								

- a. (5 pts) The profit margin (\$D\$13) for a bottle from Vineyard 1 to Restaurant 2 (\$D\$19) is  $\$67 - \$23 - \$8 = \$36$ . Suppose the profit margin increases to  $\$38$  instead of  $\$36$  per bottle. If possible, determine the new total profit. If not, explain why not.

Type of ranging (Lower & Upper Limits): \_\_\_\_\_ Range: [ \_\_\_\_\_ ]

Provide detailed computations for the range here:

- b. (5 pts) The profit margin (\$E\$14) for a bottle from Vineyard 2 to Restaurant 3 (\$E\$20) is  $\$70 - \$25 - \$8 = \$37$ . Suppose the profit margin decreases to  $\$32$ , instead of  $\$37$  per bottle. If possible, determine the new total profit. If not, explain why not.

Type of ranging (Lower & Upper Limits): \_\_\_\_\_ Range: [ \_\_\_\_\_ ]

Provide detailed computations for the range here:

- c. (5 pts) Suppose Restaurant 2 decreases its demand ( $D_1$  or  $D_2$ ) from 2300 bottles to 2000 bottles. How this change will affect the current optimal solution? If possible, determine the new optimal net profit. If not, explain why not.

Type of ranging (Lower & Upper Limits): \_\_\_\_\_ Range: [ \_\_\_\_\_ ]

Provide detailed computations for the range here:

- d. (5 pts) Suppose Vineyard 2 increases its capacity ( $G_2$  or  $H_2$ ) from 3100 bottles to 4000 bottles. If possible, determine the new optimal net profit. If not, explain why not.

Type of ranging (Lower & Upper Limits): \_\_\_\_\_ Range: [ \_\_\_\_\_ ]

Provide detailed computations for the range here and explain your result:

- e. (10 pts) As shown in the Sensitivity Report, the reduced cost for Vineyard 1 to Restaurant 3 (V1R3 in  $E_3$ ) is -3, interpret its meaning within the context to the management who is not familiar with LP modeling.

- f. (5 pts) As shown in the Sensitivity Report, the reduced cost for Vineyard 2 to Restaurant 1 (V2R1 or  $C_1$ ) is blanked out, compute the missing reduced cost and show the details.

- g. (5 pts) As shown in the Sensitivity Report, the shadow price for the demand from Restaurant 4 ( $F_1$  or  $F_2$ ) is 0 (zero), interpret its meaning within the context to the management who is not familiar with LP modeling.

6. (35 points) Valley Electronics manufactures two products that can be produced on either of two different production lines. Both products have their lowest production costs when produced on the more modern of the two production lines. However, the modern production line does not have the capacity to handle the total production. As a result, some production will have to be routed to the older production line. The following data show total production requirements, production line capacities, and production costs.

Product	Modern Line	Old Line	Minimum Production Requirements
1	3	5	500 Units
2	2.5	4	700 Units
Production line capacity	800 Units	600 Units	

(10 pts) Define the decision variables clearly:

(5 pts) State the objective function with its goal:

(20 pts) Write out constraints with their names: